Proof Complexity (11w5103)
October 2-7, 2011

MEALS

*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday
*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday
*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday
Coffee Breaks: As per daily schedule, foyer of TransCanada Pipelines Pavilion
***Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

MEETING ROOMS

All lectures will be held atTransCanada Pipelines Pavilion.

SCHEDULE

Sunday

16:00	Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
17:30–19:30	Buffet Dinner, Sally Borden Building
20:00	Informal gathering in 2nd floor lounge, Corbett Hall Beverages and a small assortment of snacks are available on a cash honor system.

Monday

7:00–8:45	Breakfast
8:45–9:00	Introduction and Welcome by BIRS Station Manager
9:00–10:00	Jakob Nordstrom *Understanding the Hardness of Proving Formulas in Propositional Logic*
10:05–10:35	Toni Pitassi *A little advice can be very helpful*
10:35–10:55	Coffee Break
11:00–11:50	Chris Beck *Time-Space Tradeoffs in Resolution: Lower Bounds for Superlinear Space*
12:00–13:00	Lunch
13:00–14:00	Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
14:00	Group Photo; meet on the front steps of Corbett Hall
14:10–15:10	Alexander Razborov *Flag algebras*
15:10–15:30	Coffee Break
17:30–19:30	Dinner

Tuesday

7:00–9:00	Breakfast
9:00–9:45	Edward A. Hirsch *Distributional proving problems, and beyond*
9:50–10:35	Iddo Tzameret*Proof Complexity of Dense Random 3CNF Formulas*
10:35–10:55	Coffee Break

10:55–11:25	Oliver Kullman *Towards a better understanding of SAT-hardness: Constructing soft SAT-representations of boolean functions, with applications to AES+DES*
11:30–12:00	Russell Impagliazzo *A satisfiability algorithm for* AC_0 *circuits*
12:00–13:30	Lunch
13:30–14:00	Sam Buss *An Improved Separation of Regular Resolution from Proof Resolution and Clause Learning.*
14:05–15:05	Jan Johannsen *Lower Bounds for Width-restricted Clause Learning*
15:05–15:30	Coffee break
17:30–19:30	Dinner
20:00	Panel discussion

Wednesday

7:00–9:00	Breakfast
9:00–10:00	Konstantinos Georgiou *Refuting CSPs require Sherali-Adams SDPs of Exponential Size, due to Pairwise Independence.*
10:00–10:20	Coffee Break
10:20–11:20	Albert Atserias *Sherali-Adams Relaxations and Indistinguishability in Counting Logics*
11:25–11:55	Moritz Müller *Some definitorial suggestions for parameterized proof theory.*
12:00–13:30	Lunch
17:30–19:30	Dinner

Thursday

7:00–9:00	Breakfast
9:00–10:00	Stephen Cook *Formalizing Randomized Matching Algorithms*
10:05–10:25	Coffee Break
10:25–10:55	Leszek Kolodziejczyk *Fragments of approximate counting*
11:00–11:30	Emil Jerabek *Root finding in* TC^0
11:35–12:05	Satoru Kuroda *Axiomatizing proof tree concepts in bounded arithmetic*
12:05–13:30	Lunch
13:30–14:00	Chris Pollett *On the Finite Axiomatizability of Prenex* R^1_2
14:05–14:35	Phuong Nguyen *Proving soundness for the quantified propositional calculus* G^*_i
14:40–15:10	Alexis Maciel *Lifting Lower Bounds for Tree-Like Proofs*
15:10–15:30	Coffee Break
17:30–19:30	Dinner

Friday

7:00–9:00	Breakfast
9:00–9:30	Alasdair Urquhart *Width and size of regular resolution proofs*
9:35–10:05	Nicola Galesi *Some results on the complexity of proofs in parameterized Resolution*
10:05–10:25	Coffee Break
10:30–11:00	Pavel Pudlak *A lower bound on resolution proofs of the Ramsey Theorem*
11:30–13:30	Lunch
Checkout by 12 noon.	

