Poisson Geometry, Lie Groupoids and Differentiable Stacks

Henrique Bursztyn (Instituto de Matemática Pura e Aplicada)
Rui Loja Fernandes (University of Illinois Urbana–Champaign)
Brent Pym (McGill University)
Jiang-Hua Lu (University of Hong Kong)

June 5–10, 2022

1 Overview of the Field

Poisson Geometry is an amalgam of three classical theories: it is *Foliation Theory* inside which *Symplectic Geometry* and *Lie Theory* interact with each other. Geometrically, a Poisson structure on a space M is, first of all, a (possibly singular) foliation of M; hence M is partitioned into leaves. Secondly, the leaves are endowed with symplectic structures. Thirdly, transversal to the leaves, we have Lie groups/algebras. Often the space M has some additional structure which leads to further connections. For example, M could be an algebraic variety in which case Poisson brackets on M are often intimately related to *Representation Theory* and *Noncommutative Geometry*. Altogether, one of the strengths of Poisson Geometry is its potential to provide often unexpected interplays between diverse fields.

Groups typically arise as the symmetries of some given object. The concept of a groupoid allows for more general symmetries, acting on a collection of objects rather than just a single one. Groupoid elements may be pictured as arrows from a source object to a target object, and two such arrows can be composed if and only if the second arrow starts where the first arrow ends. Just as Lie groups (as introduced by Lie around 1900) describe smooth symmetries of an object, Lie groupoids (as introduced by Ehresmann in the late 1950's) describe smooth symmetries of a smooth family of objects. That is, the collection of arrows is a manifold G, the set of objects is a manifold M, and all the structure maps of the groupoid are smooth. Ehresmann's original work was motivated by applications to differential equations, but since then Lie groupoids have appeared in many other branches of mathematics and physics, such as Algebraic Geometry (Grothendieck), Foliation Theory (Haefliger), Noncommutative Geometry and Index Theory (Connes-Skandalis). Nowadays, one can find many other applications of Lie groupoids, such as in geometric mechanics, equivariant differential geometry, higher gauge theory, orbifold theory, exterior differential systems, Ricci flows, and generalized complex geometry.

Motivated by quantization problems, Karasev and Weinstein introduced the symplectic groupoid of a Poisson manifold in the late 1980's, as a way to "untwist" the complicated behavior of the symplectic foliation underlying the Poisson manifold. Moreover, the infinitesimal symmetries corresponding to Lie groupoids are described by Lie algebroids, and at the same time it was realized that Lie algebroids can be characterized as vector bundles with fiberwise linear Poisson structures. Once these connections between Lie groupoid theory and Poisson geometry were established, the two fields exploded and became inseparable.

Recent days have also seen rapid developments on shifted Poisson and symplectic structures on (derived) differentiable or algebraic stacks. A differentiable stack is, roughly speaking, a Lie groupoid up to Morita equivalence, and the stack represented by a symplectic groupoid of a Poisson manifold naturally has a 1-shifted symplectic structure. There have also been remarkable recent advances in other geometries, such as Dirac geometry and generalized complex geometry, that generalize Poisson geometry and have Lie groupoids and Lie algebroids at their cores. Many basic concepts and constructions in these geometries can be rephrased using the language of differential stacks, and such reformulations put these geometric structures in vastly new perspectives and establish further connections with other fields of mathematics such as algebraic geometry, deformation theory and high category theory.

2 Recent Developments and Open Problems

This BIRS workshop was centred around Poisson geometry, Lie Groupoids and differentiable stacks. In spite of all the progresses made so far and some amazing recent advances, deep and rich interconnections between these areas remain to be discovered and the workshop brought together different groups of people and diverse viewpoints. Among the major open problems that remain, where Lie groupoids should play a fundamental role, the workshop addressed the following specific topics:

- **Integrations of Poisson and Dirac structures**: Originally conceived as a framework for Dirac's theory of second class constraints in geometric mechanics, Dirac geometry has emerged as a flexible generalization of Poisson geometry with far-reaching applications [11, 15, 32, 39, 49]. Generalizing the fact that symplectic groupoids are integrations of Poisson manifolds, the global objects integrating Dirac manifolds are presymplectic groupoids [16]. While a fundamental result established in [25, 26] describes the precise obstruction for a Lie algebroid to have an integration into a Lie groupoid, explicit integrations of given Poisson or Dirac structures into symplectic and presymplectic groupoids remain interesting and desirable, as such integrations have applications in the problems of normal forms and linearizations around their leaves [1, 24, 27, 28, 29], in quantization [8, 43], as well as in applications to topological field theory through Poisson sigma models [20]. Explicit integrations for a large class of Poisson and Dirac structures originated from the theory of quantum groups have recently been given in [18, 59].

