Model reduction in continuum thermodynamics: Modeling, analysis and computation
Sep 16 - Sep 21, 2012

MEETING ROOMS

All lectures will be held in the new lecture theater in the TransCanada Pipelines Pavilion (TCPL). LCD projector and blackboards are available for presentations.

SCHEDULE

Sunday	September 16, 2012
16:00	Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
17:30–19:30	Buffet Dinner, Sally Borden Building
20:00	Informal gathering in 2nd floor lounge, Corbett Hall Beverages and a small assortment of snacks are available on a cash honor system.
Monday	September 17, 2012
7:00–8:45	Breakfast
8:45–9:00	Introduction and Welcome by BIRS Station Manager, TCPL
9:00–10:00	Josef Málek
10:00–10:30	Coffee Break, TCPL
10:30–11:30	Rupert Klein
11:30–13:00	Lunch
13:00–13:50	Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
14:00–15:00	Eduard Feireisl
15:00–15:30	Coffee Break, TCPL
15:30–16:30	Antonín Novotný
16:30–17:30	David Silvester
17:30–18:00	Piotr Mucha
18:00–19:00	Dinner

Tuesday	September 18, 2012
7:00–9:00	Breakfast
9:00–10:00	K.R. Rajagopal
10:00–10:30	Coffee Break, TCPL
10:30–11:30	Martin Gander
11:30–12:00	Jaroslav Hron
12:05	Group Photo
12:10–13:30	Lunch
15:00–15:30	Coffee Break, TCPL
15:30–16:30	Martin Vohralík
16:30–17:00	Ondřej Šrámek
17:00–17:30	Milan Pokorný
17:30–18:30	Dinner
20:00–20:30	Martin Heida
20:30–21:00	Piotr Minakowski

Wednesday	September 19, 2012
7:00–9:00	Breakfast
9:00–10:00	Alexis Vasseur
10:00–10:30	Coffee Break, TCPL
10:30–11:00	Miroslav Bulíček
11:00–11:30	Akif Ibragimov
11:30–12:00	Agnieszka Świerczewska-Gwiazda
12:00–13:00	Lunch
	Free Afternoon
18:30–19:30	Dinner

Thursday	September 20, 2012
7:00–9:00	Breakfast
9:00–10:00	Song Jiang
10:00–10:30	Coffee Break, TCPL
10:30–11:00	Šárka Nečasová
11:00–11:30	Piotr Gwiazda
11:30–12:30	Lunch
15:00–15:30	Coffee Break, TCPL
15:30–16:00	Jan Březina
16:00–16:30	Ondřej Kreml
16:30–17:00	Jiří Neustupa
17:30–18:30	Dinner

Friday	September 21, 2012
7:00–9:00	Breakfast
9:00-10:00	ZdeněK Strakoš: Bits of scattered (summarizing?) comments
10:00-10:30	Coffee break
10:30-11:00	Discussion and closing comments
11:30–12:30	Lunch
	Checkout by 12 noon

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ABSTRACTS
(in alphabetic order by speaker surname)

Speaker: **Jan Březina** (Kyushu University, Japan)
Email: b-jan@math.kyushu-u.ac.jp
Title: *On asymptotic behavior of solutions to the compressible Navier-Stokes equation around a time-periodic parallel flow*
Abstract: Under appropriate smallness conditions on Reynolds and Mach numbers we show the global in time existence of strong solutions to the compressible Navier-Stokes equation around time-periodic parallel flows in R^n, $n \geq 2$. Furthermore, we study the asymptotic behavior of these solutions and prove that the cases $n = 2$ and $n \geq 3$ are considerably different. When $n = 2$, the asymptotic leading term is given by a solution of one-dimensional viscous Burgers equation multiplied by a predetermined time-periodic function. When $n \geq 3$, the asymptotic leading term is given by a solution of $n - 1$-dimensional heat equation with convective term multiplied by a predetermined time-periodic function.

