# Algebraic Combinatorixx
# BIRS Workshop #11w5025
# May 22-27, 2011


**ORGANIZERS**

Georgia Benkart, University of Wisconsin-Madison
Stephanie vanWilligenburg, University of British Columbia
Monica Vazirani, University of California Davis


## MEALS

*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday
*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday
*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday
*Coffee Breaks: As per daily schedule below, 2nd floor lounge, Corbett Hall
***Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

## MEETING ROOMS

**All lectures will be held in Max Bell 159 (Max Bell Building accessible by walkway on 2nd floor of Corbett Hall). LCD projector, overhead projectors and blackboards are available for presentations.** Note that the meeting space designated for BIRS is the lower level of Max Bell, Rooms 155–159. Please respect that all other space has been contracted to other Banff Centre guests, including any food and beverages in those areas.

## SCHEDULE

**Sunday**

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| **16:00** | Check-in begins (Front Desk - Professional Development Centre - open 24 hours) |
| **17:30–19:30** | Buffet Dinner, Sally Borden Building |
| **19:30–21:30** | Welcome Gathering and Introductions<br>Meet Your Group<br>(in 2nd floor lounge, Corbett Hall ) |

## Monday

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|---|---|
| **7:00–8:45** | Breakfast |
| **8:45–9:00** | Introduction and Welcome by BIRS Station Manager, Max Bell 159 |
| **9:00–9:45** | Lauren Williams |
| **9:55–10:10** | ***Poster Session 1*** |
| | Jessica Striker |
| | Huilan Li |
| | Camillia Smith Barnes |
| **10:10–10:45** | Coffee Break, 2nd floor lounge, Corbett Hall |
| **10:45–11:10** | Margaret Bayer |
| **11:10–??** | ***Groups*** |
| **??–14:00** | Lunch Break |
| **14:00–14:45** | Monica Vazirani |
| **14:45–15:15** | Coffee Break, 2nd floor lounge, Corbett Hall |
| **15:15–15:40** | Sophie Morier-Genoud |
| **15:45–??** | ***Groups*** |
| **17:30–19:00** | Dinner |
| **19:00-20:30** | ***Panel*** |

## Tuesday

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|---|---|
| **9:00–9:45** | Michelle Wachs |
| **9:55–10:20** | Josephine Yu |
| **10:20–11:20** | Coffee Break and ***Groups*** |
| **11:20–11:45** | Margaret Ready |
| **11:50–12:15** | Maud DeVisscher |
| **12:20** | ***Group Photo*** **meet on the front steps of Corbett Hall** |
| **12:30–13:30** | Lunch Break |
| **13:30–14:30** | Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall |
| **17:30–19:30** | Dinner |
| | ***Afternoon and Evening Unscheduled*** |

## Wednesday

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|---|---|
| **9:00–9:45** | Stephanie vanWilligenburg |
| **9:55–10:10** | ***Poster Session 2*** |
| | Melody Chan |
| | Karola Meszaros |
| | Megan Owen |
| **10:10–10:45** | Coffee Break, 2nd floor lounge, Corbett Hall |
| **10:45–11:10** | Sarah Mason |
| **11:10– ??** | ***Groups*** |
| **??–14:00** | Lunch Break |
| **14:00–14:45** | Anne Schilling |
| **14:50–15:00** | ***Poster Session 3*** |
| | Bridget Tenner |
| | Kelli Talaska |
| **15:05–15:30** | Caroline Klivans |
| **15:30–16:00** | Coffee Break, 2nd floor lounge, Corbett Hall |
| **16:00–??** | ***Groups*** |
| **??–19:00** | Dinner |
| **19:00–20:30** | ***Panel*** |