** 5-day workshops are welcome to use BIRS facilities (2nd Floor Lounge, Max Bell Meeting Rooms, Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. **

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ABSTRACTS
(in alphabetic order by speaker surname)

1. Speaker: **Albert Atserias** (UPC)
   Title: *Sherali-Adams Relaxations and Indistinguishability in Counting Logics*
   Abstract:

   Two graphs with adjacency matrices $\mathbf{A}$ and $\mathbf{B}$ are isomorphic if there exists a permutation matrix $\mathbf{P}$ for which the identity $\mathbf{P}^{\mathrm{T}}\mathbf{A}\mathbf{P} = \mathbf{B}$ holds. Multiplying through by $\mathbf{P}$ and relaxing the permutation matrix to a doubly stochastic matrix leads to the linear programming relaxation known as fractional isomorphism. We show that the levels of the Sherali-Adams (SA) hierarchy of linear programming relaxations applied to fractional isomorphism interleave in power with the levels of a well-known color-refinement heuristic for graph isomorphism called the Weisfeiler-Lehman algorithm, or equivalently, with the levels of indistinguishability in a logic with counting quantifiers and a bounded number of variables. This tight connection has quite striking consequences. For example, it follows immediately from a deep result of Grohe in the context of logics with counting quantifiers, that a fixed number of levels of SA suffice to determine isomorphism of planar and minor-free graphs. We also offer applications both in finite model theory and polyhedral combinatorics. First, we show that certain properties of graphs, such as that of having a flow-circulation of a prescribed value, are definable in the infinitary logic with counting with a bounded number of variables. Second, we exploit a lower bound construction due to Cai, Fürer and Immerman in the context of counting logics to give simple explicit instances that show that the SA relaxations of the vertex-cover and cut polytopes do not reach their integer hulls for up to $\Omega(n)$ levels, where n is the number of vertices in the graph.

   Joint work with Elitza Maneva.

2. Speaker: **Chris Beck** (Princeton)
   Title: *Time-Space Tradeoffs in Resolution: Lower Bounds for Superlinear Space*
   Abstract:

   We give the first time-space tradeoff lower bounds for Resolution proofs that apply to superlinear space. In particular, we show that there are formulas of size N that have Resolution refutations of space and size each roughly $N^{\log_2 N}$ (and like all formulas have Resolution refutations of space N) for which any Resolution refutation using space S and length T requires $ST \geq N^{1.16 \log_2 N}$. By downward translation, a similar tradeoff applies to all smaller space bounds.

   We also show significantly stronger time-space tradeoff lower bounds for Regular Resolution, which are also the first to apply to superlinear space. Namely, for any space bound S at most $2^{o(N^{1/4})}$ there are formulas of size $N \leq 2^S$ that have Regular Resolution proofs of space S and slightly larger size $T = O(NS)$, but for which any Regular Resolution proof of space $S^{1-\epsilon}$ requires length $T^{\Omega(\log\log N/\log\log\log N)}$.

   Joint work with Paul Beame and Russell Impagliazzo.

3. Speaker: **Sam Buss** (UCSD)
   Title: *An Improved Separation of Regular Resolution from Proof Resolution and Clause Learning.*
   Abstract:

We prove that the graph tautology principles of Alekhnovich, Johannsen, Pitassi and Urquhart have polynomial size pool resolution refutations using only input lemmas as learned clauses and without degenerate resolution inferences. Consequently, these can be shown unsatisfiable by polynomial size DPLL proofs with clause learning

4. Speaker: **Stephen Cook** (Toronto)
   Title: *Formalizing Randomized Matching Algorithms*
   Abstract:

   Using Jerabek's framework for probabilistic reasoning, we formalize the correctness of two fundamental RNC2 algorithms for bipartite perfect matching within the theory VPV for polytime reasoning. The first algorithm is for testing if a bipartite graph has a perfect matching, and is based on the Schwartz-Zippel Lemma for polynomial identity testing applied to the Edmonds polynomial of the graph. The second algorithm, due to Mulmuley, Vazirani and Vazirani, is for finding a perfect matching, where the key ingredient of this algorithm is the Isolating Lemma.

   Joint work with Dai Tri Man Le.