Speaker: **Miroslav Bulíček** (Charles University in Prague, Czech Republic)
Email: mbul8060@karlin.mff.cuni.cz
Title: *Regularity issues in systems describing flows of incompressible fluids*
Abstract: We consider a variety of models describing flows of incompressible fluids. We show that models for heat conducting fluids and for models of turbulence share some important similarities. Namely, the presence of the term with the "critical" growth on the right hand side causes a lot of difficulties in developing not only regularity but also the existence theory for given models. While these problems can be overcome in the existence theory by using a "new" proper variable - global energy, the regularity of the systems remains in almost all cases open. We present some particular results and show under which assumptions on the structure of the Cauchy stress one can established the existence of the strong or even classical solution for such models.

Speaker: **Eduard Feireisl** (Institute of Mathematics, Academy of Sciences of the Czech Republic)
Email: feireisl@math.cas.cz
Title: *Complete fluid systems, the state of art*
Abstract: We survey the recent development of the mathematical theory of complete fluid systems, meaning those satisfying all basic principles of classical thermodynamics. Emphasis is put on the results that are not restricted by the size of the data and/or the length of the time interval on which the problem is studied. In particular, we discuss several concepts of solutions and applications to problems of asymptotic behavior, singular limits and scale analysis.

Speaker: **Martin J. Gander** (University of Geneva, Swiss)
Email: Martin.Gander@unige.ch
Title: *Coarse Space Components for Domain Decomposition Methods*
Abstract: It is well known that for domain decomposition methods to be scalable, one needs to add a coarse grid. Without, the convergence of the subdomain iterations becomes slower and slower, the more processors, and hence subdomains, one uses. There are well established convergence results for these so called two level methods, for both overlapping and non-overlapping methods. These results however always contain constants which remain unspecified.

We explain in this talk how specific choices of coarse space components influence these constants. We start with a simple, one dimensional model problem and develop a coarse space correction which leads together with a Schwarz method to convergence after one coarse correction step. This needs both specific placement of grid points, and well chosen shape functions for the coarse grid correction. We then show that approximations already lead to very good coarse grid corrections, and how this idea can be generalized to two dimensional problems.

Next, we show that also interface operators can contain a coarse space component. In particular we will show a domain decomposition method which converges in two iterations independently of the subdomain decomposition, i.e. even if cross points are present. We conclude by an algebraic version for banded matrices, and show an outlook on future work.

Speaker: **Piotr Gwiazda** (University of Warsaw, Poland)
Email: pgwiazda@mimuw.edu.pl
Title: *First order hyperbolic conservation laws containing implicit relation*
Abstract: The first problem we will study is a system describing flow of granular avalanches. The derivation of considered continuum flow models essentially bases on the fact that the characteristic length in the flowing direction is in general much larger than the thickness of an avalanche. Such an approach resulted in depth-averaged equation governed by generalized system of shallow water equations (Saint-Venant equations). The evolution of granular avalanches along an inclined slope is described by the mass and momentum conservation laws. Among the variety of models capturing the dynamics of granular flow, the Savage-Hutter mode is one of the most commonly used frameworks. The Savage-Hutter model covers the process of fast moving avalanche, where the contribution of kinetic energy is significant. The equations are obtained as a multi-scale limit of the three-dimensional free surface incompressible Navier-Stokes equations through depth-averaging process. A crucial feature distinguishing the equations describing an avalanche flow from a shallow water flow is that granular media have the ability to remain static even along an inclined surface (Coulomb's friction law). This is an example of implicit relation, which now appears in a source term, cf. [4, 5]. The system contains a multi-valued term. An appearance of inclusion in the system is strictly related with the Coulomb's friction law. Next example we are interested in is a scalar conservation law in an arbitrary dimension d with a discontinuous flux F

$$\begin{aligned} u_t + \operatorname{div} F(x,u) &= 0 \quad \text{in } (0,\infty)\times\mathbb{R}^d, \\ u(0,\cdot) &= u_0 \quad \text{in } \mathbb{R}^d. \end{aligned} \tag{1}$$

The flux is supposed to be a discontinuous function in the spatial variable x and in an unknown function u. Under some additional hypothesis on the structure of possible discontinuities, we formulate an appropriate notion of entropy solution and establish its existence and uniqueness. The framework for proving the existence and uniqueness of entropy weak solutions is provided by the studies on entropy measure-valued solutions, cf. [3] and may be viewed as a corollary of the uniqueness theorem for entropy measure-valued solutions. The talk is based on joint works with M. Bulíček, P. Gwiazda and J. Málek, [1, 2].