**Thursday**

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|---|---|
| **9:00–9:45** | Christine Bessenrodt |
| **9:55–10:10** | ***Poster Session 4*** |
| | Vidya Venkateswaran |
| | Meesue Yoo |
| | Heather Dye |
| **10:10–10:45** | Coffee Break, 2nd floor lounge, Corbett Hall |
| **10:45–11:10** | Rosa Orellana |
| **11:10– ??** | ***Groups*** |
| **??–14:00** | Lunch Break |
| **14:00–14:45** | Angèle Hamel |
| **14:45–15:15** | Coffee Break, 2nd floor lounge, Corbett Hall |
| **15:15–15:40** | Patricia Hersh |
| **15:45-16:10** | Julianna Tymoczko |
| **16:10–??** | ***Groups*** |
| **??–19:15** | Dinner |
| **19:15–19:30** | ***Group Report I*** |
| **19:30–19:45** | ***Group Report II*** |
| **19:45–20:00** | ***Group Report III*** |
| **20:00–20:15** | ***Group Report IV*** |
| **20:15–20:30** | ***Group Report V*** |
| **20:30–??** | ***Social Time*** |

**Friday**

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| **8:45–9:10** | Soojin Cho |
| **9:15–9:40** | Hélène Barcelo |
| **9:45-10:00** | ***Group Report VI*** |
| **10:00–10:30** | Coffee Break, 2nd floor lounge, Corbett Hall |
| **10:30–10:45** | ***Group Report VII*** |
| **10:45–11:00** | ***Group Report VIII*** |
| **11:00-11:45** | ***Open Problems, Discussion, Future Plans*** |
| **11:45–13:30** | Lunch |
| **Checkout by 12 noon.** | |

**You are welcome to use BIRS facilities (2nd Floor Lounge, Max Bell Meeting Rooms, Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. **

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### ABSTRACTS (by day and time)

**Monday 9:00–9:45:** Lauren Williams, University of California Berkeley

*Title:* Bases for Cluster Algebras from Surfaces

*Abstract:* One of the main open problems concerning cluster algebras is the problem of finding (vector-space) bases with good positivity properties. In recent work with Gregg Musiker and Ralf Schiffler we have studied the class of cluster algebras coming from surfaces with marked points (S,M). These cluster algebras are interesting because they comprise "most" of the mutation finite cluster algebras, and because of their interpretation in terms of Teichmuller theory. I will explain how to construct a basis for each such cluster algebra, and give manifestly positive combinatorial formulas for every element of the basis.

**Monday 9:55–10:10:** *POSTER SESSION 1*

Jessica Striker, Augsburg College, *Promotion and Rowmotion*

Huilan Li, Drexel University
*An inverse approach to the Littlewood-Richardson Rule for the K-theoretic coproducts*

Camillia Smith Barnes, Sweet Briar College, *Shuffles of Permutations*

**Monday 10:45-11:10:** Margaret Bayer, University of Kansas

*Title:* Counting Faces in Polytopes

*Abstract:* The *face vector* of a convex polytope $P$ is $(f_0, f_1, f_2, \ldots, f_{d-1})$, where $f_i$ is the number of $i$-dimensional faces of $P$. The *flag vector* of $P$ is $(f_S)_{S\subseteq\{0,1,\ldots,d-1\}}$, where $f_S$ is the number of chains of faces (ordered by inclusion) with dimensions from the set $S$. Much of our knowledge of these numbers comes from the relationship between simplicial polytopes and graded rings, and from the relationship between convex polytopes and toric varieties. The talk will highlight some of the methods and results.

**Monday 14:00–14:45:** Monica Vazirani, University of California Davis

*Title:* Cores, Shi Arrangements, and Catalan Numbers

*Abstract:* Catalan numbers are known to count many mathematical objects. (See Richard Stanley's "Enumerative Combinatorics"" or
http://math.mit.edu/ rstan/ec/catalan.pdf and http://math.mit.edu/ rstan/ec/catadd.pdf
for a list of over 150 different combinatorial interpretations.) Some of the more well-known include triangulations of an $n+2$-gon or ways of closing up $n$ pairs of parentheses. In particular, the $n$-th Catalan number counts dominant regions in the Shi arrangement (of type $A_{n-1}$) and partitions that are both $n$-cores and $n+1$-cores. This fits into a more general framework, considering the $m$-Shi arrangement and partitions that are both $n$-cores and $mn+1$-cores. In joint work with Susanna Fishel, we give a bijective proof of this result, (given necessary definitions along the way) using the techniques of J. Anderson.

