Cohomological Methods in Geometric Group Theory
LECTURE TITLES

Alexander **Berglund** (University of Copenhagen) Fri 10:00am
Rational cohomology of automorphism of highly connected manifolds

Michelle **Bucher** (Université de Genève) Tues 3:15pm
Isometric properties of bounded cohomology and applications to volumes of representations

Abstract. Let G be a lattice in $\mathrm{SO}(n,1)$ and let $h\colon G \to \mathrm{SO}(n,1)$ be any representation. For cocompact lattices, the volume of a representation is an invariant whose maximal and rigidity properties have been studied extensively. We will show how the fact that the isomorphism between the bounded cohomology of a hyperbolic manifold and the relative bounded cohomology of the manifold relative its boundary is isometric can be used to define the volume of a representation (a different definition by Francaviglia and Klaff also exists). In particular, we establish a rigidity result for maximal representations, recovering Mostow rigidity for hyperbolic manifolds.

In the cocompact case, the set of values for the volume of a representation is discrete. In even dimension, this follows from the fact that the volume form is an Euler class. In odd dimension, this was proven by Besson, Courtois and Gallot. The situation changes in the noncocompact case and for example the discreteness of the set of values is not valid anymore in dimension 2 and 3. We prove that in even dimension greater or equal to 4, the set of values of the volume of a representation is, up to a universal constant, an integer.

This is joint work with Marc Burger and Alessandra Iozzi.

Marc **Burger** (ETH Zürich) Mon 3:15pm
On and around bounded cohomology

Thomas **Church** (Stanford University) Mon 2:00pm
Representation stability, FI-modules, and beyond

Abstract. I will survey the major developments in the program of representation stability, with a focus on the theory of FI-modules. FI-modules were introduced as a tool to understand and prove representation stability for sequences of S_n-representations, but they have also deepened our understanding of what representation stability means. Recently, algebraic and homological results for FI-modules have let us work integrally or over fields of positive characteristic, and give sharper explicit ranges for stability. In a different direction, homological finiteness and Koszul duality for FI-modules give new theorems on twisted homological stability. Finally, FI-modules give a new perspective on how to interpret representation stability for representations of other sequences of groups, including braid groups, mapping class groups, and $\mathrm{GL}_n(F_q)$. Joint work with Benson Farb, Jordan Ellenberg, and Rohit Nagpal.

Thomas **Church** (Stanford University) Tues 2:00pm
Homological algebra of FI-modules

Abstract. FI-modules were introduced by Church–Ellenberg–Farb to encode sequences of S_n-representations, and the algebraic properties of FI-modules govern the stable behavior of these sequences. I will prove two key theorems: the Noetherian property for FI-modules over Noetherian rings (any submodule of a finitely-generated FI-module is finitely generated), joint with J. Ellenberg, B. Farb, and R. Nagpal; and a homological bound which says that FI-modules are "approximately 1-dimensional", joint with J. Ellenberg. Both theorems have concrete applications for sequences of S_n-representations.

David Bruce **Cohen** (Rice University) Wed 11:15am
Borel stability for congruence subgroups

Abstract. Expanding on a result of Charney, we prove that under various conditions, the homology of a congruence subgroup of the special linear or symplectic group does not depend on the level.

Koji **Fujiwara** (Kyoto University) Thur 10:30am
Group actions on quasi-trees

Abstract. It is a standard method to use group actions on trees, but it turns out that group actions on quasi-trees are also useful. I'd like to discuss a construction of quasi-trees and applications using the actions. This is a joint work with Bestvina and Bromberg.

Ilya **Grigoriev** (Stanford University) Fri 11:00am
Relations in the cohomology of classifying spaces of manifold bundles

Abstract. The classical tautological subring is the image of the map from a free polynomial algebra to the cohomology of the classifying space of surface bundles $R : \mathbb{Q}[\kappa_1, \kappa_2, \ldots] \to H^*(\mathrm{BDiff}\,\Sigma_g)$. Theorems of Madsen-Weiss, Harer, and others imply that R is an isomorphism in the "stable range" of cohomological dimensions $* \leq (2g-2)/3$. The kernel of R in high cohomological dimension has been studied by Morita, Faber, Looijenga, Pandharipande and many others.