References

[1] M. Bulíček, P. Gwiazda, J. Málek, and A. Świerczewska Gwiazda. On scalar hyperbolic conservation laws with a discontinuous flux. *Math. Models Methods Appl. Sci.*, 21(1):89–113, 2011.

[2] M. Bulíček, P. Gwiazda, and A. Świerczewska Gwiazda. Multi-dimensional scalar conservation laws with fluxes discontinuous in the unknown and the spatial variable, to appear in *Math. Models Methods Appl. Sci.*.

[3] R. J. DiPerna. Measure-valued solutions to conservation laws. *Arch. Rational Mech. Anal.*, 88(3):223–270, 1985.

[4] P. Gwiazda. An existence result for a model of granular material with non-constant density, Asympt. Anal. 30 (2002), no. 1, 43-60

[5] P. Gwiazda, A. Świerczewska. Multivalued equations for granular avalanches. Nonlinear Anal., Theory Methods Appl. 62, No.5 (A), 895-912 (2005)

Speaker: **Martin Heida** (Technical University of Dortmund, Germany)
Email: martin-heida@web.de
Title: *On the Derivation of Thermodynamically Consitent Boundary Conditions for the Cahn-Hilliard equation*
Abstract: We put the assumption of maximum rate of entropy production by Rajagopal and Srinivasa to an integral framework providing us the possibility to derive thermodynamically consistent boundary conditions in continuum mechanics. We apply the method to multiphase and multiconstituent fluids in the Cahn-Hilliard framework and give an outlook on the mathematical structure and the tools that can be used to treat these equations.

Speaker: **Jaroslav Hron** (Charles University in Prague, Czech Republic)
Email: hron@karlin.mff.cuni.cz
Title: *Implicitly constituted materials: mixed formulations, numerical solutions and computations*
Abstract: Implicit constitutive theory can be viewed as generalization of classical models that is based on the idea of expressing the response of bodies by an implicit relation between the stress and appropriate kinematical variables. It is capable of describing some of the material properties that explicit models seem unable to describe. It can also be viewed as an approach that lies between the classical primal and dual formulations of explicit models.

It also provides a less standard interesting structure of the governing equations. This specific structure can be preserved in the discretization method and used into an advantage. We will present several examples emphasizing the advantages of this framework at the level of modeling of material responses and the numerical solution of resulting discrete systems by finite element method.

Speaker: **Akif Ibragimov** (Texas Tech University, Lubbock, U.S.A.)
Email: akif.ibraguimov@ttu.edu
Title: *Non-Darcy Flows in the porous media for compressible fluids and application*
Abstract: The article is dedicated to the study of generalized Forchheimer flows in porous media for slightly compressible fluid and it application in reservoir engineering. Some fundamental properties for solution of corresponding non-linear parabolic equation have been obtained for any degree in the generalized Forchheimer polynomial. This properties have been used to study different industrial problems. In particular long term asymptotic of diffusive capacity, the integral characteristic of the domain with respect to non-linear Forchheimer flow in porous media. Conditions on the boundary are given in terms of the total flux and constraints on the trace of the pressure on the boundary. It is proved that if total flux stabilizes then the difference between pressure average inside domain and on the boundary stabilizes as well. This result can be applied to calculate productivity index of the well, an important characteristic of the well performance. To obtain the main theorem refined comparison of fully transient and pseudo steady state pressure (the time derivative of pressure is constant) was performed. These results can be effectively used in reservoir engineering and can also be applied in other problems modeled by non-linear diffusive equations. This is a joint work with Eugenio Aulisa, Luan Hoang, Lidia Bloshnskaya (all from Texas Tech University, Lubbock, USA)