**Monday 15:15–15:40:** Sophie Morier-Genoud, University of Paris 6

*Title:* Frieze Patterns: Geometry and Combinatorics

*Abstract:* Frieze patterns are combinatorial objects introduced by Coxeter in the 70's. They have a nice geometric realization as moduli spaces of points on the circle. Recently (joint work with V. Ovsienko and S. Tabachnikov) we studied a version of the friezes in higher dimension to understand the moduli space of polygons in the projective plane. In this presentation, I will give the definition and main properties of friezes, explain the connection to geometry and mention combinatorial questions related to these objects.

**Monday 19:00–20:30:** *PANEL*

*Title:* What I wish I had known, but was afraid to ask

*Moderator:* Julie Beier, Mercer University

*Panelists:* Christine Bessenrodt, Hannover University; Susanna Fishel, Arizona State University; Sarah Mason, Wake Forest University; Kathryn Nyman, Willamette University; Anne Shepler, University of North Texas

**Tuesday 9:00–9:45:** Michelle Wachs, University of Miami

*Title:* $q$-Eulerian Numbers

*Abstract:* The Eulerian numbers are classical combinatorial numbers that enumerate permutations with a given descent number or equivalently a given excedance number. The descent number and excedance number are fundamental examples of Eulerian permutation statistics. The inversion number and the major index are another pair of equidistributed permutation statistics; these are fundamental examples of Mahonian permutation statistics. By considering the joint distribution of an Eulerian permutation statistic and a Mahonian permutation statistic one obtains a $q$-analog of the Eulerian numbers. I will describe classical results on various $q$-Eulerian numbers. Then I will discuss joint work with Shareshian in which the $q$-Eulerian numbers obtained from the excedance number and the major index were first considered. More recent joint work with Sagan and Shareshian on cyclic sieving and with Henderson on unimodality will also be presented. An interesting symmetric function analog of the Eulerian numbers, which specializes to our $q$-Eulerian numbers, plays a critical role in the proofs of all of our results.

**Tuesday 9:55-10:20:** Josephine Yu, Georgia Institute of Technology

*Title:* Introduction to Tropical Geometry

*Abstract:* Tropical geometry is the geometry over the max-plus semiring, where addition is taking the maximum and multiplication is the usual addition. Just as ordinary polynomial algebra give rise to algebraic geometry, max-plus polynomial algebra give rise to tropical geometry. In this talk I will introduce tropical varieties as polyhedral objects and discuss some combinatorial problems about them.

**Tuesday 11:20-11:45:** Margaret Readdy, University of Kentucky

*Title:* An Eulerian Relation for the Semisuspension

*Abstract:* The flag vector contains all the face incidence data of a polytope, and in the poset setting, the chain enumerative data. It is a classical result due to Bayer and Klapper that for face lattices of polytopes, and more generally, Eulerian graded posets, the flag vector can be written as a cd-index, a non-commutative polynomial which removes all the linear redundancies among the flag vector entries. This result holds for regular CW complexes. We relax the regularity conditions to show the cd-index exists for non-regular CW complexes and extend the notion of a graded poset to that of a quasi-graded poset.

This is work-in-progress with Richard Ehrenborg and Mark Goresky.

**Tuesday 11:50-12:15:** Maud DeVisscher, City University of London

*Title:* Diagrammatic Kazhdan-Lusztig Theory for the (walled) Brauer Algebra

*Abstract:* Classical Schur-Weyl duality relates the representations of the symmetric groups and the general linear groups via their action on tensor space. The Brauer algebra was introduced to play the role of the symmetric group in a corresponding duality with the orthogonal and symplectic groups. The walled Brauer algebra was introduced in another variation of Schur-Weyl duality, by replacing the tensor space by a mixed tensor space (made of copies of the natural module and its dual). In this talk, I will explain how the complex representation theory of these two algebras can be described using parabolic Kazhdan-Lusztig polynomials.

**Wednesday 9:00-9:45:** Stephanie vanWilligenburg, University of British Columbia

*Title:* An Introduction to Quasisymmetric Functions

*Abstract:* For over a century the Hopf algebra of symmetric functions, Sym, has been a major area of study in algebraic combinatorics with applications to other areas of mathematics including algebraic topology, representation theory, and algebraic geometry.