- **Generalized complex geometry and mirror symmetry**: Mirror symmetry suggests deep relations between complex manifolds with their symplectic "mirror" manifolds. Generalized complex structures treat symplectic and complex structures on equal footing [39, 45], as suggested by mirror symmetry and other physical dualities. They have a corresponding global object given by Lie groupoids with a multiplicative structure consisting of a symplectic form and a complex structure, satisfying certain compatibility relations [23], but this is not the full picture. The complete description of global counterparts of generalized complex structures is more intricate and has been only recently obtained in [5]. There also have been proposals to explain the origin of a monoidal structure on the Fukaya category via symplectic groupoids [62].

- **Multiplicative structures on Lie groupoids and stacks**: generalizing the previous cases, the study of multiplicative structures (differential forms, multivector fields, connections, and so on) and their infinitesimal versions [14, 16, 46, 43] has provided new insights into both classical problems of differential geometry and the geometry of moduli spaces (stacks). This study includes for example Poisson group(oid)s [54], closely connected with integrable systems and quantum groups, holomorphic structures on Lie groupoids [47], and Riemannian structures on stacks [36, 37]. Additionally, one can recast Cartan's work on Lie pseudogroups in the language of multiplicative forms on Lie groupoids, showing that the classical Spencer operator appears as the linearization data of the Cartan Pfaffian system [30].

- **Shifted symplectic geometry**: Shifted symplectic geometry is the study of symplectic structures on spaces known as derived stacks, which are generalizations of smooth manifolds and algebraic varieties. A novel aspect of symplectic structures on stacks is that they have an integer grading: usual symplectic structures are simply 0-shifted in this grading, but alternative degrees -1, 1, and 2 have been much studied over the past 10 years since the foundational work of Pantev, Toen, Vaquie, and Vezzosi [61]. They are very closely related to and tie together all the subjects listed in this proposal. For example, the symplectic form on the space G of arrows of a symplectic groupoid over the Poisson manifold M gives rise to a 1-shifted symplectic structure on the stack M/G (see [34] for a more general picture). Similarly, by work of Pym and Safronov, Dirac structures may be interpreted as 2-Lagrangians in a 2-shifted symplectic manifold. Finally, recent work [5] of Bailey and Gualtieri shows that generalized complex structures may be interpreted as holomorphic 1-shifted symplectic stacks. Perhaps most importantly, -1-shifted symplectic structures are at the heart of what is called the Batalin-Vilkovisky formalism for classical and quantum field theory in physics; much of the current research in the subject aims to make use of this relationship to make progress on current problems in geometry and physics, such as the problem of deformation quantization (after Kontsevich) of derived stacks which was solved in [19] for the case of nonzero shifts but remains open for shift zero. Another important open question is to develop more explicit forms of results such as the -1-shifted Darboux theorem of [12, 13]. This result asserts the existence of local algebraic models for the Chern-Simons potential, which are in turn used to construct various refined Donaldson-Thomas/3-manifold invariants, but are currently very difficult to calculate.

- **Higher Lie groupoids and higher gauge theory**: Higher gauge theory, as developed by Baez and coauthors (see e.g. [4]), is an extension of gauge theory that describes parallel transport not only for point particles but also for higher-dimensional objects; in particular, it treats horizontal lifts of surfaces, rather than just paths. From a physics perspective, it is motivated by string theory, but has also been applied to other fields, such as loop quantum gravity. Just as ordinary gauge theory concerns fiber bundles with structure Lie groups, higher gauge theory deals with bundles with gauge structure given by higher, or categorified, versions of Lie groups and groupoids. Particular types of higher groupoids, known as *double Lie groupoids*, arise naturally in Poisson geometry: for example, the integration of Poisson Lie group(oid)s often leads to double symplectic groupoids [52]. More general Lie 2-groupoids (or stacky groupoids) [66] can be regarded as groupoid structures on differentiable stacks, i.e., as models for groupoid structures on singular quotients. The general treatment of higher groupoids usually involves simplicial and homotopical methods, which is linked to the fact that, from a Lie theoretic perspective, higher Lie groupoids are thought of as global versions of L_∞-algebroids, though a precise connection remains elusive.