Speaker: **Song Jiang** (Institute of Applied Physics and Computational Mathematics, Beijing, China)
Email: jiang@iapcm.ac.cn
Title: *Incompressible limit of the planar compressible ideal magnetohydrodynamic equations*

Abstract: We study the incompressible limit of the planar compressible ideal magnetohydrodynamic equations with general initial data in a one dimensional torus. The existence of classical solutions on a time interval independent of the Mach number is proved and the incompressible limit is rigorously verified. (Joint work with F.C. Li and Q.C. Ju)

Speaker: **Rupert Klein** (Freie Universität Berlin, Germany)
Email: `rupert.klein@math.fu-berlin.de`
Title: *Scaling regimes and multiple scales problems of atmospheric flows*
Abstract: Typical textbooks of theoretical meteorology cover a wide range of scale-dependent atmospheric flow phenomena. For each flow regime they present a different set of reduced model equations. The first part of my lecture will provide an introductory overview of this collection of model equations from a unified mathematical perspective based on ideas of multiple scales asymptotics.

In the second and main part of the lecture I will discuss atmospheric flows on length and time scales relevant to cloud formation and other small-scale processes. These processes belong to an interesting three-time-scale limit for vanishing Mach number. Careful formal asymptotic analyses allow us to assess the regimes of validity of the Lipps-and-Hemler anelastic and Durran's pseudo-incompressible models. The pseudo-incompressbile model family turns out to cover a broader range of physical situations.

If time permits, I will also summarize the recent development of an approach to hurricane modelling that borrows ideas from the theory of slender vortex filaments in classical fluid mechanics.

Speaker: **Ondřej Kreml** (Institute of Mathematics, Academy of Sciences of the Czech Republic)
Email: `kreml@math.cas.cz`
Title: *Steady Navier-Stokes-Fourier system with nonlinear dependence of viscosity on temperature*
Abstract: We study the steady compressible Navier–Stokes–Fourier system in a bounded three–dimensional domain. The pressure p and the internal energy e are related by the constitutive law $p = (\gamma - 1)\varrho e$. Special attention is given to the case where the viscosity $\mu(\vartheta)$ does not behave like a linear function of temperature ϑ, the physically reasonable case $\mu \sim \vartheta^{\frac{1}{2}}$ is included. We prove existence of variational entropy solutions for any $\gamma > 1$. The main novelty of this work is adaptation of technique of special test functions used by Plotnikov and Sokolowski to the case of velocity bounded in $W^{1,p}$ with $p < 2$. This is a joint work with Milan Pokorný.

Speaker: **Josef Málek** (Charles University in Prague, Czech Republic)
Email: `malek@karlin.mff.cuni.cz`
Title: *Continuum Thermodynamics, complete and reduced systems*
Abstract: This introductory lecture aims to outline areas covered during the workshop. We formulate the governing equations for a single continuum as well as for the interacting continua including the balance equations, constitutive equations and the boundary conditions. We also discuss mathematical aspects concerning the complete systems as well as their various reductions that can be of various types. Some examples will be given too.

Speaker: **Piotr Minakowski** (University of Warsaw, Poland)
Email: `minak@mimuw.edu.pl`
Title: *Fluid model of crystal plasticity - numerical computations for compression of a single-slip crystal*
Abstract: The aim is to describe behaviour of the metal under severe plastic deformation (SPD). In the presented approach the material is treated as a highly viscous, incompressible, anisotropic fluid which flows through an adjustable crystal lattice. The detailed description of the model is provided. We consider the compression of a single crystal and the High Pressure torsion experiments. Mixed finite element approach and non-linear Newton solver was used to solve appropriate strongly non-linear system of equations. We compare several approaches to modelling such phenomena, namely investigate different types of constitutive

assumptions, carriers of plastic deformations (particularly dislocations) and elasticity. We will focus on comparison between explicitly and implicitly constituted models.

