

# Groups and Geometries

Sep 2 - Sep 7, 2012

## MEALS

\*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday

\*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday

\*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday

Coffee Breaks: As per daily schedule, in the foyer of the TransCanada Pipeline Pavilion (TCPL)

**\*Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

## MEETING ROOMS

All lectures will be held in the new lecture theater in the TransCanada Pipelines Pavilion (TCPL). LCD projector and blackboards are available for presentations.

## SCHEDULE

### Sunday

**16:00** Check-in begins (Front Desk - Professional Development Centre - open 24 hours)

**17:30–19:30** Buffet Dinner, Sally Borden Building

**20:00** Informal gathering in 2nd floor lounge, Corbett Hall (if desired)

Beverages and a small assortment of snacks are available on a cash honor system.

### Monday

**7:00–8:45** Breakfast

**8:45–9:00** Introduction and Welcome by BIRS Station Manager, TCPL

**9:10–10:00** R. Guralnick: Maximal Subgroups of Finite Groups

Coffee Break, TCPL

**10:30–11:20** P.H. Tiep: Linear groups and conjectures of Kollar and Larsen

**11:30–13:30** Lunch

**13:45–14:35** M. Aschbacher: Fusion systems and groups of component type

Coffee Break, TCPL

**15:10–16:00** A. Chermak: The normal structure of linking systems

**16:15–17:05** R. Solomon: Characterizing Lie Type Groups

**17:30–19:30** Dinner

### Tuesday

**7:00–9:00** Breakfast

**9:00–9:30** B. Baumeister: Permutation Groups and Applications

**9:40–10:10** R. Waldecker: Special primitive pairs in finite groups

Coffee Break, TCPL

**10:40 - 11:30** J. Hall: Algebras from groups and geometry

**11:30–13:00** Lunch

**13:00–14:00** Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall

**14:00** Group Photo; meet in foyer of TCPL (photograph will be taken outdoors so a jacket might be required).

**14:15 - 15:05** R. Weiss: The local structure of Bruhat-Tits buildings

Coffee Break, TCPL

**15:35–16:25** K. Magaard: Groups of even type which are not of even characteristic

**16:35 - 17:25** L. Pyper: Growth in linear groups

**17:30–19:30** Dinner

## Wednesday

- 7:00–9:00** Breakfast  
**9:00–9:30** T. Burness: Bases for algebraic groups  
**9:40–10:10** N. Gill: The width of a finite simple group  
Coffee Break, TCPL  
**10:40 - 11:30** Chr. Parker: Groups which are almost of Lie type  
**11:30–13:30** Lunch  
Free Afternoon  
**17:30–19:30** Dinner

## Thursday

- 7:00–9:00** Breakfast  
**9:00–9:30** A. Devillers: Automorphisms and opposition in twin buildings  
**9:40–10:10** K. Struyve: Galois descent of Bruhat-Tits buildings  
Coffee Break, TCPL  
**10:40 - 11:30** E. Henke: Cohomology  $F$ -Isomorphism and Fusion in Finite Groups  
**11:30–13:30** Lunch  
**13:30 - 14:20** Ch. Praeger: Coprime subdegrees for primitive permutation groups and completely reducible linear groups  
Coffee Break, TCPL  
**14:40–15:30** P.E. Caprace: Simple locally compact groups and branching  
**15:40 - 16:30** A. Cohen: Constructing Riemann surface models from regular maps  
**16:40–17:30** H. van Maldeghem: Characterizations of groups by geometries and geometries by groups  
**17:30–19:30** Dinner

## Friday

- 7:00–9:00** Breakfast  
**9:00–9:50** G. Malle: Variations on the Baer-Suzuki theorem  
**10:00: 10:30** F.G. Timmesfeld: Subspaces of the root-group geometry

Coffee Break, TCPL

- 11:30–13:30** Lunch

**Checkout by  
12 noon.**

\*\* 5-day workshop participants are welcome to use BIRS facilities (BIRS Coffee Lounge, TCPL and Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. \*\*

# Groups and Geometries

## Sep 2 - Sep 7, 2012

### ABSTRACTS (in alphabetic order by speaker surname)

Speaker: **Michael Aschbacher** (California Institute of Technology)

Title: *Fusion systems and groups of component type*

Abstract: I'll discuss a speculative program to use the theory of fusion systems to simplify that part of the classification of the finite simple groups dealing with the groups of component type.

Speaker: **Barbara Baumeister** (Universität Bielefeld)

Title: *Permutation Groups and Applications*

Abstract: The theory of permutation groups has many applications in quite a few mathematical areas. In this talk we will introduce and discuss the notation of a permutation polytope which is for instance of importance in combinatorial optimization or in mathematical physics. We will present examples and debate characterizations.