A nonsymmetric analogue of Sym is QSym, the Hopf algebra of quasisymmetric functions, which has been growing in importance since its introduction as a source of generating functions for P-partitions in the 1980s by Gessel. For example, quasisymmetric functions are generating functions for flags in graded posets and matroids; form the terminal object in the category of certain graded Hopf algebras; arise as characters of a degenerate quantum group; investigate the behaviour of random permutations; and simplify the calculation of symmetric functions such as Macdonald polynomials. Furthermore, dual to QSym is a noncommutative analogue of Sym called NSym, which is the Hopf algebra of noncommutative symmetric functions that in turn connects quasisymmetric functions to a variety of other algebraic objects.

In this talk we will define and discuss quasisymmetric functions, their combinatorics, and their connection to some of the above areas.

**Wednesday 9:55–10:10:** *POSTER SESSION 2*

Melody Chan, University of California Berkeley,
*Combinatorics of the Tropical Torelli Map*

Karola Meszaros, MIT
*Flow Polytopes and Kostant Partition Functions for Signed Graphs*

Megan Owen, University of California Berkeley,
*Geodesics in CAT(0) Cubical Complexes*

**Wednesday 10:45–11:10:** Sarah Mason, Wake Forest University

*Title:* Partitions and Permutations: A Tale of Two Symmetries

*Abstract:* This talk explores the role of partitions in symmetric function theory and their counterpart, compositions, in quasisymmetric function theory. We look at properties that are common to both settings, ways in which they differ, and interactions between the two. Along the way we develop several new bases and explain how these can be used to prove results in both settings.

**Wednesday 14:00–14:45:** Anne Schilling, University of California Davis

*Title:* The Murnaghan-Nakayama rule for $k$-Schur functions

*Abstract:* We prove the Murgnaghan–Nakayama rule for $k$-Schur functions of Lapointe and Morse, that is, we give an explicit formula for the expansion of the product of a power sum symmetric function and a $k$-Schur function in terms of $k$-Schur functions. This is proved using the noncommutative $k$-Schur functions in terms of the nilCoxeter algebra introduced by Lam and the affine analogue of noncommutative symmetric functions of Fomin and Greene.

In the course of the talk we will encounter several conjectures and open problems that can be worked on during the week.

This is joint work with Jason Bandlow and Mike Zabrocki, http://arxiv.org/abs/1004.4886

**Wednesday 14:50–15:00:** *POSTER SESSION 3*

Bridget Tenner, DePaul University, *Patterns and Permutations*

Kelli Talaska, University of California Berkeley, *Determinants and Path Matrices*

**Wednesday 15:05–15:30:** Caroline Klivans, University of Chicago

*Title:* Hyperplane Arrangements with Isometric Cones

*Abstract:* I will pose the following open problem: "Does there exist a real central hyperplane arrangement with all isometric cones that is not a reflection arrangement?" I will give some motivation as to how

this problem arose and thoughts on how one might answer it. (Reference: This question is posed at the end of http://people.cs.uchicago.edu/ klivans/volumes.pdf)

**Wednesday 19:00–20:30:** *PANEL*
*Title:* Advice on how to handle the etc.
*Moderator:* Stephanie vanWilligenburg, University of British Columbia
*Panelists:* *VOLUNTEERS*

**Thursday 9:00-9:45:** Christine Bessenrodt, Hannover University

*Title:* Modular Representation Theory of Symmetric Groups and $p$-Combinatorics

*Abstract:* Suitable combinatorial tools for complex representations of symmetric groups have been developed early on, starting with partitions and their Young diagrams. For the representations at characteristic $p > 0$ and their relation to the characteristic 0 representations, a crucial algebraic notion is that of a $p$-block of the group algebra. The corresponding combinatorial invariant is the $p$-core partition of the partition labels of the irreducible representations belonging to the $p$-block. Indeed, for many properties on the combinatorial side, $p$ need not be a prime, and the corresponding more general results often play a role in a generalized block theory and for related algebras. We will look at some situations in this context where representation theoretical questions have a nice combinatorial answer. In particular, a new relative hook formula will be discussed which connects the degree of a character in a $p$-block with that associated to its $p$-core.

