Banff International Research Station

for Mathematical Innovation and Discovery

Twenty-five years of representation theory of quantum groups
August 7 - August 12, 2011

MEALS

*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday
*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday
*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday
Coffee Breaks: As per daily schedule, 2nd floor lounge, Corbett Hall
***Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

MEETING ROOMS

All lectures will be held in Max Bell 159 (Max Bell Building accessible by walkway on 2nd floor of Corbett Hall). LCD projector, overhead projectors and blackboards are available for presentations. Note that the meeting space designated for BIRS is the lower level of Max Bell, Rooms 155–159. Please respect that all other space has been contracted to other Banff Centre guests, including any Food and Beverage in those areas.

SCHEDULE

Sunday

16:00	Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
17:30–19:30	Buffet Dinner, Sally Borden Building
20:00	Informal gathering in 2nd floor lounge, Corbett Hall (if desired) Beverages and a small assortment of snacks are available on a cash honor system.

Monday

7:00–8:45	Breakfast
8:45–9:00	Introduction and Welcome by BIRS Station Manager, Max Bell 159
9:00–10:00	Evgeny Mukhin, *Representations of Toroidal Quantum* $\mathfrak{gl}[1]$
10:00–10:30	Coffee Break, 2nd floor lounge, Corbett Hall.
10:30–11:30	Jacob Greenstein, *Quantum foldings*
11:30–13:00	Lunch
13:00–14:00	Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
14:00	Group Photo; meet on the front steps of Corbett Hall
14:05–15:05	Anthony Licata, *Heisenberg Categorification and Hilbert Schemes*
15:05–15:20	Coffee Break
15:20–16:20	Weiqiang Wang, *Spin fake and generic degrees for the symmetric group*
16:20–16:30	Break
16:30–17:30	Milen Yakimov, *Ring theory of quantum solvable algebras*
17:30–19:30	Dinner

Tuesday

7:00–9:30	Breakfast
9:30–10:00	Charles Young, *Extended T-systems*
10:00–10:30	Coffee Break
10:30–11:30	Anne Schilling, *Crystal energies via the charge in types A and C*
11:30–13:30	Lunch
13:30–14:30	Maxim Nazarov, *Generalized Harish-Chandra isomorphism*
14:30–15:00	Coffee Break
15:00–16:00	Masato Okado, *Open problems related to Kirillov-Reshetikhin crystals*
16:00–16:30	Break
16:30–17:30	Joel Kamnitzer, *Components of quiver varieties and affine Mirkovic-Vilonen polytopes*
17:30–19:30	Dinner

Wednesday

7:00–9:00	Breakfast
9:00–10:00	Benjamin Enriquez, *Solutions of some problems in the quantization of Lie bialgebras*
10:00–10:15	Coffee Break
10:15–11:15	Valerio Toledano Laredo *Yangians, quantum loop algebras and trigonometric connections*
11:15–11:30	Break
11:30–12:00	Sachin Gautam *Monodromy of the trigonometric Casimir connection for* $\mathfrak{sl}(2)$
12:00–13:30	Lunch
	Free Afternoon
17:30–19:30	Dinner

Thursday

7:00–9:00	Breakfast
9:00–10:00	Catharina Stroppel *Fractional Euler characteristic and categorified coloured Jones polynomial*
10:00–10:30	Coffee Break
10:30–11:30	Wolfgang Soergel, *Koszul duality in positive caracteristic*
11:30–13:30	Lunch
13:30–14:30	Igor Frenkel *Quantum groups associated to the split real Lie groups, their representations and future perspectives*
14:30–15:00	Coffee Break
15:00–16:00	Alexander Molev, *Feigin-Frenkel center for classical types*
16:00–16:30	Break
16:30–17:30	Eric Vasserot, *Cyclotomic rational double affine Hecke algebras and categorification*
17:30–19:30	Dinner

Friday

7:00–9:00	Breakfast
9:00–9:30	David Jordan, *Quantized multiplicative quiver varieties*
9:30–9:35	Break
9:35–10:05	Peter Tingley, *Combinatorics of affine* $\mathfrak{sl}(2)$ *MV polytopes*
10:05–10:15	Coffee break
10:15–11:15	Ben Webster, *Categorification, Lie algebras and topology*
11:30–13:30	Lunch
Checkout by 12 noon.	

