5th Workshop on GRAph Searching, Theory and Applications (GRASTA 2012)

October 7–12, 2012

Banff International Research Station, Banff, AB, Canada

Organizers

Fedor Fomin (University of Bergen)
Richard Nowakowski (Dalhousie University)
Pawel Pralat (Ryerson University)
Dimitrios M. Thilikos (National and Kapodistrian University of Athens)

PROGRAM

Monday, October 8

8:50 - 9:00	Opening remarks
9:00 - 10:00	**Anthony Bonato**, Ryerson University *Cops and Robbers: Directions and Generalizations*
10:00 - 10:30	COFFEE BREAK
10:30 - 11:00	**Jan Kratochvil**, Charles University *Cops and robbers in special graph classes*
11:00 - 11:30	**Lawrence Erickson**, University of Illinois at Urbana-Champaign *Locating a robber on a graph via distance queries*
11:30 - 12:00	**Josep Diaz**, Universitat Politecnica de Catalunya *Metric dimension*
	LUNCH BREAK
from 2:00	Open problem session

Tuesday, October 9

9:00 - 10:00	**Douglas B. West**, University of Illinois *Revolutionaries and spies: Spy-good and spy-bad graphs*
10:00 - 10:30	COFFEE BREAK
10:30 - 11:00	**Przemyslaw Gordinowicz**, Technical University of Lodz *Let us play the cleaning game*
11:00 - 11:30	**Nicolas Nisse**, MASCOTTE team-project *On the surveillance game*
11:30 - 11:45	**Dimitrios M. Thilikos**, National and Kapodistrian University of Athens *Report on GRASTA and Special Issue of TCS*
11:45 - 12:00	**Nicolas Nisse**, MASCOTTE team-project *Presentation of the next GASTA*
	LUNCH BREAK
	Free afternoon – hiking

Wednesday, October 10

9:00 - 10:00	**Peter Widmayer**, ETH Zurich *Polygon Reconstruction with Little Information*
10:00 - 10:30	COFFEE BREAK
10:30 - 11:00	**Ladislav Stacho**, Simon Fraser University *Graph traversal with constant number of pebbles*
11:00 - 11:30	**Dariusz Dereniowski**, Gdansk University of Technology *Minimum length path decompositions*
11:30 - 12:00	**Pawel Pralat**, Ryerson University *Revolutionaries and spies on random graph*
12:00	Workshop photo
1:15 - 2:00	Campus tour
	LUNCH BREAK
from 2:00	Open problem session

Thursday, October 11

9:00 - 10:00	**Nicolas Nisse**, MASCOTTE team-project *Routing reconfiguration and processing games*
10:00 - 10:30	COFFEE BREAK
10:30 - 11:30	**Tobias Muller**, Mathematical institute of Utrecht University *Cops and robbers on random geometric graphs*
11:30 - 12:00	**Douglas B. West**, University of Illinois *Revolutionaries and spies: Spy-good and spy-bad graphs—part 2*
	LUNCH BREAK
from 2:00	Open problem session

Friday, October 12

from 9:00	Open problem session

PARTICIPANTS

Bonato, Anthony (Ryerson University)
Clarke, Nancy (Acadia University)
Dereniowski, Dariusz (Gdansk University of Technology)
Diaz, Josep (Universitat Politecnica de Catalunya)
Dudek, Andrzej (Western Michigan University)
Dyer, Danny (Memorial University of Newfoundland)
Erickson, Lawrence (University of Illinois at Urbana-Champaign)
Finbow, Stephen (St. Francis Xavier University)
Fitzpatrick, Shannon (University of Prince Edward Island)
Gavenciak, Tomas (Charles University)
Gordinowicz, Przemyslaw (Technical University of Lodz)
Hahn, Gena (University of Montreal)
Kinnersley, Bill (University of Illinois at Urbana-Champaign)
Kratochvil, Jan (Charles University)
Messinger, Margaret-Ellen (Mount Allison University)
Muller, Tobias (Utrecht University)
Nisse, Nicolas (INRIA Sophia Antipolis)
Nowakowski, Richard (Dalhousie University)
Pardo Soares, Ronan (MASCOTTE)
Pralat, Pawel (Ryerson University)
Seamone, Ben (Universite de Montreal)
Stacho, Ladislav (Simon Fraser University)
Thilikos, Dimitrios (National and Kapodistrian University of Athens)
West, Douglas (University of Illinois Urbana-Champaign)
Widmayer, Peter (ETH)
Yang, Boting (University of Regina)