3 Presentation Highlights

3.1 Paired lectures

The morning sessions on Monday–Thursday consisted of "paired lectures", in which two researchers were invited to coordinate lectures related to specific topics where recent advances have suggested significant opportunities for future developments.

Ana Bălibanu and **Ioan Mărcuţ** delivered a pair of lectures on the problem of desingularizing the symplectic foliation of a Poisson manifold to make it regular (i.e. such that all symplectic leaves have the same dimension). The goal, given a Poisson manifold M, is to find a Poisson manifold $\tilde{M}$ whose leaves are equidimensional, and a proper Poisson map $\tilde{M} \to M$ that is an isomoprhism over the locus where M is regular. Mărcuţ explained how, in the context of Poisson manifolds of compact type(s) [27, 28], a sequence of blowups along closed submanifolds can be used to desingularize the foliation; in the case where M is the dual of a Lie algebra of compact type, this yields a desingularization of the coadjoint orbits. Bălibanu explained the complex algebraic counterpart of this construction (the Grothendieck–Springer alteration for complex semisimple Lie algebras, which is generically a covering rather than an isomorphism) and gave an overview of the basic theory of symplectic singularities and their versal deformations, following Namikawa [57].

Andrew Harder and **Mykola Matviichuk** spoke about holomorphic log symplectic manifolds: these are holomorphic Poisson manifolds that have an open dense symplectic leaf whose symplectic form has logarithmic poles on the boundary. Harder explained his work [42] concerning the properties of the mixed Hodge structure on the cohomology ring of the open leaf, and its role in the study of semi-stable degenerations of compact hyprekähler manifolds, where log symplectic structures naturally appear on the irreducible components of the singular fibre. Matviichuk discussed his recent work [56] with Pym and Schedler on local normal forms and deformations of log symplectic structures, giving a conjectural condition for them to be "holonomic" (meaning that the Poisson cohomology sheaves governing the deformation are locally finite-dimensional), and sketching a proof that this property holds for Hilbert schemes of log Calabi–Yau surfaces, based on a novel construction of the corresponding symplectic groupoid.

Francis Bischoff and **Charlotte Kirchhoff-Lukat** spoke about applications of Fukaya categories to problems in generalized complex and Poisson geometry. Bischoff explained his recent work [7] with Gualtieri, giving a general proposal for quantizing holomorphic Poisson manifolds using the generalized Kähler metrics and the Fukaya category of the symplectic groupoid and fully realizing it in the case of Poisson structures generated by torus actions. Kirchhoff-Lukat explained her work in progress on the Lagrangian Floer theory of two-dimensional real log symplectic manifolds (known as Radko surfaces), in which one has to modify the usual construction of Floer homology by allowing disks that intersect the boundary of the symplectic leaves in a controlled fashion.

Miquel Cueca and **Chris Rogers** spoke about various aspects of higher Lie theory. Cueca gave physical motivations and explained different approaches to describe higher Lie groupoids and their infinitesimal counterparts by means of simplicial methods and graded geometry, with concrete focus on the case of higher cotangent bundles [33]. Rogers explained joint work in progress with Jesse Wolfson concerning a homotopy-theoretic toolkit for constructing explicit integrations and differentiations in higher Lie theory, enjoying good geometric properties. Their work improves the earlier results of E. Getzler's [35] and A. Henriques' [44], proving that every finite-type Lie n-algebra integrates to a finite dimensional Lie n-group. More important, they propose an inverse to this construction, which was missing in those earlier works. The construction of the inverse builds upon the work of A. Beilinson on Chern-Weil theory, and the work of J. Pridham on the cosimplicial Dold-Kan correspondence.

3.2 Research talks

The workshop also included several sessions of research talks, covering a wide variety of topics related to Poisson geometry and stacks, and grouped loosely by theme.

