

# Triangulated Categories and Applications

## 2011 June 12-17

### MEALS

- Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday
- Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday
- Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday
- Coffee Breaks: As per daily schedule, 2nd floor lounge, Corbett Hall

**Please remember to scan your meal card at the entrance of the dining room for each meal.**

### MEETING ROOMS

All lectures will be held in **Max Bell 159** (Max Bell Building accessible by walkway on 2nd floor of Corbett Hall). LCD projector, overhead projectors and blackboards are available for presentations. Note that the meeting space designated for BIRS is the lower level of Max Bell, **Rooms 155–159**. Please respect that all other space has been contracted to other Banff Centre guests, including any food and beverage in those areas.

### SCHEDULE

#### SUNDAY

- 16:00** Check-in begins (Front Desk - Professional Development Centre - open 24 hours)  
**Dinner** (Buffet) in Sally Borden Building – see “Meals” above.
- 19:30** Informal gathering in 2nd floor lounge, Corbett Hall  
Beverages and a small assortment of snacks are available on a cash honor system.

#### MONDAY

- Breakfast
- 9:00–9:15** Introduction and Welcome by BIRS Station Manager, Max Bell 159
- 9:15–10:15** M. Hovey: *Introduction to triangulated categories in homotopy theory.*  
Coffee Break, 2nd floor lounge, Corbett Hall
- 10:45–11:45** Y. Kawamata: *Introduction to triangulated categories in algebraic geometry.*  
Lunch
- 13:00–14:00** Guided tour of the Banff Centre; meet in the 2nd floor lounge, Corbett Hall
- 14:00** Group photo; meet on the front steps of Corbett Hall
- 14:15–15:15** H. Krause: *Introduction to triangulated categories in representation theory.*  
Coffee Break, 2nd floor lounge, Corbett Hall
- 15:30–16:15** R. Meyer: *Bivariant K-theory as a triangulated category.*  
Break
- 17:00–17:45** G. Tabuada: *Noncommutative motives.*  
Dinner

## TUESDAY

- Breakfast
- 9:00–9:45** N. Strickland: *Interaction between tensor structure and triangulation.*  
Coffee Break, 2nd floor lounge, Corbett Hall
- 10:15–11:00** G. Stevenson: *Actions by tensor triangulated categories and applications.*
- 11:15–12:00** I. Dell’Ambrogio: *From equivariant K-theory to equivariant KK-theory.*  
Lunch  
Free Afternoon  
Dinner

## WEDNESDAY

- Breakfast
- 9:00–9:45** J. Carlson: *Endotrivial modules.*  
Coffee Break, 2nd floor lounge, Corbett Hall
- 10:15–11:00** J. Pevtsova: *Representations of elementary abelian  $p$ -groups and bundles on Grassmannians.*
- 11:15–12:00** S. Iyengar: *Stratifying triangulated categories.*  
Lunch
- 14:15–15:00** B. Shipley: *Rigidity and algebraic models for rational equivariant stable homotopy theory.*  
Coffee Break, 2nd floor lounge, Corbett Hall
- 15:30–16:15** J. Greenlees: *Homotopy invariant notions of complete intersection.*  
Free time  
Dinner

## THURSDAY

- Breakfast
- 9:00–9:45** D. Stanley:  *$t$ -structures on the derived category of a hereditary category.*  
Coffee Break, 2nd floor lounge, Corbett Hall
- 10:15–11:00** F. Muro: *Obstructions to Adams representability.*
- 11:15–12:00** A. Yekutieli: *Cohomologically complete complexes.*  
Lunch
- 14:15–15:00** T. Licata: *Equivalences from  $sl(2)$  categorification.*  
Coffee Break, 2nd floor lounge, Corbett Hall
- 15:30–16:15** S. Cautis: *Braid group actions on derived categories.*  
Break
- 17:00–17:45** Discussion and open problem session.  
Dinner

## FRIDAY

- Breakfast and check-out preparation
- 9:15–10:00** W. Lowen: *On compact generation of some deformed surfaces.*  
Coffee Break, 2nd floor lounge, Corbett Hall
- 10:45–11:30** D. Murfet: *Pushing forward matrix factorisations.*
- 12:00** **Check-out.**  
Lunch

\*\* 5-day workshops are welcome to use BIRS facilities (2nd Floor Lounge, Max Bell Meeting Rooms, Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. \*\*

# Triangulated Categories and Applications

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### ABSTRACTS

(in alphabetic order by speaker surname)

Speaker: **Jon Carlson** (U. of Georgia)

Title: *Endotrivial modules*

Abstract: I will give a survey of recent work on endotrivial modules. For a finite group or group scheme, an endotrivial module is one whose  $k$ -endomorphism ring is isomorphic in the stable category to the trivial module. Tensoring with an endotrivial gives a self-equivalence of Morita type of the stable category. I will discuss what is known about classifying endotrivial modules and how exotic endotrivial modules can be constructed.

Speaker: **Sabin Cautis** (Columbia U.)