OPEN PROBLEMS

Pawel Pralat, Ryerson University
Meyniel's conjecture

The biggest open conjecture in the area of cops and robbers is the one of Meyniel, which asserts that for some absolute constant C, the cop number of every connected graph G is at most $C\sqrt{n}$, where $n = |V(G)|$. Today we only know that the cop number is at most $n2^{-(1+o(1))\sqrt{\log_2 n}}$ (which is still $n^{1-o(1)}$) for any connected graph on n vertices.

Pawel Pralat, Ryerson University
Revolutionaries and spies on random graphs

The behaviour of the spy number is analyzed for dense graphs (that is, graphs with average degree at least $n^{1/2+\varepsilon}$ for some $\varepsilon > 0$). For sparser graphs, only some bounds are provided and the picture is far from clear.

Pawel Pralat, Ryerson University
The firefighter problem

Consider the following *k-many firefighter problem* on a finite graph $G = (V, E)$. Suppose that a fire breaks out at a given vertex $v \in V$. In each subsequent time unit, a firefighter protects k vertices which are not yet on fire, and then the fire spreads to all unprotected neighbours of the vertices on fire. The objective of the firefighter is to save as many vertices as possible.

The surviving rate $\rho_k(G)$ of G is defined as the expected percentage of vertices that can be saved when a fire breaks out at a random vertex of G. Let

$$\tau_k = \begin{cases} \frac{30}{11} & \text{if } k = 1 \\ k + 2 - \frac{1}{k+2} & \text{if } k \geq 2. \end{cases}$$

It is known that there exists a constant $c > 0$ such that for any $\varepsilon > 0$ and $k \geq 1$, each graph G on n vertices with at most $(\tau_k - \varepsilon)n$ edges is not flammable; that is, $\rho_k(G) > c \cdot \varepsilon > 0$. Moreover, a construction of a family of flammable random graphs is proposed to show that the constants τ_k cannot be improved.

It would be nice to find the threshold for other families of graphs, including planar graphs.

Problem 1: Determine the largest real number M such that every planar graph G with $n \geq 2$ vertices and $\frac{2m}{n} \leq M - \varepsilon$ edges has $\rho_1(G) \geq c \cdot \varepsilon$ for some $c > 0$. It is known that $\frac{30}{11} \leq M \leq 4$.

One can generalize this question to any number of firefighters. We know that all planar graphs are not k-flammable for $k \geq 4$. However, it is conjectured that, in fact, planar graphs are not 2-flammable but the techniques are too local to show it. Therefore, it seems that the question does not make sense for $k \geq 2$ (unless the conjecture is false).

Problem 2: Determine the least integer g^* such that there is a constant $0 < c < 1$ such that every planar graph G with girth at least g^* has $\rho(G) \geq c$. It is known that $5 \leq g^* \leq 7$.

Pawel Pralat, Ryerson University
Chipping away at the edges: how long does it take?

We introduce the single-node traffic flow process, which is related to both the chip-firing game and the edge searching process. Initially, real-valued weights (instead of chips) are placed on some vertices of a graph G, and all the edges have zero weight. When a vertex is "fired", the whole content accumulated in this vertex is sent uniformly to all its neighbours, and each edge increases its weight by the amount that is sent through this edge. We would like to discover the shortest firing sequence such that the total amount of traffic that has passed through each edge is at least some fixed value.

Suppose that initially each vertex has weight of ω. Let $f(G)$ be the number of rounds of the shortest firing sequence such that the total amount of traffic that has passed through each edge is at least one. It is known that

$$\begin{aligned} \frac{f(K_n)}{|E(K_n)|} \cdot \omega &= \frac{1}{2} + o(1), \\ \frac{f(K_{n,n})}{|E(K_{n,n})|} \cdot \omega &\leq 1 + o(1), \\ \frac{f(K_{1,n})}{|E(K_{1,n})|} \cdot \omega &\leq \frac{1}{4} + o(1), \end{aligned}$$

for ω small enough. In particular, complete bipartite graphs and stars are not fully investigated.