**Thursday 9:55-10:10:** *POSTER SESSION 4*

Vidya Venkateswaran, California Institute of Technology
*Vanishing Results for Hall-Littlewood Polynomials*

Meesue Yoo, Seoul National University
*Combinatorial interpretations of symmetric function operators*

Heather Dye, McKendree University, *Parity Based Invariants of Virtual Knots*

**Thursday 10:45–11:10:** Rosa Orellana, Dartmouth College

*Title:* The Kronecker Coefficients

*Abstract:* The Kronecker coefficients are the multiplicities obtained when we take the tensor product of two irreducible representations of the symmetric group. In this talk I will present quasi-polynomial formulas for these coefficients when the indexing partitions have two parts. I will then give applications of these results to complexity theory.

This is joint work with E. Briand and M. Rosas.

**Thursday 14:00–14:45:** Angèle Hamel, Wilfrid Laurier University

*Title:* Alternating Sign Matrices and Schur Functions

*Abstract:* There are many connections between alternating sign matrices and Schur functions. We explore several of these, stopping along the way to visit Weyl denominator formulas, six vertex models, and Yang-Baxter equations. We also reveal connections to analytic number theory.

**Thursday 15:15–15:40:** Patricia Hersh, North Carolina State University

*Title:* Discrete Morse Theory for Posets

*Abstract:* This talk will be a hands-on introduction to discrete Morse theory, including some discussion of the speaker's work on how to apply discrete Morse theory effectively to order complexes of partially ordered sets. The focus will be on what types of things discrete Morse functions can tell you and on nuts and bolts of how to use discrete Morse theory. Essentially a discrete Morse function is a graph matching,

so this enables topological questions to be transformed into combinatorial ones. Numerous examples will be given.

**Thursday 15:50–16:15:** Julianna Tymoczko, University of Iowa

*Title:* The Combinatorics of GKM Theory

*Abstract:* GKM theory describes a combinatorial algorithm that produces torus-equivariant cohomology of suitable algebraic varieties out of a labeled graph. We will briefly summarize GKM theory, and then discuss some computational tools that generalize classical tools from algebraic combinatorics. We will end with open combinatorial problems in Schubert calculus.

**Friday 8:45-9:10:** Soojin Cho, Ajou University

*Title:* Littlewood-Richardson rule for Schur $P$-functions

*Abstract:* A new description of shifted Littlewood-Richardson coefficients will be introduced in terms of semistandard decomposition tableaux which were recently introduced by L. Serrano. We also show that the set of semistandard decomposition tableaux is invariant under the action of Lascoux-Schutzenberger involution, providing a combinatorial proof of the symmetry of Schur $P$-functions.

We find counter examples to the conjecture made by L. Serrano on skew Schur P-functions, leaving us problems to consider.

Many combinatorial properties of semistandard decomposition tableaux will also be introduced.

**Friday 9:15-9:40:** Hélène Barcelo, MSRI and Arizona State University

*Title:* $k$-Equal Subspace Arrangements revisited

*Abstract:* The $k$-equal arrangement is the collection of subspaces in $R^n$ given by equations of the form $x_{i_1} = x_{i_2} = \cdots = x_{i_k}$ , over all indices $1 \leq i_1 < i_2 < \cdots < i_k < n$. In this talk, we describe the $k$-parabolic arrangement, a generalization of the $k$-equal arrangement for any finite real reflection group. When $k = 2$, these arrangements correspond to the well-studied Coxeter arrangements, including the Braid arrangement when W is of type A. In 1963, Fadell, Fox, and Neuwirth showed that the complement of the complex braid arrangement is a $K(\pi, 1)$ space, and that its fundamental group is isomorphic to the pure braid group. Brieskorn (1971) generalized the last result to complexified $W$ Coxeter arrangements by showing that the fundamental group is isomorphic to the pure Artin group of type W. Khovanov (1996) gave a real counterpart to Fadell, Fox and Neuwirth's result when $W$ is of type A (and B) by showing that the complement of the 3-equal arrangement (over $\mathbb{R}$) is a $K(\frac{1}{4}; 1)$ space, and by giving an algebraic description of its fundamental group. We generalize Khovanov's result and obtain an algebraic description of the fundamental group of the complement of the 3-parabolic arrangement for arbitrary finite reflection group. Our description is a real analogue of Brieskorn's one. We conjecture that for $W$ of any type, the complement of the 3-parabolic arrangement is a $K(\pi, 1)$ space. This is joint work with Christopher Severs and Jacob White.