** 5-day workshops are welcome to use BIRS facilities (2nd Floor Lounge, Max Bell Meeting Rooms, Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. **

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ABSTRACTS
(in alphabetic order by speaker surname)

Speaker: **Benjamin Enriquez** (Université de Strasbourg)
Title: *Solutions of some problems in the quantization of Lie bialgebras*
Abstract:

Speaker: **Igor Frenkel** (Yale University)
Title: *Quantum groups associated to the split real Lie groups, their representations and future perspectives*
Abstract:

Speaker: **Sachin Gautam** (Columbia University)
Title: *Monodromy of the trigonometric Casimir connection for* $\mathfrak{sl}(2)$
Abstract: We will prove that the monodromy of the trigonometric Casimir connection, for a tensor product of evaluation modules of the Yangian of $\mathfrak{sl}(2)$, is described by the quantum Weyl group operators of the quantum loop algebra. In the course of the proof, we will also obtain an explicit expression for the lattice part of the affine braid group.

This talk is based on a joint work with Valerio Toledano Laredo.

Speaker: **Jacob Greenstein** (University of California at Riverside)
Title: *Quantum foldings*
Abstract: A classical result in Lie theory stipulates that a simple finite dimensional Lie algebra that is not simply laced can be constructed as the subalgebra of a Lie algebra of type ADE fixed by an admissible diagram automorphism of the latter. This construction is called "folding" and extends to Kac-Moody Lie algebras. Although it is well-known that foldings do not admit direct quantum analogues, it can be shown that there exists an embedding of crystals or Lusztigs canonical bases for the corresponding Langlands dual Lie algebras. Our aim is to introduce algebraic analogues and generalizations of foldings in the quantum setting which yield new flat quantum deformations of non-semisimple Lie algebras and of Poisson algebras. Perhaps the most spectacular example is an algebra that can be regarded as a new algebra of quantum n by n matrices. If time permits, a link with Hall algebras will be discussed. (joint work with A. Berenstein).

Speaker: **David Jordan** (University of Texas at Austin)
Title: *Quantized multiplicative quiver varieties*
Abstract: We introduce a new class of algebras $D_q(Mat_d(Q))$ associated to a quiver Q and dimension vector d, which yield a flat (PBW) q-deformation of the algebra of differential operators on the space of matrices associated to Q. This algebra admits a q-deformed moment map from the quantum group $U_q(\mathfrak{gl}_d)$, acting by base change at each vertex. The quantum Hamiltonian reduction, $A_d^\xi(Q)$, of D_q by μ_q at the character ξ is simultaneously a quantization of the multiplicative quiver variety, and a q-deformation of the quantized quiver variety, associated to Q.

Specific examples of the data (Q, d, ξ) yield q-deformations of important algebras in representation theory: for example, the spherical DAHA of type A_n may be obtained in this way as a deformation of

a construction of the spherical rational Cherednik algebra by Gan and Ginzburg. Given the ubiquity of quiver varieties in geometric representation theory, we anticipate further connections, which we will outline.

Speaker: **Joel Kamnitzer** (University of Toronto)
Title: *Components of quiver varieties and affine Mirkovic-Vilonen polytopes*
Abstract: Lusztig introduced quiver varieties whose components index the semicanonical basis for symmetric Kac-Moody Lie algebras. I will explain a method for understanding these components in finite and affine types using the combinatorics of Mirkovic-Vilonen polytopes. In the affine type, this gives a new combinatorics to describe crystals of affine Lie algebras, generalizing ideas of Beck-Chari-Pressley, Dunlap, and others. This is joint work with Pierre Baumann and Peter Tingley.

Speaker: **Anthony Licata** (Stanford University)
Title: *Heisenberg Categorification and Hilbert Schemes*
Abstract: Given a finite subgroup G of $SU(2)$, we define a monoidal category whose Grothendieck group is isomorphic to a q-deformation of the homogeneous Heisenberg algebra corresponding to G. This category acts on the derived categories of Hilbert schemes of points on the resolution of the corresponding simple singularity. (This is joint work with Sabin Cautis.)

Speaker: **Alexander Molev** (University of Sydney)
Title: *Feigin-Frenkel center for classical types*
Abstract: For each simple Lie algebra $\mathfrak{g}$ consider the corresponding affine vertex algebra $V(\mathfrak{g})$ at the critical level. The center of this vertex algebra is a commutative associative algebra whose structure was described by a remarkable theorem of Feigin and Frenkel about two decades ago. However, only recently simple formulas for the generators of the center were found for the Lie algebras of type A following Talalaev's discovery of explicit higher Gaudin Hamiltonians. We give explicit formulas for generators of the centers of the affine vertex algebras $V(\mathfrak{g})$ associated with the simple Lie algebras $\mathfrak{g}$ of types B, C and D. The construction relies on the Schur-Weyl duality involving the Brauer algebra, and the generators are expressed as weighted traces over tensor spaces and, equivalently, as traces over the spaces of singular vectors for the action of the Lie algebra $\mathfrak{sl}(2)$ in the context of the Howe duality. This leads to an explicit construction of a commutative subalgebra of the universal enveloping algebra $U(\mathfrak{g}[t])$ and to higher order Hamiltonians in the Gaudin model associated with each Lie algebra g. We also introduce analogues of the Bethe subalgebras of the Yangians $Y(\mathfrak{g})$ and show that their graded images coincide with the respective commutative subalgebras of $U(\mathfrak{g}[t])$.