On Monday afternoon, the focus was on representations and cohomology for Poisson structures and Lie algebroids. **Maria Amelia Salazar** discussed a definition of relative cohomology for a Lie subalgebroid, and its application to the construction of characteristic classes of representations. **Florian Zeiser** explained his calculation, joint with Hoekstra and Mărcuţ, of the Poisson cohomology of all 3-dimensional Lie algebras. **Linhui Shen** described his work with Casals, Gorsky, Gorsky, Le and Simental, in which they construct cluster structures on braid varieties of complex simple groups of ADE (confirming a conjecutre of Leclerc) and use them to construct and quantize Poisson structures on these varieties.

Tuesday afternoon concerned the application of Lie algebroids to the study of Poisson and generalized complex structures. **Aldo Witte** spoke about his joint work with Cavalcanti and Klaasse on so-called "elliptic symplectic structures" (related to the log symplectic structures from Harder and Matviichuk's morning session above). In paricular he explained a connected sum procedure that enables to construction of many examples of elliptic symplectic structures on non-complex manifolds. **Marco Gualtieri** and **Yucong Jiang** gave a pair of talks on the theory of generalized Kähler (GK) manifolds, giving a description of the latter in terms of holomorphic Manin triples, extending earlier work Bischoff–Gualtieri–Zabzine to cover arbitrary GK manifolds.

On Thursday afternoon, the focus was on (higher) categorical structures. **Daniel Alvarez** described joint work with Bursztyn and Cueca in which they apply Pantev–Toën–Vaquié–Vezzosi's theory of shifted symplectic structure to elucidate the problem of groupoid integrations of various Poisson-like structures, such as Poisson homogeneous spaces (building on earlier work to Bursztyn–Iglesias–Lu) and quasi-Poisson manifolds. **Frank Neumann** discussed his work with Szymik concerning the characteristic map on the Hochshild cohomology of differential graded categories, interpreting it as an edge map in a spectral sequence and providing concrete examples illustrating various (in)finite dimenisionality phenomena. **Cristian Ortiz** explained a version of Morse–Bott theory for groupoids, which yields a Morse-style complex that computes the cohomology of the associated quotient stack.

Finally, Friday morning feaatured talks about geometric structures on Lie algebroids and Lie groupoids. **Clarice Netto** decribed a notion of Courant–Nijenhuis algebroids and outlined several examples related to Kähler geometry and Poisson–Nijenhuis structures. **Joel Villatoro** discussed an extension of the theory of Lie groupoids and Lie algebroids to the category of diffeological spaces, which enables integration of Lie algebroids to groupoids in diffeological spaces even when a groupoid in manifolds cannot be found.

The conference closed with a talk by **Reyer Sjamaar**, who explained joint work with Lin, Loizides and Song that generalizes the index-theoretic “quantization commutes with reduction” theorem to the context of transversaly symplectic Riemannian foliations.

4 Scientific Progress Made

In order to stimulate progress, the schedule included afternoon discussion sessions led by the “paired speakers” from the morning sessions. These discussions produced some of the most directly visible scientific progress at the meeting, which we now summarize.

Desingularization: In the first discussion session, Bălibanu and Mărcuţ set forth a number of directions for future investigation, such as possible generalizations of the Grothendieck–Springer alteration based on symplectic groupoids, extending the blowup construction for Poisson submanifolds to the context of Dirac structure, and generalizations of the notion of symplectic singularity in which we ask that the resolution has a Poisson structure instead of (pre)symplectic structure. The discussion led to refinements of several of these questions and ideas to address them; for instance, Matviichuk immediately produced nontrivial examples of non-symplectic Poisson resolutions from elliptic curves.

Log symplectic manifolds: Harder and Matviichuk higlighted a number of open problems, including the problem of constructing/reversing toric/semi-stable degenerations of (log) symplectic varieties to obtain new examples, and ways to relax the smoothness hypothesis often imposed in log symplectic geometry, e.g. allowing singularities of the manifold itself (a logarithmic generalization of symplectic singularities) and allowing the boundary to by divisorial log terminal instead of normal crossings. By clarifying the key features of these problems, the discussion opened some exciting new avenues for interactions between Poisson geometry, birational geometry, and hyperhähler geometry.

