The Traveling Salesman Problem: Algorithms & Optimization

Joseph Cheriyan, University of Waterloo
Sylvia Boyd, University of Ottawa
Amin Saberi, Stanford University
Ola Svensson, Ecole Polytechnique Federale de Lausanne

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1 Overview of the Field

The Traveling Salesman Problem (TSP) is one of the central and well-known problems in combinatorial optimization. It has been a source of inspiration and intrigue for decades. In the words of Schrijver [8, Ch. 58], "it belongs to the most seductive problems in combinatorial optimization, thanks to a blend of complexity, applicability, and appeal to imagination". Also see Vazirani [12] and Williamson & Shmoys [13]. TSP has several applications in planning, logistics, and manufacturing. Slightly modified, it appears as a sub-problem in many other areas, such as genome sequencing.

The problem has been studied intensely for over 60 years. Almost all of the emerging new ideas and techniques in the areas of algorithms and optimization have been applied to the TSP. In turn, these efforts have given rise to important new sub-disciplines such as survivable network design.

Despite the attention paid to this problem, its tractability from the point of view of approximation remains poorly understood. For example, the best known approximation algorithm for the symmetric case is a 3/2-approximation algorithm due to Christofides from 1976. On the other hand, existing results on its hardness of approximation seem to be loose (i.e., far from tight), and there is a substantial gap between upper bounds and lower bounds.

2 Recent Developments and Open Problems

Over the past decade, deep new ideas have been applied to the TSP and closely related problems, and there have been major recent advances. Many of these advances are based on new and beautiful connections with probability theory, coupled with technically difficult exploitation of methods and structures that are studied in combinatorial optimization.

Oveis Gharan, Saberi, and Singh [7] used properties of strongly Rayleigh measures coupled with an elaborate analysis of the structure of near-minimum cuts to obtain the first improvement on the 3/2-approximation guarantee for a key special case called the graphic TSP. Currently, the best result known on this special case is a 7/5-approximation algorithm of Sebő and Vygen [9] that hinges on a probabilistic lemma of Momke and Svensson [5] coupled with an in-depth and novel analysis of structures that are well known in combinatorial optimization.

At the workshop, there was an evening session on open problems, led by Goemans.

Open problem 1: Find a 4/3-approximation algorithm for symmetric TSP. The integrality ratio of the subtour LP relaxation of TSP is at least 4/3.

An, Kleinberg, and Shmoys [1] improved on a 20-year old 5/3-approximation guarantee of Hoogeveen [4] for the s-t path TSP, which is a variant of the TSP; they use a randomized rounding algorithm, and their improvement uses probabilistic methods coupled with an analysis of near-minimum cuts. Very recently, the combination of results by Traub and Vygen [11] and Zenklusen [14] gave a 3/2-approximation algorithm for this problem. Although the approximation factor of 3/2 matches the known lower bound on the integrality ratio (of a well-known LP relaxation), the analysis does not imply an upper bound on the integrality ratio.

Open problem 2: Prove an upper bound of 3/2 on the integrality ratio of the s-t path TSP.

For the Asymmetric TSP (ATSP), Asadpour et al. [3] gave an $O(\log n/\log\log n)$ approximation algorithm by exploiting a connection to thin trees. This connection was further advanced by Oveis Gharan and Saberi [6], and later by Anari and Oveis Gharan [2]. The latter made very interesting connections to the Kadison-Singer problem and proved an $O(poly\log\log n)$ integrality ratio for the problem. It is conjectured that there is a factor 2 approximation algorithm for the ATSP. A key recent breakthrough comes from a fascinating result of Svensson et al. [10] that finally gave a constant factor approximation for the ATSP. The constant factor achieved in that paper is far from the conjectured lower bound of 2.

Open problem 3: Design and analyze a factor 2 approximation algorithm for ATSP. A more modest goal is an approximation algorithm for ATSP with double-digit approximation ratio.

Despite decades of research by some of the best researchers in algorithms and optimization, there are many other tantalizing open questions. We refer the interested reader to Bill Cook's talk in the workshop (see the video posted on the workshop web page) for a comprehensive list and mention only a few here:

Open problem 4: Determine whether there exists a polynomial-time simplex algorithm for optimally solving the subtour-elimination polytope of the TSP. A related question is to determine if there exists a polynomial-time cutting-plane algorithm for this polytope.

Open problem 5: Design and analyze an algorithm for sampling near-optimal node-separators, similar to Karger's method for sampling near-optimal edge-cuts. Such an algorithm would be useful in identifying structures that could be exploited in the cutting-plane method for the TSP.

Open problem 6: Design an algorithm that given an n-city instance of TSP, finds an optimum solution in time $O^*((2-\epsilon)^n)$.

3 Presentation Highlights

Bill Cook gave a plenary talk on "Open Problems on TSP." A plenary talk was given by Jakub Tarnawski on his recent breakthrough result (joint with Svensson and Vegh) on a "Constant-factor approximation algorithm for the Asymmetric Traveling Salesman Problem."