Speaker: **Tim Burness** (University of Southampton)

Title: *Bases for algebraic groups*

Abstract: Let  $G$  be a permutation group on a set  $X$ . A subset  $B$  of  $X$  is a base for  $G$  if the pointwise stabilizer of  $B$  in  $G$  is trivial. The minimal size of a base is called the base size of  $G$ . Bases for finite permutation groups have been widely studied since the nineteenth century, with many new results and applications in recent years. In contrast, very little is known about bases for infinite permutation groups, apart from a few special cases.

In joint work with Guralnick and Saxl we study bases for primitive actions of simple algebraic groups over algebraically closed fields, computing the exact base size in almost all cases. We also introduce two new base measures, which arise naturally in this setting, and we compute these numbers too. As an application we obtain new results on base sizes for the corresponding finite groups of Lie type. In this talk I will present the main results of this joint work, and I will briefly explain some of the key ingredients.

Speaker: **Pierre-Emanuel Caprace** (Universite Catholique de Louvain)

Title: *Simple locally compact groups and branching*

Abstract: The global structure of a Lie group is largely determined by its local structure, encoded in the Lie algebra. In the case of locally compact groups that are non-discrete and totally disconnected, the local structure is given by a commensurability class of profinite groups. The goal of the talk is to illustrate that, when the ambient group is simple and compactly generated, there is also a local-to-global correspondence in that case, although it is not as tight as in Lie theory. This is a joint work with Colin Reid and George Willis.

Speaker: **Andy Chermak** (Kansas State University)

Title: *The normal structure of linking systems*

Abstract: We discuss the category of partial groups and the subcategory of objective partial groups. Among these are the finite group-like "centric linking systems". The talk will focus on normal subsystems of linking systems and on the problem of how to view these as linking systems in their own right. The results to be outlined are in analogy with, Aschbacher's theory of normal subsystems of fusion systems.

Speaker: **Arjeh Cohen** (Technische Universiteit Eindhoven)

Title: *Constructing Riemann surface models from regular maps*

Abstract: We describe computational methods for finding algebraic models of the smooth complex projective curves (Riemann surfaces) connected to regular maps. The methods have been applied to all regular maps of genus at most 6 that occur in the list of Marston Conder and Peter Dobcsanyi. Also, the first Hurwitz triplet (curves of genus 14 with automorphism group  $\mathrm{PSL}(2, 13)$ ) has been handled successfully; this led to the closure of an open end in the classification by Magaard and Volklein of Hurwitz groups acting transitively on the Weierstrass points on the curve. The work reported on is joint with Maxim Hendriks.

Speaker: **Alice Devillers** (University of Western Australia)

Title: *Automorphisms and opposition in twin buildings*

Abstract: Opposition in twin buildings generalises the notion of opposition in spherical buildings. With James Parkison and Hendrik Van Maldeghem, we looked at automorphisms mapping some/all residues to opposite residues. For instance we proved that an automorphism of a thick twin building (swapping the two halves) always maps at least one spherical residue to an opposite. However, if the building is also locally finite and 2-spherical, then not every residue of a given type can be mapped to an opposite.

Speaker: **Nick Gill** (Open University)

Title: *The width of a finite simple group*

Abstract: I describe recent work with Pyber, Short and Szabo on the "Product Decomposition Conjecture" of Liebeck, Nikolov and Shalev. Given a finite simple group  $G$  and a subset  $A$  of  $G$  we want to write  $G$  as a product of few conjugates of  $A$ . To do this we appeal to the recent Product Theorem, and we uncover some interesting new connections between classical additive combinatorics and normal subsets of a group.

Speaker: **Robert Guralnick** (University of Southern California)

Title: *Maximal Subgroups of Finite Groups*

Abstract: We will discuss some results about counting the number of maximal subgroups and the number of conjugacy classes of maximal subgroups of finite groups. The problem naturally splits up into first looking at maximal subgroups of almost simple groups and secondly getting certain bounds on the size of first cohomology groups. We will discuss some recent results (with Larsen and Tiep) on the first problem as well as some results about cohomology. In particular, Wall's conjecture (that the number of maximal subgroups of a finite group  $G$  is less than  $|G|$ ) is false.

Speaker: **Jon Hall** (Michigan State University)

Title: *Algebras from groups and geometry*

Abstract: In 1913 Study discussed the triality properties of hyperbolic 8-space. In 1925 Cartan discussed this geometric triality in the context of the automorphism groups of Lie groups of type  $D_4$  and noted a connection with the Cayley-Graves octonions. In 1935 Moufang further connected alternative algebras with projective planes possessing many relations. We will discuss these and more general concepts of triality along with other situations where algebras are presented or explained by geometries and groups.