Fukaya categories: Bischoff and Kirchhoff-Lukat led a discussion outline open problems around the appearance of Fukaya categories in Poisson geometry. A particularly active discussion centred around an idea of Kirchoff-Lukat to relate her logarithmic Fukaya category of surfaces to the ordinary Fukaya category of the symplectic groupoid; this led to some progress in understanding the exact mechanisms of such a correspondence, which is likely to shed significant light on the connection between Fukaya categories and quantization.

Higher Lie theory: In the final discussion session, Cueca discussed the problem of defining cotangent bundles of Lie 2-groupoids. These objects must be VB Lie 2-groupoids that carry a 2-shifted symplectic structure, so that the corresponding Lie 2-algebroid can be obtained from a particular 2-shifted lagrangian. These cotangent bundles are also related to coadjoint orbits of Lie 2-algebras, toric symplectic groupoids, and higher Hamiltonian actions. Other topics of discussion were the monoidal properties of the Dold-Kan functor and the internal Hom in the category of higher VB-groupoids. Meanwhile, Rogers gave more details on the integration procedure for Lie n-algebras explained in his talk, discussing the existence of the Lie group cover that allows the integration and explaining conceretely how it works for Lie 2-algebras, by integrating abelian pieces and cocycles. Roger also explained that the integration functor respects the structure of ICFO (Incomplete Category of Fibrant Objects) and the sense in which this is the adjoint to the differentiation defined by Pridham. There were particularly active discussions centred around the relationship with the Van Est map and possible extensions to Lie n-algebroids.

5 Outcome of the Meeting

For several participants, this workshop was the first in-person scientific meeting since the beginning of the COVID-19 pandemic, and consequently there were many opportunities for new discussions and collaborations that had been sorely lacking in recent years. We thank BIRS for this opportunity, and for the chance to host the meeting at an increased capacity, which made it possible for a much larger number of junior researchers to attend and network in a way that had not been possible remotely. We are also grateful for the flexibility of the staff at BIRS, who gracefully accommodated last-minute changes to the in-person attendance list and provided support for talks and discussion sessions that involved remote participants.

References

[1] A. Alekseev and E. Meinrenken, Linearization of Poisson Lie group structures. J. Symplectic Geom. 14 (2016), 227–267.

[2] I. Androulidakis and G. Skandalis, The holonomy groupoid of a singular foliation, J. Reine Angew. Math., 626 (2009), 1–37.

[3] C. Arias Abad and M. Crainic, The Weil algebra and the Van Est isomorphism. Ann. Inst. Fourier (Grenoble) 61 (2011), 927–970.

[4] J. Baez and U. Schreiber, Higher gauge theory. In: *Categories in Algebra, Geometry and Mathematical Physics*, eds. A. Davydov et al, Contemp. Math. **431**, AMS, Providence, Rhode Island, 2007, pp. 7–30

[5] M. Bailey and M. Gualtieri, Integration of generalized complex structures. ArXiv:1611.03850.

[6] K. Behrend and P. Xu, Differentiable stacks and gerbes, J. Symplectic Geom. 9 (2011), 285–341.

[7] F. Bischoff and M. Gualtieri, Brane quantization of toric Poisson varieties, Comm. Math. Phys. 391 (2022), n. 2, 357–400.

[8] F. Bonechi, N. Ciccoli, J. Qiu and M. Tarlini, Quantization of Poisson manifolds from the integrability of the modular function. Comm. Math. Phys. 331 (2014), 851–885.

[9] R.L. Bryant, Bochner-Kaehler Metrics, J. of Amer. Math. Soc. 14 (2001), 623–175.

[10] R.L. Bryant, Notes on Exterior Differential Systems, preprint arXiv:1405.3116.

[11] H. Bursztyn, A brief introduction to Dirac manifolds, Geometric and topological methods for quantum field theory, 4–38, Cambridge Univ. Press, Cambridge, 2013.

[12] O. Ben-Bassat, C. Brav, V. Bussi and D. Joyce, A 'Darboux theorem' for shifted symplectic structures on derived Artin stacks, with applications, Geom. Topol. 19 (2015), no. 3, 1287–1359.