Speaker: **Piotr Mucha** (University of Warsaw, Poland)
Email: P.Mucha@mimuw.edu.pl
Title: *Inhomogeneous Navier-Stokes equations and jumps of density*
Abstract: I plan to talk about minimal assumptions on regularity of the initial density for the issue of the well posedness to the inhomogeneous Navier-Stokes system for incompressible flows with variable density. Thanks to an approach via the Lagrangian coordinate system we are able to show the existence of unique solutions with positive L_∞ initial density, admitting arbitrary jumps. For global in time existence we are obligated to assume same smallness conditions.

The talk will be based on results joint with Raphael Danchin (Paris):

R. Danchin, P.B. Mucha: A Lagrangian Approach for the Incompressible Navier-Stokes Equations with Variable Density, CPAM 65, 2012,14581480

R. Danchin, P.B. Mucha: Incompressible flows with piecewise constant density, arXiv:1203.1131 (2012).

Speaker: **Šárka Nečasová** (Institute of Mathematics, Academy of Sciences of the Czech Republic)
Email: matus@math.cas.cz
Title: *Weak solutions to the barotropic Navier-Stokes system with slip boundary conditions in time dependent domains*
Abstract: This is a joint work with E. Feireisl, O. Kreml, J. Neustupa and J. Stebel.

We consider the compressible barotropic Navier-Stokes system on time-dependent domains with slip boundary conditions. The global in time weak solutions are obtained.

Speaker: **Jiří Neustupa** (Institute of Mathematics, Academy of Sciences of the Czech Republic)
Email: neustupa@math.cas.cz
Title: *On the Inhomogeneous Steady NavierStokes Problem with the NavierType Boundary Conditions in a 2D Multiply Connected Bounded Domain*
Abstract: We assume that Ω is a bounded domain in $\mathbb{R}^2$ with N holes. We show that there exists an infinite-dimensional manifold $\mathcal{M}$ in $L2(\Omega)$, whose codimension is equal to N, such that the steady NavierStokes problem with inhomogeneous Naviertype boundary conditions has a solution if the body force $\mho$ belongs to $\mathcal{M}$. We discuss the structure of the set of solutions and its dependence on certain N parameters. We show that the problem is not solvable for some configurations of given boundary data and body forces.

Speaker: **Antonín Novotný** (Université du Sud Toulon-Var, France)
Email: novotny@univ-tln.fr
Title: *Navier-Stokes-Fourier system: dissipative solutions, relative entropies, applications*
Abstract: We investigate the Navier-Stokes-Fourier system describing the motion of a compressible, viscous and heat conducting fluid. We discuss existence of dissipative solutions for arbitrary large data. We show that any dissipative solution satisfies a relative entropy inequality. Finally we mention some applications of the relative entropy inequality to the investigation of the weak-strong uniqueness property of dissipative solutions or to the investigation of some types of singular limits.The talk is based on the joint works with Eduard Feireisl.

Speaker: **Milan Pokorný** (Charles University in Prague, Czech Republic)
Email: pokorny@karlin.mff.cuni.cz
Title: *On steady compressible Navier–Stokes–Fourier system*
Abstract: We review the recent results concerning the existence theory for the system of partial differential equations describing steady flow of Newtonian compressible heat conducting fluid in a bounded region $\Omega \subset R^3$

$$\begin{gathered} \operatorname{div}(\varrho \mathbf{u}) = 0, \\ \operatorname{div}(\varrho \mathbf{u} \otimes \mathbf{u}) - \operatorname{div} \mathbf{S}(\vartheta, \nabla \mathbf{u}) + \nabla p(\varrho, \vartheta) = \varrho \mathbf{f}, \\ \operatorname{div}(\varrho E(\varrho, \vartheta, \mathbf{u})\mathbf{u}) = \varrho \mathbf{f} \cdot \mathbf{u} - \operatorname{div}(p(\varrho, \vartheta)\mathbf{u}) + \operatorname{div}(\mathbf{S}(\vartheta, \nabla \mathbf{u})\mathbf{u}) - \operatorname{div} \mathbf{q}(\vartheta, \nabla \vartheta) \end{gathered} \tag{2}$$

