Banff International Research Station
for Mathematical Innovation and Discovery

"Advances in hyperkähler and holomorphic symplectic geometry"
Sunday, 11-03-2012 to Friday, 16-03-2012

MEALS

*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday
*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday
*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday
Coffee Breaks: As per daily schedule, in the foyer of the TransCanada Pipeline Pavilion (TCPL)
*Please remember to scan your meal card at the host/hostess station in the dining room for each meal.

MEETING ROOMS

All lectures will be held in the new lecture theater in the TransCanada Pipelines Pavilion (TCPL). LCD projector and blackboards are available for presentations.

SCHEDULE

Sunday, 11-03-2012

16:00	Check-in begins (Front Desk – Professional Development Centre – open 24 hours)
17:30–19:30	Buffet Dinner, Sally Borden Building
20:00	Informal gathering in 2nd floor lounge, Corbett Hall
	Beverages and small assortment of snacks available on a cash honour-system.

Monday, 12-03-2012

07:00–08:45	Breakfast
08:45–09:00	Introduction and Welcome to BIRS by the BIRS Station Manager, Room 201, TCPL
09:00–10:00	**Kieran G. O'Grady**
	"Vector-bundles and zero-cycles on K3 surfaces"
10:00–10:30	Coffee Break, TCPL
10:30–11:30	**Daniel Huybrechts**
	"Chow groups and stable maps"
11:30–13:00	Lunch
13:00–14:00	Guided Tour of the Banff Centre; meet in the 2nd floor lounge, Corbett Hall.
14:00–15:00	**Sergey Cherkis**
	"Doubly-periodic monopoles and their moduli spaces"
15:00–15:30	Coffee Break, TCPL
15:30–16:30	**Andrew Neitzke**
	"Spectral networks"
16:30–17:30	Informal discussions / Recreation
17:30–19:30	Dinner

Tuesday, 13-03-2012

07:00–09:00	Breakfast
09:00–10:00	**Tamás Hausel**
	"Symmetries of SL(n) Hitchin fibres"
10:00–10:30	Coffee Break, TCPL
10:30–11:30	**Dimitri Markushevich**
	"Some examples of Prym Lagrangian fibrations"
11:30–11:45	Group Photo; meet in foyer of TCPL (photograph will be taken outdoors so a jacket might be required).
11:45–13:30	Lunch
14:00–15:00	**Yoshinori Namikawa**
	"On the structure of homogeneous symplectic varieties of complete intersection"
15:00–15:30	Coffee Break, TCPL
15:30–16:30	**Sabin Cautis**
	"Flops and about"
16:30–17:30	Informal discussions / Recreation
17:30–19:30	Dinner
19:30–20:30	**Nicholas Proudfoot**
	"Quantizations of conical symplectic resolutions"

Wednesday, 14-03-2012

07:00–09:00	Breakfast
09:00–10:00	**Philip Boalch**
	"Irregular connections, Dynkin diagrams, and fission"
10:00–10:30	Coffee Break, TCPL
10:30–11:30	**Brent Pym**
	"Residues of Poisson structures and applications"
11:30–13:30	Lunch
13:30–17:30	Informal discussions / Recreation
17:30–19:30	Dinner

Thursday, 15-03-2012

07:00–09:00	Breakfast
09:00–10:00	**Jun-Muk Hwang**
	"Webs of Lagrangian tori in projective symplectic manifolds"
10:00–10:30	Coffee Break, TCPL
10:30–11:30	**Giulia Saccà**
	"Fibrations in abelian varieties and Enriques Surfaces"
11:30–13:30	Lunch
14:00–15:00	**Keiji Oguiso**
	"K3 surface automorphisms and hyperkähler automorphisms inspired by complex dynamics"
15:00–15:30	Coffee Break, TCPL
15:30–16:30	**François Charles**
	"Some arithmetic aspects of specialization of Néron-Severi groups for holomorphic symplectic varieties"
16:30–17:30	Informal discussions / Recreation
17:30–19:30	Dinner
19:30–20:30	**Emanuele Macrì**
	"Projectivity and birational geometry of Bridgeland moduli spaces"

Friday, 16-03-2012

07:00–09:00	Breakfast
09:00–10:00	**Andrew Dancer**
	"Implosion for hyperkähler manifolds"
10:00–10:30	Coffee Break, TCPL
10:30–11:30	**Misha Verbitsky**
	"Trisymplectic manifolds"
11:30–13:30	Lunch
13:30–15:00	Informal discussions / Departures

Please remember to checkout by 12:00 noon.