Speaker: **Eugene Mukhin** (Indiana University - Purdue University Indianapolis)
Title: *: Representations of Toroidal Quantum* $\mathfrak{gl}[1]$
Abstract: We a construct tame irreducible representations on the quantum toroidal $\mathfrak{gl}[1]$ with natural bases parameterized by plane partitions with various boundary conditions. We study the corresponding characters using various degenerations of parameters. As a byproduct we construct Gelfand-Zetlin like bases in a family of highest weight $\mathfrak{gl}_\infty$ modules. This is a joint work with B. Feigin, M. Jimbo, and T. Miwa.

Speaker: **Maxim Nazarov** (University of York)
Title: *Generalized Harish-Chandra isomorphism*
Abstract: This is a joint work with S.Khoroshkin and E.Vinberg. For any complex reductive Lie algebra $\mathfrak{g}$ and any locally finite $\mathfrak{g}$-module V, we extended to the tensor product of $U(\mathfrak{g})$ with V the Harish-Chandra description of $\mathfrak{g}$-invariants in the universal enveloping algebra $U(\mathfrak{g})$. In our subsequent work with S. Khoroshkin, this result was used to give explicit realizations of all simple finite-dimensional modules of Yangians and their twisted analogues.

Speaker: **Masato Okado** (Osaka University)
Title: *Open problems related to Kirillov-Reshetikhin crystals*

Abstract: It is widely known that Kirillov-Reshetikhin modules of quantum affine algebras have extremely fruitful structures, such as T-systems, fermionic character formulas, existence of crystal bases (Kirillov-Reshetikhin crystals), Kyoto path realization of affine highest weight crystals, existence of the corresponding geometric crystals, and positive birational Yang-Baxter maps (tropical R maps). There were many conjectures related to Kirillov-Reshetikhin crystals. Although some conjectures were settled recently, many are still open. I will review recent progress and attempt to survey important open problems on this subject.

Speaker: **Anne Schilling** (University of California at Davis)
Title: *Crystal energies via the charge in types A and C*
Abstract: The energy function of affine crystals is an important grading used in one-dimensional configuration sums of statistical mechanical models and generalized Kostka polynomials. It is defined by the action of the affine Kashiwara crystal operators through a local combinatorial rule and the R-matrix.

Nakayashiki and Yamada have related the energy function in type A to the charge statistic of Lascoux and Schuetzenberger. Computationally, it is much more efficient to compute charge than energy since its definition involves a recursive definition of local energy and the combinatorial R-matrix, for which not in all cases efficient algorithms exist. In this talk we relate energy to a new charge statistic in type C which comes from the Ram-Yip formula for Macdonald polynomials. This involves in particular the generalization of parts of the Kyoto path model for perfect crystals to the nonperfect setting, which yields an isomorphism between affine highest weight crystals and tensor products of Kirillov-Reshetikhin crystals.

This is joint work with Cristian Lenart.

Speaker: **Wolfgang Soergel** (University of Freiburg)
Title: *Koszul duality in positive characteristic*
Abstract: The main new point is a formality result for the derived category of sheaves on the complex analytic flag variety with coefficients in a finite field, constructible along the Bruhat stratification: The extension algebra of parity sheaves as a dg-ring with trivial differential already describes this triangulated category. The method to prove this is splitting by action of the Frobenius, which can be done under very mild and explicit restrictions on the characteristic.

Speaker: **Catharina Stroppel** (Universität Bonn)
Title: *Fractional Euler characteristic and categorified coloured Jones polynomial*
Abstract: In Khovanov's categorification of the Jones polynomial, a polynomial invariant of links is upgraded to an invariant with values in complexes of graded vector spaces such that taking the graded Euler characteristics gives back the original polynomial. One would like to extend this construction to other invariants like coloured Jones or Turaev-Viro 3-manifold invariants. The problem hereby is that the polynomial invariant (or at least its construction) is not defined integrally anymore, but rather over the rational numbers. Hence one would like to interpret rational numbers as Euler characteristics and linear maps with not necessarily integral matrix entries as maps induced by functors on the Grothendieck group. These questions and their relevance in existing categorifications are addressed in the talk.