Speaker: **Ellen Henke** (University of Copenhagen)

Title: *Cohomology  $F$ -Isomorphism and Fusion in Finite Groups*

Abstract: This talk is about a shared project with Dave Benson and Jesper Grodal. We prove an elementary group theoretical result which has implications for mod  $p$  cohomology and higher chromatic cohomology theories. More precisely, we prove the following result: Suppose we are given a finite group  $G$ , an odd prime  $p$ , and a subgroup  $H$  of  $G$  containing a Sylow  $p$ -subgroup of  $G$ . Then  $H$  controls fusion in  $G$  if and only if it controls fusion of elementary abelian subgroups. The analogous result is true for  $p = 2$  if one considers abelian subgroups of exponent at most 4 instead of elementary abelian subgroups. The proof is more easily carried out in the category of fusion systems than in the category of groups.

Speaker: **Kay Magaard** (University of Birmingham)

Title: *Groups of even type which are not of even characteristic*

Abstract: In classifying the finite simple groups one distinguishes between "even" and "odd" groups. Exactly where the "even/odd" subdivision is made depends on whether semisimple, or unipotent elements, are the primary focus of investigation. While semisimple methods naturally lead to the concept of groups of even type, unipotent methods, notably the amalgam method, naturally lead to define groups of even characteristic. We clarify the relationship between the two concepts by showing that there are exactly six exceptions to the statement that "Simple groups of even type in which all 2-local composition factors are known finite simple groups are of even characteristic". Consequently both, the signalizer functor method and the amalgam method, are now available for use in the classification of groups of even type. This is joint work with Gernot Stroth.

Speaker: **Gunter Malle** (TU Kaiserslautern)

Title: *Variations on the Baer-Suzuki theorem*

Abstract: The well-known Baer-Suzuki theorem asserts that in a finite group any conjugacy class for which all pairs of elements generate a nilpotent group already lies inside a nilpotent subgroup itself. We discuss extensions of this result to pairs of conjugacy classes of finite groups and of algebraic groups. This is joint work with R. Guralnick and P.H. Tiep.

Speaker: **Chris Parker** (University of Birmingham)

Title: *Groups which are almost Lie type*

Abstract: This is a report on joint work with Gernot Stroth. For a prime  $p$ , a  $p$ -subgroup  $Q$  of a finite group  $G$  is called *large* if and only if  $Q = F(N_G(Q))$  and, for all  $1 \neq UZ(Q)$ ,  $N_G(U) \leq N_G(Q)$ . In this talk, I will discuss research which determines those groups  $G$  which have a large subgroup and which in addition have a proper subgroup  $H$  which contains a Sylow  $p$ -subgroup of  $G$  with  $F(H)$  a group of Lie type in characteristic  $p$  and rank at least 3.

Speaker: **Cheryl Praeger** (University of Western Australia)

Title: *Coprime subdegrees for primitive permutation groups and completely reducible linear groups*

Abstract: This work was inspired by a question of Gabriel Navarro about orbit lengths of groups acting on finite vector spaces, and is joint work with Pablo Spiga, Silvio Dolfi and Bob Guralnick. If a finite group  $H$  acts irreducibly on a finite vector space  $V$ , then we proved that for every pair of non-zero vectors, their orbit lengths  $a, b$  have a non-trivial common factor.

This could be interpreted in the context of permutation groups. The group  $VH$  is an affine primitive group on  $V$  and  $a, b$  are orbit lengths of the point stabiliser  $H$ , that is,  $a$  and  $b$  are subdegrees of  $VH$ . This raises a question about subdegrees for more general primitive permutation groups. Coprime subdegrees can arise, but (we show) only for three of the eight types of primitive groups. Moreover it is never possible to have as many as three pairwise coprime subdegrees. All proofs depend on the finite simple group classification.

Speaker: **Laci Pyber** (Renyi Institute of Mathematics Budapest)

Title: *Growth in linear groups*

Abstract: Let  $S$  be an inverse-closed subset of  $GL(n, F)$  satisfying  $|S^3| < K|S|$  for some  $K > 1$ , where  $F$  is an arbitrary field. Then  $S$  is contained in the union of polynomially many (more precisely  $K^c(n)$ ) cosets of a finite-by-soluble subgroup  $G$  normalised by  $S$ . Moreover  $G$  has a finite subgroup  $P$  normalised by  $S$  such that  $G/P$  is soluble and  $S^3$  contains a coset of  $P$ . This includes the Product theorem for finite simple groups of bounded rank proved in 2010 by Breuillard-Green-Tao and Pyber-Szabo' and various other earlier results. Joint work with Endre Szabo'.

Speaker: **Ron Solomon** (Ohio State University)

Title: *Characterizing Lie Type Groups*

Abstract: I will provide a progress report on joint work with Richard Lyons directed towards a proof of the following theorem.