You are welcome to use the BIRS facilities (BIRS Coffee Lounge, TCPL and Reading Room) until 15:00 on Friday, although participants are still required to checkout of the guest rooms by 12:00 noon.

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ABSTRACTS
(in alphabetical order by speaker surname)

Speaker: **Philip Boalch** (Centre National de la Recherche Scientifique)
Title: "Irregular connections, Dynkin diagrams, and fission"

Abstract: I'll survey some results (both old and new) related to the geometry of hyperkahler moduli spaces of irregular connections on curves. If time permits this will include: 1) new nonlinear group actions generalising the well known actions of the mapping class/braid groups on character varieties, 2) new ways to glue Riemann surfaces together to obtain (symplectic) generalisations of the complex character varieties of surfaces, and 3) a precise conjecture that the Hilbert scheme of points on any 2d meromorphic Hitchin system is again a Hitchin system.

Speaker: **Sabin Cautis** (Columbia University)
Title: "Flops and about"

Abstract: Stratified flops of type A, D and E show up in the birational geometry of holomorphic symplectic varieties. For example, by a result of Namikawa all Springer resolutions of the closure of nilpotent orbits are related by a sequence of such flops.

Two varieties related by such a flop are expected to have equivalent derived categories. Concentrating on the A and D types, I will discuss the geometry of such flops, explain how they induce derived equivalences and speculate on various open questions.

Speaker: **François Charles** (IRMAR – Université de Rennes 1)
Title: "Some arithmetic aspects of specialization of Néron-Severi groups for holomorphic symplectic varieties"

Abstract: For a given family of smooth projective complex varieties, the theory of variations of Hodge structures gives a precise description of the variation of the Picard number of the members of the family. In the case of families of holomorphic symplectic varieties, the two following properties are well-known. On the one hand, the Picard number of a very general member of the family is equal to the Picard number of the generic fiber of the family. This is an easy consequence of Baire's theorem. On the other hand, if the family is not isotrivial, the locus in the parameter space of varieties with Picard number strictly bigger than that of the generic fiber is topologically dense. This is a consequence of the local Torelli theorem and of Lefschetz' theorem on $(1, 1)$-classes which was pointed out by M. Green.

The goal of this talk is to investigate the extent to which the behaviour described above still appears in the arithmetic situation where the parameter space is replaced with the ring of integers of a number fields. This amounts to investigating specialization of Néron-Severi groups for holomorphic symplectic varieties defined over number fields after reduction to a finite field.

In this situation, the first result above does not hold. However, we will describe precisely the extent to which the Picard number can be forced to jump after specialization to a finite field. If time allows, we will describe a proof of the arithmetic analog of the theorem of Green in the special case of products of elliptic curves and discuss its arithmetic significance.

These problems have implications outside of arithmetic geometry, as was pointed out by recent results on the existence of rational curves on K3 surfaces by Bogomolov-Hassett-Tschinkel and Li-Liedtke. We will describe how they can be used to get an algorithm that allows one to compute the Picard number of any holomorphic symplectic variety.

Speaker: **Sergey Cherkis** (University of Arizona)
Title: "Doubly-periodic monopoles and their moduli spaces"

Abstract: A monopole wall is a solution of the Bogomolny equation on $\mathbb{R} \times T^2$; in other words it is a doubly periodic monopole. Moduli spaces of monopole walls are hyperkähler and, when the dimension is minimal, deliver examples of gravitational instantons. We formulate spectral description of a monopole wall of any given charges and use it to compute the dimension of its moduli space.

The Nahm transform maps a monopole wall to a monopole wall establishing the isometry between their respective moduli spaces. We find $SL(2,\mathbb{Z})$ group action on monopole walls, such that the Nahm transform is its S element. We conclude by identifying all monopole walls with four real moduli, up to this $SL(2,\mathbb{Z})$ equivalence.

These results are obtained in collaboration with Richard Ward.

Speaker: **Andrew Dancer** (Oxford University)
Title: "Implosion for hyperkähler manifolds"

Abstract: Implosion is an abelianisation construction in symplectic geometry, due to Guillemin, Jeffrey and Sjamaar. In this talk we describe joint work with Frances Kirwan and Andrew Swann on developing an analogous construction for hyperkähler spaces.

Speaker: **Tamás Hausel** (Oxford University)
Title: "Symmetries of SL(n) Hitchin fibres"

Abstract: In this talk we show how the computation of the group of components of Prym varieties of spectral covers leads to cohomological results motivated by mirror symmetry on the cohomology of moduli space of Higgs bundles and in turn to cohomological results on the moduli space of stable bundles on curves originally due to Harder-Narasimhan. This is joint work with Christian Pauly.