5. Speaker: **Nicola Galesi** (Universita degli Studi di Roma La Sapienza)
   Title: *Some results on the complexity of proofs in parameterized Resolution*
   Abstract:

   Parameterized Resolution was recently introduced by Dantchev, Martin, and Szeider in the context of parameterized proof complexity, an extension of the proof complexity approach of Cook and Reckhow to parameterized complex- ity. Analogously to the case of Fixed Parameter Tractable (FPT) algorithms for optimization problems. After a brief introduction, we look inside the structure of Parameterized DPLL giving a new information-theoretical characterization of proofs in terms of a two- player game, the Asymmetric Prover-Delayer (APD) game. We present a completely diff erent analysis of APD-games based on an information-theoretical argument

   It is a natural question what happens when we equip a parametrized proof system with a more effi cient way of encoding the exclusion of assignments with hamming weight k, than just adding all possible clauses with k + 1 negated variables. Dantchev et al. proved that this is a signicant point. They presented a di fferent and more e cient encoding, and showed that under this encoding PHP admits e cient FPT Parameterized Resolution proofs, while in previous work we proved that PHP does not admit efficent proofs when using the first encoding. In the same work we investigated this question further and noticed that for propositional encodings of prominent combinatorial problems like k- independent set or k-clique, the separation between the two encodings vanishes.

   The k-clique principle simply says that a given graph contains a clique of size k. When applied on a graph not containing a k-clique it is a contradiction. We prove signicant lower bounds for the k-clique principle in the case of Parameterized DPLL using the APD game. Our k-clique formula is based on random graphs distributed according to a simple variation of the Erdos-Renyi model $G(n; p)$.

6. Speaker: **Konstantinos Georgiou** (Waterloo)
   Title: *Refuting CSPs require Sherali-Adams SDPs of Exponential Size, due to Pairwise Independence.*
   Abstract:

   This work considers the problem of approximating fixed predicate constraint satisfaction problems (MAX k-CSP(P)). We show that if the set of assignments accepted by P contains the support of a balanced pairwise independent distribution over the domain of the inputs, then such a problem on n variables cannot be approximated better than the trivial (random) approximation, even after augmenting the natural semidefinite relaxation with Omega(n) levels of the Sherali-Adams hierarchy.

It was recently shown that under the Unique Game Conjecture, CSPs for predicates satisfying this condition cannot be approximated better than the trivial approximation. Our results can be viewed as an unconditional analogue of this result in a restricted computational model. Alternatively, viewing the Sherali-Adams SDP system as a proof system, our result states that a proof of exponential size is required in order to refute highly unsatisfiable instances. For our result we introduce a new generalization of techniques to define consistent local distributions over partial assignments to variables in the problem, which is often the crux of proving lower bounds for such hierarchies.

This is joint work with Siavosh Benabbas, Avner Magen and Madhur Tulsiani.

7. Speaker: **Edward A. Hirsch** (Steklov Institute of Mathematics at St. Petersburg)
   Title: *Distributional proving problems, and beyond*
   Abstract:

   A distributional proving problem is a language and a probability distribution on its complement. In order to solve it, an algorithm has to accept every word ("theorem") in the language while accepting only a small number of words outside it ("false theorems"), where "a small number" is with respect to the probability distribution. Krajicek and Pudlak (1989) showed that TAUT has p-optimal proof systems iff it has optimal acceptors, i.e., algorithms that accept tautologies as fast (up to a polynomial) as any other algorithm. We show that for every polynomial-time samplable distribution, the corresponding distributional proving problem has optimal randomized heuristic acceptors unconditionally. We also show that if the distribution is an injective pseudo-random generator, then the construction can be derandomized. If time permits, we will discuss a non-heuristic optimal acceptor for graph non-isomorphism and numerous related questions.

   This is joint work with D.Itsykson, I.Monakhov, V.Nikolaenko, A.Smal.

8. Speaker: **Russell Impagliazzo** (UCSD/IAS)
   Title: *A satisfiability algorithm for AC_0 circuits*
   Abstract:

   We give a new Satisfiability algorithm for constant-depth circuits. More generally, our algorithm describes the set of 1's of the circuit as a disjoint union of restrictions under which the circuit becomes constant. This allows us to count the number of solutions as well as decide Satisfiability. Let d denote the depth of the circuit and cn denote the number of gates. The algorithm runs in expected time $|C|2^{n(1-\mu_{c.d})}$ where $|C|$ is the size of the circuit and $\mu_{c,d} \geq 1/O[\lg c + d \lg d]^{d-1}$.