[13] C. Brav, V. Bussi and D. Joyce, A Darboux theorem for derived schemes with shifted symplectic structure, J. Amer. Math. Soc. 32 (2019), no. 2, 399–443.

[14] H. Bursztyn and A. Cabrera, Multiplicative forms at the infinitesimal level, Math. Annalen 353 (2012), 663–705.

[15] H. Bursztyn and M. Crainic, Dirac geometry, quasi-Poisson actions and D/G-valued moment maps, J. Diff. Geom. 82 (2009), 501–566.

[16] H. Bursztyn, M. Crainic, A. Weinstein and C. Zhu, Integration of twisted Dirac brackets, Duke Math. J. 123 (2004), 549–607.

[17] H. Bursztyn and T. Drummond, Lie theory of multiplicative tensors. Math. Annalen 375 (2019), 1489-1554.

[18] H. Bursztyn, D. Iglesias-Ponte and J.-H. Lu, Dirac geometry and integration of Poisson homogeneous spaces, arXiv:1905.11453. To appear in J. Differential Geom.

[19] D. Calaque, T. Pantev, B. Toën, M. Vaquiè and G. Vezzosi, Shifted Poisson structures and deformation quantization, J. Topology, 10 (2017), 483-584.

[20] A.S. Cattaneo and G. Felder,Poisson sigma models and symplectic groupoids Quantization of singular symplectic quotients, Progr. Math., vol. 198, Birkhaeuser, Basel, 2001,pp. 61–93.

[21] A. Connes, Noncommutative Geometry, Academic Press, San Diego, 1994.

[22] M. Crainic, Differentiable and algebroid cohomology, van Est isomorphisms, and characteristic classes, Comment. Math. Helv. 78 (2003), no. 4, 681–721.

[23] M. Crainic, Generalized complex structures and Lie brackets, Bulletin of the Brazilian Mathematical Society, 42 (2011), 559–578.

[24] M. Crainic and R.L. Fernandes, A geometric approach to Conn's linearization theorem. Annals of Math 173 (2011), 1119–1137.

[25] M. Crainic and R.L. Fernandes, Integrability of Lie brackets, Annals of Math. 157 (2003), 575–620.

[26] M. Crainic and R.L. Fernandes, Integrability of Poisson brackets, J. Diff. Geom. 66 (2004), no. 1, 71–137.

[27] M. Crainic, R. L. Fernandes, D. Martinez-Torres, Poisson manifolds of compacts types (PMCT I), J. Reine Angew. Math. 756 (2019), 101–149.

[28] M. Crainic, R. L. Fernandes, D. Martinez-Torres, Regular Poisson manifolds of compacts types (PMCT II), Astérisque, 413 (2019).

[29] M. Crainic and I. Marcut, A normal form theorem around symplectic leaves. J. Diff. Geom. 92 (2012), 417–461.

[30] M. Crainic, A. Salazar and I. Struchiner, Multiplicative forms and Spencer operators. Mathematische Zeitschrift 279 (2015), 939–979.

[31] M. Crainic and O. Yudilevich, Lie Pseudogroups á la Cartan, preprint arXiv:1801.00370.

[32] T. Courant, Dirac structures, Trans. Amer. Math. Soc. 319 (1990), 631–661.

[33] M. Cueca, The geometry of graded cotangent bundles, J. Geom. Phys. 161 (2021), 104055.

[34] M. Cueca and C. Zhu, Shifted symplectic higher Lie groupoids and classifying spaces. Arxiv: 2112.01417.

[35] E. Getzler, Lie theory for nilpotent L_∞-algebras, Ann. of Math. (2) 170, 1 (2009) 271–301.

[36] M. del Hoyo and R.L. Fernandes, Riemannian Metrics on Lie Groupoids. J. Reine Angew. Math., 735 (2018), 143?173.

[37] M. del Hoyo and R.L. Fernandes, Riemannian metrics on differentiable stacks, Mathematische Zeitschrift 292 (2019), Issue 1?2, pp 103?132.

[38] C. Debord and J.-M. Lescure : Index theory and groupoids. In: Geometric and topological methods for quantum field theory, 86-158, Cambridge Univ. Press, Cambridge, 2010

[39] M. Gualtieri, Generalized complex geometry, Annals of Math. 174 (2011), 75–123.