Speaker: **Daniel Huybrechts** (University of Bonn)
Title: "Chow groups and stable maps"

Speaker: **Jun-Muk Hwang** (Korea Institute for Advanced Study)
Title: "Webs of Lagrangian tori in projective symplectic manifolds"

Abstract: For a Lagrangian torus A in a simply-connected projective symplectic manifold M, we prove that M has a hypersurface disjoint from a deformation of A. This implies that a Lagrangian torus in a compact hyperkaehler manifold is a fiber of an almost holomorphic Lagrangian fibration, giving an affirmative answer to a question of Beauville's. Our proof employs two different tools: the theory of action-angle variables for algebraically completely integrable Hamiltonian systems and Wielandt's theory of subnormal

subgroups. This is a joint-work with Richard Weiss.

Speaker: **Emanuele Macrì** (The Ohio State University)
Title: "Projectivity and birational geometry of Bridgeland moduli spaces"

Abstract: In this talk we will present a construction of a family of nef divisor classes on every moduli space of stable complexes in the sense of Bridgeland. For a generic stability condition on a K3 surface, we will prove that these classes are ample, thereby generalizing a recent result of Minamide, Yanagida, and Yoshioka.

We will apply this construction to describe a region in the ample cone of a moduli space of Gieseker-stable sheaves on a K3 surface and to study its birational geometry.

This is joint work in progress with Arend Bayer.

Speaker: **Dimitri Markushevich** (Universite Lille 1)
Title: "Some examples of Prym Lagrangian fibrations"

Abstract: The objective of the talk is to describe several constructions of holomorphically symplectic varieties equipped with Lagrangian fibrations. The constructions are related to the variations of mixed Hodge structures, and the fibers of the obtained Lagrangian fibrations are intermediate Jacobians of algebraic varieties. Several examples are produced when the latter algebraic varieties are conic bundles and the intermediate Jacobians are Prym varieties of double covers. A work is in progress on their (partial) compactification.

Speaker: **Yoshinori Namikawa** (Kyoto University)
Title: "On the structure of homogeneous symplectic varieties of complete intersection"

Abstract: If X is a symplectic variety embedded in an affine space as a complete intersection of homogeneous polynomials, then X coincides with a nilpotent orbit closure of a semisimple Lie algebra. Moreover, if X is a homogeneous symplectic hypersurface, then $\dim X = 2$ and X is an A_1-surface singularity.

Speaker: **Andrew Neitzke** (University of Texas at Austin)
Title: "Spectral networks"

Abstract: I will describe some objects called "spectral networks." A spectral network is a set of paths drawn on a punctured Riemann surface, obeying some local conditions. Spectral networks arise naturally in a new construction of the hyperkähler structure on moduli spaces of Higgs bundles with gauge group SU(N). This is joint work with Davide Gaiotto and Greg Moore.

Speaker: **Kieran G. O'Grady** (Rome (Sapienza))
Title: "Vector-bundles and zero-cycles on $K3$ surfaces"

Abstract: Let X be a projective complex $K3$ surface. Let $A^q(X)$ be the Chow group of codimension-q cycles on X modulo rational equivalence. Beauville and Voisin singled out a class $c_X \in A^2(X)$ of degree 1: it is represented by any point lying on an arbitrary rational curve (an irreducible curve whose normalization is rational). The class c_X has the following remarkable property:

$$\text{Let } D_1, D_2 \in A^1(X): \text{ then } D_1 \cdot D_2 \in \mathbb{Z}c_X.$$