Speaker: **Peter Tingley** (Massachusetts Institute of Technology)
Title: *Combinatorics of affine* $\mathfrak{sl}(2)$ *MV polytopes*
Abstract: MV polytopes give a useful realization of finite type crystals (combinatorial objects related to representations of complex simple Lie algebras). Recent work of Baumann and Kamnitzer constructs MV polytopes from Lusztig's quiver varieties, which are well defined outside of finite type. This work has now been extended to give a definition of MV polytopes in all symmetric affine cases, and to show that understanding the resulting combinatorics reduces to understanding the $\mathfrak{sl}(3)$ and affine $\mathfrak{sl}(2)$ cases. In this talk I will give a simple characterization of the polytopes and a description of the crystal operators in the affine $\mathfrak{sl}(2)$ case, thereby completing the picture in all symmetric affine cases. I will also explain what the combinatorics means in terms of quiver varieties. This is joint work with Pierre Baumann, Thomas Dunlap and Joel Kamnitzer.

Speaker: **Valerio Toledano Laredo** (Northeastern University)
Title: *Yangians, quantum loop algebras and trigonometric connections*
Abstract: I will describe monodromy representations of affine braid groups arising from a flat connection with values in the Yangian of a simple Lie algebra g. These representations are related to those arising from the quantum Weyl group operators of the quantum loop algebra of g. Matching these two classes of representations involves in particular the construction of a functor relating finite-dimensional modules of those two quantum groups.

This is based on joint work with Sachin Gautam.

Speaker: **Eric Vasserot** (Université de Paris VII)
Title: *Cyclotomic rational double affine Hecke algebras and categorification*
Abstract: The goal of this talk is to explain briefly some aspects of the category $\mathcal{O}$ of cyclotomic rational double affine Hecke algebras. In particular the relation with the affine category $\mathcal{O}$ and the classification of the finite dimensional modules.

Speaker: **Weiqiang Wang** (University of Virginia)
Title: *Spin fake and generic degrees for the symmetric group*
Abstract: The fake degrees are graded multiplicities of an irreducible module of a Weyl group in its coinvariant algebra. The generic degrees arise from Hecke algebras, and their evaluation at a prime power are degrees of irreducible characters of finite Chevalley groups. In this talk, we formulate and compute the spin analogues of fake and generic degrees for the symmetric group and related Hecke algebra. This is joint work with Jinkui Wan.

Speaker: **Ben Webster** (University of Oregon)
Title: *Categorification, Lie algebras and topology*
Abstract: It's a long established principle that an interesting way to think about numbers as the sizes of sets or dimensions of vector spaces, or better yet, the Euler characteristic of complexes. You can't have a map between numbers, but you can have one between sets or vector spaces. For example, Euler characteristic of topological spaces is not functorial, but homology is.

One can try to extend this idea, by taking a vector space, and trying to make a category by defining morphisms between its vectors. This approach (interpreted suitably) has been a remarkable success with the representation theory of semi-simple Lie algebras (and their associated quantum groups). I'll give an introduction to this area, with a view toward applications in topology; in particular to replacing polynomial invariants of knots that come from representation theory with vector space valued invariants that reduce to knot polynomials under Euler characteristic.

Speaker: **Milem Yakimov** (Louisiana State University)
Title: *Ring theory of quantum solvable algebras*
Abstract: The area of quantum groups supplied a very large number of examples where general methods for studying noncommutative rings can be tested and developed, using representation theoretic methods. Joseph, Hodges-Levasseur and Soibelman obtained a great deal of information about the spectra of quantized coordinate rings of simple Lie groups, and Goodearl and Letzter developed a general stratification theory putting the area in the framework of quantum affine toric varieties. Consequently the De Concini-Kac-Procesi quantum nilpotent algebras were another large class of algebras which were heavily investigated.

This talk will be an overview of the known results on spectra of quantum solvable algebras, normal separation, the extension of Gabber's catenarity theorem to these classes, and the classification of their automorphism groups along the Andruskiewitsch-Dumas conjecture. We will finish with an overview of the open problems in the ring theoretic side of quantum groups.

Speaker: **Charles Young** (University of York)
Title: *Extended T-systems*

Abstract: I will present some systems of short exact sequences in the categories of finite-dimensional representations of quantum affine algebras of types A and B. These systems contain the T-system of relations among Kirillov-Reshetikhin modules, and extend it to include, for example, all minimal affinizations. I will outline the proofs, which use the theory of q-characters, and comment on what can be expected in other types. This is joint work with E. Mukhin.