I will discuss the analogues of the map R and the tautological ring in the case when the surface of genus g is replaced with the connect sum of a product of spheres, $M = \#_g S^k \times S^k$, with k odd. I will present a large family of elements in the kernel of R that implies that the image of R is a finitely-generated ring and includes elements of cohomological dimension independent of k. This contrasts with the recent work of Søren Galatius, Oscar Randal-Williams, Ib Madsen, and Alexander Berglund who have shown the map R in this setting (and $k > 2$) to be an isomorphism in a range, just as it is for surfaces ($k = 1$).

Allen **Hatcher** (Cornell University) Thur 9:15am
Stable homology of spaces of graphs

Aditi **Kar** (Oxford University) Fri 9:15am
A cohomological approach to splittings of groups

Abstract. I will begin with a brief introduction to the Relative Ends Invariants and then survey results from my joint work with Graham Niblo about splitting manifold groups and their relatives. I will conclude with applications to Algebraic Topology — namely, for a large class of manifolds, every codimension-1 π_1-injective map is homotopically a finite cover of an embedding.

Wolfgang **Lück** (Universität Bonn) Mon 4:30pm
L^2-invariants and applications to group theory

Abstract. We give a brief and gentle introduction to L^2-Betti numbers and L^2-torsion, and discuss their applications to group theory. These range from questions about the deficiency of groups, constructions of exotic groups, structure of group rings, K-theory of group rings, homological growth in zero characteristic and prime characteristic, and torsion invariants for automorphisms of groups.

Igor **Mineyev** (University of Illinois at Urbana-Champaign) Thur 2:00pm
The deep-fall property, submultiplicativity, and the Atiyah problem

Andrew **Putman** (Rice University) Mon 9:30am
Stability in the homology of congruence subgroups

Abstract. We will discuss stability phenomena in the mod p homology group of congruence subgroups.

Andrew **Putman** (Rice University) Tues 9:30am
Unstable homological stability

Abstract. We will discuss patterns in the rational homology groups of $\mathrm{SL}_n(\mathbb{Z})$ and related groups that occur near the top of their cohomological dimensions. This is joint work with Tom Church and Benson Farb.

Holger **Reich** (Freie Universität Berlin) Tues 4:30pm
$\mathrm{GL}(n,\mathbb{Z})$ *and algebraic* K*-theory*

Abstract. The talk will give a brief introduction to the Farrell-Jones conjecture and discuss the case $\mathrm{GL}(n,\mathbb{Z})$.

Thomas **Schick** (Universität Göttingen) Thur 3:15pm
Hodge theory of metric spaces

Abstract. Hodge theory is a beautiful synthesis of geometry, topology, and analysis which has been developed in the setting of Riemannian manifolds. However, many spaces important in applications do not fit this framework. This motivates us to develop a version of Hodge theory on metric spaces with a probability measure.

The goal here is to obtain a theory which is sensitive to features which are present at a given scale α. Our approach is based on an Alexander-Spanier replacement of differential forms, with a localization with parameter α.

We discuss that, unfortunately, for completely general metric spaces the theory does not have all nice analytic features one would hope to have.

On the other hand, as the main test case, we provide conditions for the local structure of the metric measure space which implies that the α-scale cohomology is isomorphic with ordinary homology. In particular, this applies to Riemannian manifolds (if we choose the scale small enough in terms of the injectivity radius and the sectional curvature).

Karen **Vogtmann** (Cornell University) Mon & Tues 10:45am
Unstable homology of $\mathrm{Out}(F_n)$

Jennifer C. H. **Wilson** (University of Chicago) Wed 10:30am
FI–Modules and representations of the classical Weyl groups

Abstract. The theory of FI–modules developed by Church–Ellenberg–Farb gives new tools and a new point of view for studying sequences of representations of the symmetric group. In this talk, I will describe how their theory extends to sequences of representations of the other two families of classical Weyl groups: the signed permutation groups in type B/C, and the even-signed permutation groups in type D. As an application, I will give some stability results for the cohomology of the group of pure string motions of circles embedded in 3-space, a group related to the pure braid group.

Kevin **Wortman** (University of Utah) Wed 9:15am
Survey of finiteness properties of arithmetic groups