with the boundary conditions

$$\begin{gathered} -\frac{\partial \mathbf{q}(\vartheta, \nabla \vartheta)}{\partial \mathbf{n}} + L(\vartheta)(\vartheta - \Theta_0), \\ \mathbf{u} \cdot \mathbf{n} = 0, \\ \alpha(\mathbf{S}(\vartheta, \nabla \mathbf{u})\mathbf{n}) \cdot \boldsymbol{\tau} + (1 - \alpha)\mathbf{u} \cdot \boldsymbol{\tau} = 0, \end{gathered} \tag{3}$$

$\alpha \in [0, 1]$. Here $\mathbf{S}$ is the stress tensor for the Newtonian fluid with possible dependence of viscosity coefficients on temperature, p, the pressure, is assumed in the form

$$p(\varrho, \vartheta) \sim \varrho\vartheta + \varrho^\gamma.$$

We explain the ideas how to prove existence of a variational entropy solution (for any $\gamma > 1$) and weak solution (for any $\gamma > \frac{5}{4}$) under suitable assumptions on the data and the form of temperature dependence of viscosity coefficients and heat conductivity.

Speaker: **K.R. Rajagopal** (Texas A&M University, College Station, U.S.A.)
Email: krajagopal@tamu.edu
Title: *On implicit constitutive equations*
Abstract: After introducing implicit constitutive relations to describe the response of both non-linear fluids and solids, I will discuss applications wherein they can be gainfully exploited. It will be shown that such implicit relations can explain phenomena that have hitherto defied adequate explanation.

Speaker: **David Silvester** (University of Manchester, United Kingdom)
Email: d.silvester@manchester.ac.uk
Title: *Fast iterative solvers for buoyancy driven flow problems*
Abstract: We outline a new class of robust and efficient methods for solving the Navier–Stokes equations with a Boussinesq model for buoyancy driven flow. We describe a general solution strategy that has two basic building blocks: an implicit time integrator using a stabilized trapezoid rule with an explicit Adams–Bashforth method for error control, and a robust Krylov subspace solver for the spatially discretized system.

We present numerical experiments illustrating the efficiency of the chosen preconditioning schemes with respect to the discretization parameters.

Speaker: **Ondřej Šrámek** (University of Maryland, U.S.A.)
Email: sramek@umd.edu
Title: *Modeling of two-phase flow in geophysics: compaction, differentiation, partial melting, and melt migration*
Abstract: Many important geophysical processes involve flow and deformation of multiphase environments. Partial melting and migration of melts play a crucial role in the formation and evolution of the Earth and other terrestrial bodies. Transport of heat, rock rheology, and distribution of major, minor, as well volatile chemical species are all affected by the presence and migration of magmas. Partial melting and melt extraction are processes central to the formation of the Earth's crust, and are responsible for global scale chemical fractionation. Migration of molten material played a major role in the dynamic evolution of the early Earth and even now plays a fundamental role in the transport of matter as well as heat in partially molten regions of deeper mantle. The early separation of the denser metal from the lighter silicates is the most extensive differentiation event in the course of Earths evolution. This process also implies the presence of distinct phases, in solid and liquid states. Gravitational energy which is released upon differentiation is a major source of heat that must be considered when assessing the thermal history of a forming planet. It is therefore essential to properly take into account the energy exchange that takes place in a multiphase medium on a large spatial scale in order to investigate early planetary evolution and to constrain the differentiation time scales. In my talk I will review the recent progress in geophysical modeling of multiphase flow. I will present a general model of two-phase flow and deformation widely used in geophysics. The model offers a self-consistent description of the mechanics and thermodynamics of a mixture of two viscous fluids, in the form of continuum mechanical equations in the limit of a slow creeping flow. The difference in pressures that exists between the two phases is generated by the surface tension at the interfaces between the phases, and by the isotropic deformation (i.e., compaction or dilation) of the individual phases upon flow. In most geological applications, one of the phases is much less viscous than the other phase, which greatly simplifies the equations. I will show several examples of two-phase flow modeling in geophysical settings.