Theorem: Let  $G$  be a  $K$ -proper finite simple group of  $p$ -generic type for some prime  $p$ . If  $p$  is odd, suppose also that  $G$  is of even type and does not contain a strong  $p$ -uniqueness subgroup. Then either  $G$  is an alternating group of degree  $n > 12$ , or  $G$  is a simple group of Lie type in characteristic  $p$  of BN-rank at least 3.

I will also report on the following theorem proved recently with Andy Woldar.

Theorem: Let  $G$  be a finite group and  $S$  a finite simple group of Lie type. If  $G$  and  $S$  have isomorphic commuting graphs, then they are isomorphic as groups.

Speaker: **Koen Struyve** (Ghent University)

Title: *Galois descent of Bruhat-Tits buildings*

Abstract: Bruhat-Tits buildings are a class of affine buildings associated to certain classical, algebraic and mixed groups defined over (skew) fields with a complete valuation. An open question in Bruhat-Tits theory concerned the existence of Bruhat-Tits buildings for these groups, with the remaining open cases pertaining to certain exceptional groups of relative rank 1 and 2.

In this talk we provide a general solution of this problem using combinatorial and geometric methods. (Joint work with B. Mühlherr and H. Van Maldeghem)

Speaker: **Pham Huu Tiep** (University of Arizona)

Title: *Linear groups and conjectures of Kollar and Larsen*

Abstract: I will describe my joint work with R. M. Guralnick on some conjectures of Kollar and Larsen, which are motivated by various applications in algebraic geometry, in particular, stability of vector bundles, quotients of Calabi-Yau varieties, and (non-)existence of crepant resolutions.

Speaker: **Franz Timmesfeld** (Universität Gießen)

Title: *Subspaces of the root-group geometry*

Abstract: The root-group geometry of a Lie-type group has as points the long root-groups and lines 2-dim. subgroups partitioned by points. A geodesically closed subset of points is called a complete subspace, if it generates an abelian subgroup. The definition of arbitrary subspaces is slightly more technical. We will discuss: Opposition of subspaces Subgroups generated by opposite subspaces Consequences for the embedding of classical in exceptional groups.

Speaker: **Hendrik van Maldeghem** (Ghent University)

Title: *Characterizations of groups by geometries and geometries by groups*

Abstract: We present two recent characterizations of groups and geometries related to buildings. The first one characterizes the "standard orbits" of certain modular representations of the groups of the second row of the Freudenthal-Tits Magic Square by a simple extension of the Mazzocca-Melone axioms that were designed for quadric Veronesean varieties (corresponding, however, to the first cell of the second row of the FT Magic Square!). Recall that this second row consists of groups, building and Lie algebras of types  $A_2$ ,  $A_2 \times A_2$ ,  $A_5$  and  $E_6$ . The last case is still ongoing research at the time of this writing.

The second characterization is one of the finite Hermitian unital in the spirit of the "Moufang property" for projective planes (and, more generally, twin buildings). It says that an abstract finite unital is Hermitian if and only if every point is the center of a transitive translation group, where a translation fixes all blocks through the center, and transitivity refers to the set of points on any block through the center, except for the center itself. Just as in the usual Moufang case for buildings, this implies, although not so immediate, that this action is regular.

Speaker: **Rebecca Waldecker** (Universität Halle)

Title: *Special primitive pairs in finite groups*

Abstract: This talk is about a particular kind of primitive pairs in finite groups. Primitive pairs play a role whenever local analysis leads to pairs of distinct maximal subgroups  $M_1$  and  $M_2$  of a finite group  $G$  such that there exists a prime  $p$  with  $F^*(M_1) = O_p(M_1)$  and  $F^*(M_2) = O_p(M_2)$ . There are results for such

a situation (for example Glauber's  $ZJ$ -Theorem or Flavell's  $K^\infty$ -Theorem), but sometimes neither is applicable. I will give an example of a problematic case and explain how a new result about primitive pairs with special properties solved this problem.

Speaker: **Richard Weiss** (Tufts University)

Title: *The local structure of Bruhat-Tits buildings*

Abstract: Roughly speaking, Bruhat-Tits buildings are classified by spherical buildings defined over a field  $K$  that is complete with respect to a discrete valuation. In particular, each spherical building  $\Delta$  defined over a complete field  $K$  is the "building at infinity" of a unique Bruhat-Tits building  $X$ , and the residues of the building  $X$  are spherical buildings defined over the residue field  $\bar{K}$ . Bruhat-Tits buildings are not uniquely determined by their residues (in contrast to spherical buildings), but their residues are nevertheless an important structural feature of these buildings. We will discuss efforts to give a complete description of all the possibilities that arise for each family of spherical buildings  $\Delta$  defined over a complete field  $K$ . This is joint work with Bernhard Mühlherr and Holger Petersson.