Let $G(n)$ be a family of connected graphs on n vertices. It it natural to ask whether the following limits exist, and if so to find their values.

$$\begin{aligned} M &= \lim_{n\to\infty} \max_{G \in G(n)} \frac{f(G)}{|E(G)|} \cdot \omega, \\ m &= \lim_{n\to\infty} \min_{G \in G(n)} \frac{f(G)}{|E(G)|} \cdot \omega. \end{aligned}$$

In particular, is it true that $0 < m < M = O(1)$?

Gena Hahn, University of Montreal

Let G be a finite graph, $c(G)$ its cop-number and $g(G)$ its genus. Schroeder proved that $c(G) \leq \lceil \frac{3}{2} g(G) \rceil + 3$ and conjectured that $c(G) \leq g(G) + 3$.

Find a toroidal graph that needs 4 cops to catch a robber. Note that Andreae thinks that $c(G) \leq 3$ for toroidal G, so another way to approach the question is to prove that he is right.

Prove (or disprove) Schroeder's conjecture.

Let T be a tournament obtained from a Steiner triple system by orienting the edges of each of the triples in a triangle decomposition of the appropriate complete graph in a cycle. Nowakowski asked if $c(T) \leq 2$ and Thériault found - by computer search - that this is not the case. Is there a constant c such that $c(T) \leq c$ for each tournament obtained in the way described?

Lawrence Erickson, University of Illinois at Urbana-Champaign
Cops and robbers with distance queries

A cop and robber game is played on a graph with the following rules:

- A robber is hiding at a vertex.
- At the beginning of a round, the robber moves distance 0 or 1.
- The cop scans a vertex and receives the distance to the robber.
- The cop wins if it determines the robber's location. Otherwise a new round begins.
- The robber wins if it can hide indefinitely.

Let $G^{1/m}$ be the graph formed by replacing each edge of G with a path of length m. Let $n = |V(G)|$. Let $\mu(G)$ be the metric dimension of G.

It is known that:

- The cop wins in $G^{1/m}$ if $m > \min(\max(\mu(G) + 2^{\mu}(G), \Delta(G)), n - 1)$.
- The cop wins in $G^{1/m}$ if G is a grid and $m \geq 2$.
- The cop wins in $K_{a,b}^{1/m}$ if $m \geq a \geq b$ and $m > b$.
- The robber wins in G if G has girth 5 or less.

Open questions:

- Does the robber win in $K_n^{1/m}$ if $m \leq n - 1$?
- If the cop wins in G, does the cop win in every subdivision of G?
- Does the robber win in G if G has girth 6?

This game is studied in Carraher et al. (2012, Theoretical Computer Science). A very similar game, with a slightly stronger cop, is studied in Seager (2012, Discrete Mathematics).

Richard Nowakowski, Dalhousie University
Complementary Cops and Robber (Hill-Nowakowski)

Give a graph G the cops move along the edges of G and the robber along the non-edges or edges of $\bar{G}$. Given this move set, let $CCR(G)$ be the number of cops required to capture the robber on G. In general, $\gamma(G) - 1 \leq CCR(G) \leq \gamma(G)$. (See Hill PhD thesis (2008) and also Neufeld-Nowakowski, A vertex-to-vertex pursuit game played with disjoint sets of edges, Finite and infinite combinatorics in sets and logic, Kluwer Acad. Publ., Dordrecht, 1993)

Characterize G such that $CCR(G) = 1$. On the last move, the cop and robber are on adjacent vertices that also dominate the graph.

Richard Nowakowski, Dalhousie University
Boolean Cops and Robber (Hill-Nowakowski)

Given a boolean lattice B_n, i.e., the partial order of all subsets of an n-element set. The robber starts at the top and moves downwards, the cops start on the bottom and move upwards. No-one is allowed pass and all cops must move. (Hill PhD Thesis (2008) see also Nowakowski. Search and sweep numbers of finite directed acyclic graphs. Discrete Appl. Math., 41(1):111, 1993)

How many cops are required to capture the robber?

Known: for $n = 1, 2, 3, 4, 5, 6, 7, 9$ the number of cops required is 2, 2, 4, 3, 9, 6, 9 respectively.