Speaker: **Mark Steinhauer** (University Koblenz-Landau, Germany)
Email: steinhauerm@uni-koblenz.de
Title: *Expected contribution to the summarizing talk*

Speaker: **Zdeněk Strakoš** (Charles University in Prague, Czech Republic)
Email: strakos@karlin.mff.cuni.cz
Title: *Bits of scattered (summarizing?) comments*

Speaker: **Agnieszka Świerczewska-Gwiazda** (University of Warsaw, Poland)
Email: aswiercz@mimuw.edu.pl
Title: *On the generalized Stokes equation in Orlicz spaces*
Abstract: Our interest is directed to unsteady generalized Stokes equation

$$u_t - \operatorname{div} T(Du) + \nabla p = f, \ \operatorname{div} u = 0.$$

where u, p are the velocity and the pressure respectively, T is the Cauchy stress tensor. The growth and the coercivity of the tensor T are prescribed by an N–function. Since the case of fast and moderately growing (w.r.t. shear rate) tensors have been already considered by us in previous papers ([1], [4]), we concentrate on slowly growing tensors. The generality of the growth conditions implies the formulation of the problem in Orlicz spaces. The problems related to density of smooth functions in Orlicz spaces, embeddings and

Korn inequality will be presented. We will discuss the case of $x-$dependent stress tensor, and consequently $x-$dependent $N-$function. Such formulation of the problem allows to include an influence of some outer fields for the flow, like magnetic or electric field. We shall also present the case of implicit constitutive relations between T and Du, cf. [2]. The talk will concentrate on results of existence of weak solutions, cf. [3].

References

[1] M. Bulicek, P. Gwiazda, J. Malek and A. Świerczewska–Gwiazda.On unsteady flows of implicitly constituted incompressible fluids, SIAM J. Math. Anal, SIAM J. MATH. ANAL., Vol. 44, No. 4, pp. 27562801

[2] M. Bulicek, P. Gwiazda, J. Malek, K.R. Rajagopal and A. Świerczewska–Gwiazda. On flows of fluids described by an implicit constitutive equation characterized by a maximal monotone graph, accepted to London Math. Soc. Lecture Note, Cambridge Univ. Press

[3] P. Gwiazda, A. Świerczewska–Gwiazda and A. Wróblewska. Generalized Stokes system in Orlicz spaces, Discrete Contin. Dyn. Syst. Ser. A Vol. 32 (6), 2012

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Speaker: **Alexis Vasseur** (University of Texas at Austin, U.S.A.)
Email: `vasseur@math.utexas.edu`
Title: *Relative entropy method applied to model reduction in fluid mechanics*
Abstract: We will present the relative entropy method, and some recent applications to model reduction in fluid mechanics. We will consider, in particular, the case involving shocks (discontinuities) on the limit problems.

Speaker: **Martin Vohralík** (INRIA Paris-Rocquencourt, Paris, France)
Email: `vohralik@ann.jussieu.fr`
Title: *Adaptive regularization, linearization, and numerical solution of unsteady nonlinear problems*
Abstract: We show how computable a posteriori error estimates can be obtained for model nonlinear and unsteady problems, namely the nonlinear Laplace problem, the Stefan problem, and the two-phase porous media flow problem. Regularization of the nonlinear functions, iterative linearizations, and iterative solutions of the arising linear systems are typically involved in the numerical approximation procedure. We show how the corresponding error components can be distinguished and estimated separately. A fully adaptive solution procedure, with adaptive choices of the regularization parameter, the number of nonlinear and linear solver steps, the time step size, and the computational mesh, are presented.