   As a corollary, we get a tight bound on the maximum correlation of such a circuit with the parity function. The tightness of this bound shows that the time our algorithm takes matches the size of the smallest description as above. By recent results of Ryan Williams, any substantial improvement in our running time, even for Satisfiability, would show that $NEXP \notin NC^1$.

9. Speaker: **Emil Jerabek** (Institute of Mathematics, Prague)
   Title: *Root finding in TC^0*
   Abstract:

   We show that for any constant d, there is a uniform TC^0 algorithm computing approximations of complex zeros of degree-d univariate rational polynomials (given by a list of coefficients in binary). Equivalently, the theory VTC^0 + the set of all true $\forall\Sigma_0^B$ sentences includes $IOpen$ (for the string sort).

10. Speaker: **Jan Johannsen** (Institut für Informatik of the Ludwig-Maximilians-Universität München)
Title: *Lower Bounds for Width-restricted Clause Learning*
Abstract:

Clause learning is a technique used by propositional satisfiability solvers where some clauses obtained by an analysis of conflicts are added to the formula during backtracking. It has been observed empirically that clause learning does not significantly improve the performance of a solver when restricted to learning clauses of small width only. We survey several lower bound theorems supporting this experience.

11. Speaker: **Leszek Kolodziejczyk** (University of Warsaw)
Title: *Fragments of approximate counting*
Abstract:

We study the low-complexity consequences of Jerabek's theory of approximate counting, that is, T^1_2 plus the surjective weak pigeonhole principle for $P^N P$ functions, with the goal of showing that it does not prove all the Σ^b_1 sentences provable in full bounded arithmetic. This is inspired by the question of whether the levels of the bounded arithmetic hierarchy can be separated by a sentence of fixed low complexity, and the related question of whether the CNFs provable in constant depth Frege systems form a hierarchy with depth. We give some partial results. Joint work with Sam Buss and Neil Thapen.

12. Speaker: **Oliver Kullmann** (Swansea)
Title: *Towards a better understanding of SAT-hardness: Constructing soft SAT-representations of boolean functions, with applications to AES+DES*
Abstract:

Applying and extending the work from [Ku99b,Ku00g] on the "hardness" of clause-sets (CNF's), we propose to use the (extended) notion of "hardness" for finding good representations of boolean functions:

- The **hardness** $hd(F) \in \mathbb{N}_0$ of a clause-set F is the smallest $k \in \mathbb{N}_0$ such that for all clauses C with $F \models C$ the reduction r_k (as introduced in [Ku99b]) derives C from F.
- r_k is generalised unit-clause propagation (r_1 is unit-clause propagation, r_2 is failed-literal reduction).
- Deriving C from F means to show unsatisfiability of $F \wedge \neg C$ via r_k.
- Basic is the notion of hardness of unsatisfiable F, and here $hd(F)$ is a special case of the general measures introduced in [Ku99b,Ku00g]. On satisfiable F it goes into a different direction than the original work, coinciding with one of two measures briefly considered in [Ansotegui-BonetLevyManya2008Hardness].
- "Good representation" of a boolean function means a clause-set F such that SAT solvers are efficient on F.

Our motivating examples (ongoing work) is attacking AES (Advanced Encryption Standard) and DES (Data Encryption Standard) via SAT. However in this talk I will only consider the theoretical side, with the Pigeonhole Principle and its Extended Resolution refutation ([Co76]) yielding examples for the notion of hardness. See [GwynneKullmann2011HardnessPrelim] and the forthcoming [GwynneKullmann2011AES] for the consideration of AES/DES.

13. Speaker: **Satoru Kuroda** (Gunma)
Title: *Axiomatizing proof tree concepts in bounded arithmetic*
Abstract:

We define bounded arithmetic systems for complexity classes LOGCFL and LOGDCFL using their circuit characterizations. Our technical tools is the proof tree size concept for certain circuit classes. We then discuss provability of theorem(s) in formal language theory in these systems.