[40] M. Gualtieri, S. Li and B. Pym, The Stokes groupoids. J. Reine Angew. Math. 739 (2018), 81-119

[41] A. Haefliger, Homotopy and integrability, Lecture Notes in Mathematics 197 (1971), 133–163.

[42] A. Harder, Mixed Hodge structures in log symplectic geometry, arXive 2005.11367.

[43] E. Hawkins, A groupoid approach to quantization. J. Symplectic Geom. 6 (2008), 61–125.

[44] A. Henriques, Integrating L_∞-algebras, Compos. Math. 144 (2008), no. 4, 1017–1045.

[45] N. Hitchin, Generalized Calabi-Yau manifolds, Quart. J.Math. 54 (2003), 281-308.

[46] D. Iglesias, C. Laurent-Gengoux and P. Xu, Universal lifting theorem and quasi-Poisson groupoids, J. Eur. Math. Soc. 14 (2012), 681–731.

[47] C. Laurent-Gengoux, M. Stienon, P. Xu, Integration of holomorphic Lie algebroids. Math. Annalen. 345 (2009), 895–923.

[48] J.-M. Lescure, D. Manchon, S. Vassout, About the convolution of distributions on groupoids, J. Noncommut. Geom. 11 (2017), 757–789.

[49] D. Li-Bland, E. Meinrenken, Dirac Lie groups, Asian Journal of Mathematics 18 (2014), 779–816

[50] D. Li-Bland, E. Meinrenken, On the Van Est homomorphism for Lie groupoids. Enseign. Math. 61 (2015), 93–137.

[51] J. Lott, Dimensional reduction and the long-time behavior of Ricci flow, Comment. Math. Helv. 85 (2010), 485–534.

[52] J.-H. Lu and A. Weinstein, Groupoïdes symplectiques doubles des groupes de Lie-Poisson, C. R. Acad. Sci. Paris Ser. I Math. 309 (1989), no. 18, 951–954.

[53] K. Mackenzie, Lie groupoids and Lie algebroids in differential geometry Cambridge Univ. Press, 2005.

[54] K. Mackenzie and P. Xu, Lie bialgebroids and Poisson groupoids. Duke Math. J. 73 (1994), 415-451.

[55] E. Martinez, Lagrangian Mechanics on Lie Algebroids, Acta Appl. Math. 67 (2001), 295–320.

[56] M. Matviichuk, B. Pym and T. Schedler, A local Torelli theorem for log symplectic manifolds, arXive 2010.08692.

[57] Y. Namikawa, Poisson deformations of affine symplectic varieties, Duke Math. J., 156 (2011), n.1, 51–85.

[58] I. Moerdijk. Orbifolds as groupoids: an introduction. In: Orbifolds in mathematics and physics (Madison, WI, 2001), Contemp. Math. 310 (2002), 205–222.

[59] V. Mouquin, Local Poisson groupoids over mixed product Poisson structures and generalised double Bruhat cells, arXiv:1908.04044.

[60] V. Nistor, A. Weinstein, and P. Xu, Pseudodifferential operators on differential groupoids, Pacific Journal of Mathematics 189 (1999), 117–152.

[61] T. Pantev, B. Toën, M. Vaquié and G. Vezzosi, Shifted symplectic structures, Publ. Math. Inst. Hautes E'tudes Sci. 117 (2013) 271–328

[62] J. Pascaleff, Poisson geometry, monoidal Fukaya categories, and commutative Floer cohomology rings. Preprint arXiv:1803.07676.

[63] M. Pflaum, H. Posthuma, X. Tang, The localized longitudinal index theorem for Lie groupoids and the van Est map, Adv. Math. 270 (2015), 223–262.

[64] A. Weinstein, Symplectic groupoids and Poisson manifolds. Bull. Amer. Math. Soc. 16 (1987), no. 1, 101–104.

[65] A. Weinstein, Lagrangian mechanics and groupoids. Mechanics day (Waterloo, ON, 1992), 207-231, Fields Inst. Commun., 7, Amer. Math. Soc., Providence, RI, 1996.

[66] C. Zhu, n-groupoids and stacky Lie groupoids. Int. Math. Res. Not. IMRN 2009, no. 21, 4087–4141.