Moreover $c_2(X) = 24c_X$. (Conjecturally the Chow ring of Hyperkähler varieties has similar properties.) In particular one has the *Beauville-Voisin ring* $A^0(X) \oplus A^1(X) \oplus \mathbb{Z}c_X$. Huybrechts proved that if E is a spherical object in the bounded derived category of X then the Chern character of E belongs to the Beauville-Voisin ring provided $\mathrm{Pic}(X)$ has rank greater than 2 or $c_1(E) \equiv \pm 1 \pmod{\mathrm{rk}(E)}$ if $\mathrm{Pic}(X) = \mathbb{Z}$. A rigid simple vector-bundle on X is a particular case of spherical object - in fact the key case in the proof of Huybrecths' result. One may summarize Huybrechts' result as follows. Let F_1, F_2 be rigid vector-bundles on X (the additional hypotheses mentioned above are in force): then $c_2(F_1) = c_2(F_2) + ac_X$ where $a := (\deg c_2(F_1) - \deg c_2(F_2))$. We believe that the following more general statement (with no additional hypotheses) holds. Let $\mathfrak{M}_1^{st}$ and $\mathfrak{M}_2^{st}$ be moduli spaces of stable pure sheaves on X with Mukai vectors v_1 and v_2 respectively. Suppose that $\dim \mathfrak{M}_1^{st} = \dim \mathfrak{M}_2^{st}$ i.e. v_1 and v_2 have equal norm with respect to Mukai's pairing: then the subset of $A^2(X)$ whose elements are $c_2(F_1)$ where $[F_1] \in \overline{\mathcal{M}}_1^{st}$ (the closure of $\mathfrak{M}_1^{st}$ in the moduli space of semistable sheaves) is equal to the subset of $A^2(X)$ whose elements are $c_2(F_2) + ac_X$ where $[F_2] \in \overline{\mathcal{M}}_2^{st}$ and $a := (\deg c_2(F_1) - \deg c_2(F_2))$ (notice that $(\deg c_2(F_1) - \deg c_2(F_2))$ is independent of F_1 and F_2). We will prove that the above statement holds under some additional assumptions.

Speaker: **Keiji Oguiso** (Keio University)
Title: "K3 surface automorphisms and hyperkähler automorphisms inspired by complex dynamics"

Abstract: I would like to discuss some nature of automorphisms of K3 surfaces and compact hyperkähler manifolds from the following basic and natural aspects in complex dynamics with concrete examples:
  (1) topological entropy;
  (2) Tits' alternatives;
  (3) fixed point set;
  (4) relations with ambient spaces.

Speaker: **Nicholas Proudfoot** (University of Oregon)
Title: "Quantizations of conical symplectic resolutions"

Abstract: The most studied example of a conical symplectic resolution is the cotangent bundle M of the flag manifold G/B, which resolves the nilpotent cone in Lie(G). Much of what goes under the name "geometric representation theory" is the study of this resolution, called the Springer resolution. Here are two cool features of this subject:
   - If you construct a deformation quantization of M and take global sections, you get the ring of global (twisted) differential operators on the flag variety, which is isomorphic to a central quotient of the universal enveloping algebra of Lie(G). This allows you to study representations of Lie(G) in terms of sheaves on M.
   - There is a natural action of "convolution operators" on the cohomology of M which provides a geometric construction of the regular representation of the Weyl group of G. This action can be promoted to a braid group action on a category by replacing cohomology classes with sheaves.
   I will make the case that these two phenomena fit neatly into a theory that applies to arbitrary conical symplectic resolutions, including (for example) quiver varieties, hypertoric varieties, and Hilbert schemes of points on ALE spaces.
   This is joint work with Braden, Licata, and Webster.

Speaker: **Brent Pym** (University of Toronto)
Title: "Residues of Poisson structures and applications"

Abstract: A holomorphic Poisson manifold is foliated by symplectic leaves, and the locus consisting of all leaves of dimension $2k$ or less is called the $2k^{th}$ degeneracy locus. In recent work with Marco Gualtieri, we explain that a Poisson structure has natural residues along its degeneracy loci, which are direct analogues

of the Poincaré residue of a meromorphic volume form. As applications, we prove that the anti-canonical divisor along which a generically symplectic Poisson structure degenerates is singular in codimension two, and provide new evidence in favour of Bondal's conjecture that the $2k^{th}$ degeneracy locus of a Poisson Fano variety has dimension $\geq 2k + 1$.

Speaker: **Giulia Saccà** (Princeton University)
Title: "Fibrations in abelian varieties and Enriques Surfaces"
Abstract: I will discuss two classes of fibrations in abelian varieties that can be associated to a linear system on an Enriques surface. The first class corresponds to Lagrangian subvarieties of certain HK manifolds, whereas the second class corresponds to singular symplectic subvarieties of certain singular moduli spaces of sheaves. The second class is a joint work in progress with E. Arbarello and A. Ferretti.

Speaker: **Misha Verbitsky** (SU-HSE, Faculty of Maths)
Title: "Trisymplectic manifolds"

Abstract: A trisymplectic structure on a complex 2n-manifold is a triple of holomorphic symplectic forms such that any linear combination of these forms has rank 2n, n or 0. We show that a trisymplectic manifold is equipped with a holomorphic 3-web and the Chern connection of this 3-web is holomorphic, torsion-free, and preserves the three symplectic forms. We construct a trisymplectic structure on the moduli of regular rational curves in the twistor space of a hyperkähler manifold.