14. Speaker: **Alexis Maciel** (Clarkson)
Title: *Lifting Lower Bounds for Tree-Like Proofs*
Abstract:

It is known that constant-depth Frege proofs of some tautologies require exponential size. No such lower bound result is known for more general proof systems. We consider tree-like Sequent Calculus proofs in which formulas can contain modular connectives and only the cut formulas are restricted to be of constant depth. Under a plausible hardness assumption concerning small-depth Boolean circuits, we prove exponential lower bounds for such proofs. We prove these lower bounds directly from the computational hardness assumption. We start with a lower bound for cut-free proofs and "lift" it so it applies to proofs with constant-depth cuts. By using the same approach, we obtain the following additional results. We provide a much simpler proof of a known unconditional lower bound in the case where modular connectives are not used. We establish a conditional exponential separation between the power of constant-depth proofs that use different modular connectives. We show that these tree-like proofs with constant-depth cuts cannot polynomially simulate similar dag-like proofs, even when the dag-like proofs are cut-free. We present a new proof of the non-finite axiomatizability of the theory of bounded arithmetic $I\Delta_0(R)$. Finally, under a plausible hardness assumption concerning the polynomial-time hierarchy, we show that the hierarchy G_i^* of quantified propositional proof systems does not collapse.

Joint work with Phuong Nguyen and Toni Pitassi.

15. Speaker: **Moritz Müller** (CRM Centre de Recerca Matemtica)
Title: *Some definitorial suggestions for parameterized proof theory.*
Abstract:

Parameterized complexity theory allows for a more fine-grained complexity analysis than classical complexity theory in that many NP-complete problems such as Vertex Cover, Clique or Dominating Set have natural parameterizations that are not equivalent under parameterized reductions (under standard hypotheses from parameterized complexity). In proof complexity Extension Frege and Substitution Frege systems are equivalent under p-simulations and the question arises whether also in this context parameterized complexity allows for a more fine-grained analysis. We propose a conceptual framework for such an analysis, provide some negative answers and ask some questions.

16. Speaker: **Phuong Nguyen** (University of Toronto)
Title: *Proving soundness for the quantified propositional calculus G_i^**
Abstract:

Buss's S_2 hierarchy is related to the quantified propositional proof system G in the following way: for each $i \geq 1$, proofs in S_2^i translate to polynomial size proofs in G_i^*, and the soundness of G_i^* with respect to proving Σ_i^q tautologies are provable in S_2^i. Under Cook and Morioka's extended definition of G_i^* that allows proofs of tautologies of complexity higher than Σ_i^q, Steven Perron proved a nontrivial theorem which says that S_2^i proves the soundness of G_i^* even with respect to Σ_{i+1}^q tautologies. The most difficult part of this is when soundness is stated for nonprenex formulas. Close examination of

Perron's proof here reveals some gaps that can be fixed. There is, however, a basic problem with the (standard) way that Perron formalizes Tarski's definition of truth. Essentially, for Perron's theorem to be true we need a "faithful" formalization. This roughly means that the formalization must respect the order of boolean connectives and quantifiers in nonprenex formulas. We show how to do this for a class of what we call well structured formulas, and leave open the problem for general nonprenex formulas.

17. Speaker: **Jakob Nordstrom** (KTH)
Title: *Understanding the Hardness of Proving Formulas in Propositional Logic*
Abstract: Proving formulas in propositional logic is believed to be theoretically intractable in general, and the importance of deciding whether this is so has been widely recognized, e.g., by this being listed as one of the famous million dollar Millennium Problems. On the practical side, however, these days SAT solvers are routinely used to solve large-scale real-world SAT instances with millions of variables. This is in contrast to that there are also known small example formulas with just hundreds of variables that cause even state-of-the-art SAT solvers to stumble.

    What lies behind the spectacular success of SAT solvers, and how can one determine whether a particular formula is hard or tractable? In this talk, we will discuss if proof complexity can say anything interesting about these questions.

    In particular, we propose that the space complexity of a formula could be a good measure of its hardness. We prove that this would have drastic implications for the impossibility of simultaneously optimizing time and memory consumption, the two main resources of SAT solvers. Somewhat surprisingly, our results are obtained by relatively elementary means from combinatorial pebble games on graphs, studied extensively in the 70s and 80s.

    Joint work with Eli Ben-Sasson.