Richard Nowakowski, Dalhousie University
Stephen Finbow, St. Francis Xavier University
Mafia

Given a graph G, all the vertices are originally 'dark'. The cops choose vertices and the robbers choose vertices. If a cop passes the vertex he is on becomes light. If a robber is on a light vertex and he passes then the vertex turns dark. The cops only have information about the robber's whereabouts from light vertices. Hence, a cop and robber can be on the same dark vertex and the robber is not caught. (For example, the robber is on the light vertex x and the robber is on an adjacent vertex y and the robber passes, x goes dark and even if the cop moves to y the robber is now 'hidden' on x even though the cop knows where the robber is.)

a) Characterize those graphs in which one cop can capture 1 robber.
b) Characterize those graphs in which one cop can capture any number of robbers.

Richard Nowakowski, Dalhousie University
Shannon Fitzpatrick, University of Prince Edward Island
Cops and Robber with signal delay

a) Play Cops and Robber but the Immediately before the cops move they are informed of the robber's position on the previous turn. They do know immediately if they are on the same vertex as the robber.

i) Characterize the copwin graphs

b) The robber sends out a signal that propagates out to the k-th neighbourhood each time the robber moves. The cop that intercepts the signal knows the distance of the robber but not necessarily the position. For example on a path, if the robber at distance 4 from the cop moves toward the cop and the cop remains static, he will get two signals at once.

i) Characterize the copwin graphs.
ii) Is the number of cops smaller than a smallest resolving set?

Nicolas Nisse, INRIA Sophia Antipolis
Cops and Fast Robber

Consider the cops and robber game with speed. Rules are the same as usual but at each step, the robber can move along at most $s \geq 1$ edges and each cop can move along at most $s' \geq 1$ edges. If $s = s' = 1$, this is the classical game of Quilliot/Nowakowski and Winkler. Let $c_{s,s'}(G)$ be the smallest number of cops with speed s' needed to capture a robber with speed s.

a) Let G_n be the n-square grid ($n \geq 2$), i.e., with n^2 nodes. What is the value of $c_{2,1}(G_n)$? (It is known that, for any $s > s'$, $\Omega(\sqrt{\log n}) = c_{s,s'}(G_n) = O(n)$)

b) $c_{s,1}(G)$ can be computed in polynomial-time in interval graphs. What about other graph classes?

Nicolas Nisse, INRIA Sophia Antipolis
Another variant of graph searching

Consider the following variant of graph searching. An edge is cleared either if an agent slides along it or if both its ends are occupied (as in mixed-search). A clear edge is re-contaminated if it is incident to a contaminated edge and their common node is not occupied (classical recontamination). Let G be a graph and $k \geq 1$. A strategy for k agents starts by placing the k agents on k **distinct** nodes of G. Then, sequentially, an agent can slide from node u to node v **only if** v **is not occupied**. In other words, a strategy for k agents is defined by a set of k initial nodes and by a sequence of sliding (one agent slides at each step) that ensures that no two agents can simultaneously occupy a same node.

Let $xs(G)$ be the smallest k such that there is a strategy that clears all edges of G using k agents. It is known that, for any graph G, $s(G) - 1 \leq xs(G) \leq (\Delta - 1)s(G)$ where Δ is the maximum degree of G and $s(G)$ is the mixed-search number of G. There is a Parson-like characterization of trees T with $xs(T) = k$ and thus $xs(T)$ can be computed in polynomial-time in trees.

a) What is the complexity of computing xs?

b) If it is NP-hard can you give a polynomial-time approximation?

c) What about other graph classes?

Ben Seamone, Universite de Montreal
Cops and Robbers on Geometric Spanners

We construct graphs and digraphs on a set S of n points (vertices) in the plane. Adjacency is defined for $p \in S$ by dividing the plane into k regular cones having apex p, and an arc is added from p to q if q is the "nearest point" in C to p. The **directed Yao graph**, $\overrightarrow{Y_k}$, defines the "nearest point" to be the one with minimal distance in the L_2 metric (i.e. shortest Euclidean distance). In the **directed Theta-graph** $\overrightarrow{\Theta_k}$, the "nearest point" to p is the one whose projection onto the bisecting ray of C is minimal in the L_2 metric.