3.1 Variations of the TSP problem

Stephan Held presented a talk on vehicle routing with subtours, and Zachary Friggstad gave a presentation on linear programs for orienteering and regret-bounded vehicle routing. Viswanath Nagarajan gave a fascinating talk on Stochastic k-TSP. The stochastic version of the k-TSP assumes independent random rewards at vertices and the objective is to minimize the expected length of a tour that collects total reward at least k. Nagarajan discussed both adaptive and non-adaptive solutions. Katarzyna Paluch presented new approximation algorithms for (1,2)-TSP which is a well-studied variant of the TSP where all distances between cities

are either 1 or 2. Tobias Moemke presented a talk on the maximum scatter TSP in doubling metrics. This variation has applications in manufacturing and medical imaging.

3.2 Thin trees and Asymmetric TSP

Neil Olver, Shayan Oveis Gharan, and Nima Anari presented the thin tree conjecture, its relationship with the ATSP, and different approaches for solving it. The thin tree conjecture is due to Luis Goddyn, and it implies a constant factor approximation for ATSP; see [3] for details. Olver's talk was on pipage rounding, pessimistic estimators and matrix concentration. Anari and Oveis Gharan presented their approach using "rounding by sampling," as well as geometric methods for solving a weaker version of the conjecture.

3.3 Path TSP

Martin Naegele, Vera Traub, and Jens Vygen gave presentations on recent exciting advances on the path TSP. Naegele gave an excellent presentation of Zenklusen's beautiful 3/2-approximation algorithm for path TSP [14], that resolves an intriguing open problem. Traub presented a remarkable recent result (joint with Vygen) on the graphic special case of the s-t path TSP where they achieve an approximation factor of 1.497, thus beating the 3/2 lower bound on the integrality ratio of the general (i.e., non-graphic) problem.

Andras Sebő presented deep connections between the TSP, the postman problem and matroids.

3.4 Related problems

There were several presentations on topics related to network design, scheduling, and integer programming. Tom McCormick presented results on strongly polynomial algorithms for problems related to "parametric global minimum cuts," and Thomas Rothvoss presented results on a matroid version of the Santa Claus problem which is a well-known problem in the area of scheduling.

4 Scientific Progress Made and Outcome of the Meeting

The schedule of the workshop provided ample free time for participants to work on joint research projects. A number of new research projects were initiated during the workshop, while some other researchers used the opportunity to continue to work on projects started earlier. The research talks and the plenary talks were very well received.

Marcin Mucha and Anupam Gupta collaborated on rounding 1/2-integral points of the Held-Karp linear programming relaxation of the TSP. Inspired by Andras Sebő's talk, they focused on the special case where the underlying graph is a 2-cycle and solved that case.

Shayan Oveis Gharan continued his research project with Nima Anari. They had several new insights and resolved some questions they had about approximation algorithms on problems related to counting bases of matroids. Several talks of the workshop also gave them new directions for future research, such as Ravi's talk.

R. Ravi reports that the talks in the workshop gave him a clear overview of the state-of-the-art techniques in designing approximation algorithms for the TSP. This in turn helped him develop and submit (for funding) a new research proposal based on studying some of these techniques in detail and identifying new open problems.

Viswanath Nagarajan reports that the presentations on directed network design problems at the workshop inspired him to focus on the directed Steiner tree problem and the orienteering problem. After the workshop, jointly with a graduate student, he obtained a simple and tight quasi-polynomial-time algorithm for the directed tree orienteering problem.

Hyung-Chan An reports that during the workshop he was able to engage in valuable discussions with his colleagues. He also reports that Naegele's talk on the path TSP [14] was particularly helpful and inspired some new research ideas.

Vera Traub and Jens Vygen made progress on their TSP book project, by learning new techniques during the talks and by discussions with many other participants in the breakout sessions. For example, during one

of the discussions, Michel Goemans found a much shorter proof of a result of Henke-Traub-Vygen that says that certain LP relaxations for ATSP are equivalent.

Following Kent Quanrud's talk on fast algorithms for (approximations of) the Christofides heuristic for TSP, Neil Olver and Quanrud investigated whether they can develop even faster algorithms. They are still in the early stages of this research project and they are optimistic.

During the workshop, Hung Viet Le and Vincent Cohen-Addad discussed several research problems on planar graphs and minor-free graphs. They are now working together on designing a PTAS for one of these problems on planar graphs. They report that the workshop provided an invaluable opportunity to learn the state-of-the-art techniques in approximation algorithms for ATSP.

Tobias Moemke started work on a project with Zachary Friggstad to obtain bounds on the integrality ratio for a specific flow problem. The discussions at the workshop led to fresh ideas worth exploring and might lead to substantial improvements.

Vincent Cohen-Addad, Kamyar Khodamoradi, and Zachary Friggstad began joint work on local search algorithms. Inspired by Vygen's talk, Friggstad and Chaitanya Swamy are focusing on an improved approximation algorithm for a well-known problem related to the TSP.

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