18. Speaker: **Toni Pitassi** (University of Toronto)
Title: *A little advice can be very helpful*
Abstract:

    Patrascu recently introduced a variation on the classical two-party communication model of Yao, where at the start of the protocol, one player is given advice of a certain type. He showed that lower bounds for this model imply lower bounds for a variety of dynamic data structure problems. In this talk, we will introduce the model, and prove some surprising upper bounds. We will also prove some lower bounds in restricted versions of Patrascu's model. This is joint work with: Arkadev Chattopadhyay, Faith Ellen, and Jeff Edmonds.

19. Speaker: **Chris Pollett** (San Jose State University)
Title: *On the Finite Axiomatizability of Prenex R^1_2*
Abstract:

    The question of whether the bounded arithmetic theories S^1_2 and R^1_2 are equal is closely connected to the complexity question of whether P is equal to NC. In this talk, we examine the still open question of whether the prenex version of R^1_2, $\hat{R}^1_2$, is equal to S^1_2. We give new dependent choice-based axiomatizations of the $\forall\hat{\Sigma}^b_1$-consequences of S^1_2 and $\hat{R}^1_2$. We use these axiomatizations to give an alternative proof of the finite axiomatizability of $\forall\hat{\Sigma}^b_1(S^1_2)$. On the other hand, we show that our theory for $\forall\hat{\Sigma}^b_1(\hat{R}^1_2)$ splits into a natural hierarchy of theories and in the remainder of the talk give some attempts to separate these.

20. Speaker: **Pavel Pudlak**(Mathematical Institute AS CR, Prague)
Title: *A lower bound on resolution proofs of the Ramsey Theorem*
Abstract:

Let $n \geq 1$. The Ramsey theorem for pairs on an n-element set is presented by the following unsatisfiable set of clauses. The variables are x_{ij}, for $1 \leq i < j \leq n$. The clauses are $\bigvee_{i,j \in K} x_{ij}$ and $\bigvee_{i,j \in K} \neg x_{ij}$, for all sets $K \subset \{1, \ldots, n\}$, $|K| = \lfloor \frac{1}{2} \log n \rfloor$. All logarithms are to the base 2. We will prove:

Theorem. *Resolution proofs of the Ramsey theorem have size at least* $2^{n^{\frac{1}{4}-o(1)}}$.

21. Speaker: **Alexander Razborov** (University of Chicago)
Title: *Flag algebras*
Abstract:

A substantial part of extremal combinatorics studies relations existing between densities with which given combinatorial structures (fixed size "templates") may appear in unknown (and presumably very large) structures of the same type. Using basic tools and concepts from algebra, analysis and measure theory, we develop a general framework that allows to treat all problems of this sort in an uniform way and reveal mathematical structure that is common for most known arguments in the area. The backbone of this structure is made by commutative algebras defined in terms of finite models of the associated first-order theory.

In this talk I will give a general impression of how things work in this framework, and we will pay a special attention to concrete applications of our methods.

22. Speaker: **Iddo Tzameret** (Tsinghua)
Title: *Proof Complexity of Dense Random 3CNF Formulas*
Abstract:

I will discuss propositional refutations and refutation algorithms for random k-CNF formulas. In particular, I will show that already TC^0-Frege admits short refutations for random 3CNF formulas with n variables and $cn^{1.4}$ clauses. This is better than, e.g., resolution, where exponential lower bounds are known for random 3CNF's with up to $cn^{1.5-\epsilon}$ clauses. The argument is a propositional characterization of the unsatisfiability witnesses given by Feige, Kim and Ofek (2006). In particular, we shall see how to carry out certain spectral arguments inside weak propositional proof systems such as TC^0-Frege.

(Based on a joint work with Sebastian Muller.)

23. Speaker: **Alasdair Urquhart** (Toronto)
Title: *Width and size of regular resolution proofs*
Abstract:

Recent results provide exponential separations between the size of general and regular resolution refutations of sets of clauses. The proofs of these results also show that there are contradictory sets of clauses exhibiting a strong separation between the width of general and regular resolution refutations.

A well known theorem of Ben-Sasson and Wigderson shows that the minimum size of a resolution refutation of a k-CNF formula F is exponential in $(Width(F)^2)/V$, where Width(F) is the minimum width of a resolution refutation of F, and V the number of variables in F.

It is natural to conjecture that this theorem could be adapted to prove a version where we substitute "minimum size of a regular resolution refutation" and "minimum width of a regular resolution refutation." In this talk, I give a set of examples showing that this conjecture fails.