The underlying undirected graphs of $\overrightarrow{Y_k}$ and $\overrightarrow{\Theta_k}$ are denoted Y_k (**Yao graph**) and Θ_k (**Theta-graph**), respectively. These undirected graphs are of particular interest since the shortest paths in Y_k and Θ_k between two points have length no more than a constant times the Euclidean distance between them for large enough k (i.e. Y_k and Θ_k are **geometric spanners**).

Note that every vertex of $\overrightarrow{Y_k}$ has out-degree at most k but may have unbounded in-degree. The **directed Yao-Yao graph**, $\overrightarrow{YY_k}$, is the subdigraph of $\overrightarrow{Y_k}$ having bounded in-degree that is constructed as follows: for each $p \in S$ and each cone C with apex p, all but the shortest incoming arcs are removed. The underlying undirected graph of $\overrightarrow{YY_k}$ is denoted YY_k. It is not known whether or not YY_k is a geometric spanner.

Problem 1. For a given $k \in \mathbb{Z}^+$, determine the cop number of G if $G \in \{\Theta_k, Y_k, YY_k, \overrightarrow{\Theta_k}, \overrightarrow{Y_k}, \overrightarrow{YY_k}\}$.

Each of these graphs generalize naturally to higher dimension $d > 2$ and to arbitrary metric spaces.

Problem 2. Solve Problem 1 for higher dimensions and/or other metrics.

Lawrence Erickson, University of Illinois at Urbana-Champaign
Counting moving bodies with sparse sensor beams

Consider a directed graph G with no sinks. A set of m bodies is distributed among the vertices of G. The locations of these m bodies are initially unknown. When a body moves between vertices, the edge traversed is returned as a sensor reading. Given G, an initial distribution d of m bodies, and a movement model for the bodies, what is the expected number of sensor readings required to determine the number of bodies in each vertex? Let this number be denoted by the random variable $X(G, d)$.

It is known that the accumulated sensor readings provide enough data to determine a count of the bodies in each vertex if and only if each vertex has been empty at least once.

If the movement model causes each body to have an equal chance of being the next one to move, regardless of earlier movements, then

$$mH_m \leq E[X(G,d)] \leq \frac{m^{\frac{3}{2}} e^m}{\sqrt{2\pi} e^{\frac{1}{12m+1}}}$$

with $H_m = \sum_{i=1}^{m} \frac{1}{m}$. The lower bound is sharp, as m disjoint directed 2-cycles with one body in each produces an expectation of mH_m. If, however, the initial distribution puts all bodies in a single 2-cycle, then the expectation is exponential in m, indicating the existence of a "phase transition" between polynomial and exponential for different distributions of bodies in the same graph.

Open problems include determining the properties (expectation, variance, etc.) of $X(G,d)$ for different movement models, specific graphs, and specific starting distributions. Also, for the movement model in which each body has an equal probability of being the next to move, what conditions on d cause $E[X(G,d)]$ to become exponential (in m) as opposed to polynomial?

This problem was studied in Erickson and LaValle (2012, WAFR).

Dimitrios M. Thilikos, National and Kapodistrian University of Athens

Consider two version of searching on a graph:

- Inert invisible fugitive game.
- Agile Visible fugitive game.

Both problems are known to be monotone and the minimum number of searchers of a winning strategy is equal to the tree width of the graph. However, when the same games are defined in directed graphs the two parameters are different and both non-monotone. That ways three questions appear:

- What is the difference between the parameter corresponding to the Inert invisible fugitive game on directed graphs and its monotone counterpart?
- What is the difference between the parameter corresponding to the Agile Visible fugitive game on directed graphs and its monotone counterpart?
- What is the difference between between the parameter corresponding to the monotone Inert invisible fugitive game on directed graphs and its monotone Agile Visible counterpart?

PROGRESS

- Capture time for cubes. (Bonato, Gordinowicz, Kinnersley, Pralat)
- Cleaning process in which more than one brush can traverse an edge. (Gordinowicz, Pralat)
- Cops and robbers playing on edges. (Dudek, Gordinowicz, Pralat)
- Searching for a round trip in a simple grid (Diaz, Stacho, Widmayer)
- Boolean Cops and Robber (Kinnersley, West)
- A firefighter variation (Finbow, Messinger, Seamone)
- Fractional chip firing (Finbow, Messinger, Seamone)