Title: *Braid group actions on derived categories*

Abstract: I will explain a general method for constructing braid group actions on (derived) categories. The starting point is categorical Lie algebra actions which are also of independent interest. Applications include geometry (e.g. braid group actions on derived categories of coherent sheaves on cotangent bundles to partial flag varieties) and knot theory (e.g. algebro-geometric constructions of Khovanov homology).

Speaker: **Ivo Dell’Ambrogio** (U. Bielefeld)

Title: *From equivariant K-theory to equivariant KK-theory*

Abstract: We explain the following equivariant generalization of the very useful universal coefficient and Künneth theorems of Rosenberg and Schochet for complex  $C^*$ -algebras. Let  $G$  be a finite group. For any  $G$ - $C^*$ -algebra  $A$ , the collection of all  $H$ -equivariant topological K-theory groups of  $A$ , with  $H$  a subgroup of  $G$ , assemble to form a Mackey functor  $k(A)$ . The assignment of  $k(A)$  to  $A$  lifts to a stable homological functor defined on the  $G$ -equivariant Kasparov category. We obtain from this construction, and from general relative homological algebra for triangulated categories, the existence of previously unobserved universal coefficient and Künneth spectral sequences abutting to  $KK^G(A, B)$  and  $K^G(A \otimes B)$ , respectively. We have convergence if  $A$  belongs to a certain rather nice localizing (“bootstrap”) subcategory.

Speaker: **John Greenlees** (U. of Sheffield)

Title: *Homotopy invariant notions of complete intersection*

Abstract: The talk will discuss three different characterizations of complete intersections (regular ring mod regular sequence, eventually multiperiodic resolutions for all modules, and polynomial growth of Ext algebra), and homotopy invariant versions of them. We discuss this in the context of rational homotopy theory and mod  $p$  homotopy theory, explaining the relationships between them and giving examples. (Joint work with Benson, Hess and Shamir).

Speaker: **Mark Hovey** (Wesleyan U.)

Title: *Introduction to triangulated categories in homotopy theory*

Abstract: This is a survey talk designed to inform the auditors of how triangulated categories arise in homotopy theory, some of the known results, and some of the outstanding conjectures. We describe the global stable homotopy theory of Hopkins and Smith, and we also explain how the stable homotopy category plays a similar role in the study of triangulated categories as the category of abelian groups plays in the study of abelian categories.

Speaker: **Srikanth Iyengar** (U. of Nebraska, Lincoln)

Title: *Stratifying triangulated categories*

Abstract: My plan is to give an introduction to on-going work with Dave Benson and Henning Krause concerning, what we have called, stratification of triangulated categories. The emphasis will be on explaining what stratification means and some of the consequences that follow from it, and not on how it has been achieved in any specific context.

Speaker: **Yujiro Kawamata** (The U. of Tokyo)

Title: *An introduction to triangulated categories in algebraic geometry*

Abstract: This is a survey talk on the structure of the bounded derived categories of coherent sheaves on algebraic varieties, viewed as a new kind of spaces. The topics include the following: the explicit structure results concerning the exceptional collections and semi-orthogonal decompositions, the cases of abelian varieties and K3 surfaces, the invariants of the categories such as Hochschild (co)homologies and the moduli spaces of stability conditions, and the relationship between the semiorthogonal decompositions and the minimal model program.

Speaker: **Henning Krause** (U. of Bielefeld)

Title: *An introduction to triangulated categories in representation theory*

Abstract: Starting with the pioneering work of Happel in the 1980s, triangulated categories belong today to the standard repertoire of representation theory of finite dimensional algebras. In my talk, I will present some of the main ideas of this development. The selection of topics depends to some extent on personal taste, but the guiding principle is in each case to highlight the specific nature of finite dimensional representations.

Speaker: **Anthony Licata** (Stanford U.)

Title: *Equivalences from  $sl(2)$  categorification*

Abstract: We explain how categorical actions of  $sl(2)$  give rise to equivalences of categories. We'll also describe applications to modular representation theory and algebraic geometry.

Speaker: **Wendy Lowen** (U. Antwerpen)

Title: *On compact generation of some deformed surfaces*

Abstract: We discuss applications of Rouquier's cocovering theorem, which allows to prove compact generation of triangulated categories based upon certain coverings by localizations, to the setting of Grothendieck categories. We obtain some positive results for Grothendieck categories arising as non-commutative deformations of certain surfaces.

Speaker: **Ralf Meyer** (U. Göttingen)

Title: *Bivariant  $K$ -theory as a triangulated category*

Abstract: Noncommutative topology studies topological invariants of  $C^*$ -algebras, which are viewed as generalised spaces. The analogue of the stable homotopy category is  $KK$ -theory. Since it seems hard to lift  $KK$ -theory to a model category, I have been trying to get as much mileage out of the triangulated structure as possible. Localisation of triangulated categories and general recipes for doing homological algebra in triangulated categories turned out to be particularly fruitful.

Speaker: **Daniel Murfet** (UCLA)

Title: *Pushing forward matrix factorisations*

Abstract: Functors between triangulated categories are often described in terms of integration against a kernel. In the categorical setting this integration takes the form of a pushforward. I will describe a new approach to constructing finite models of pushforwards for matrix factorisations and some applications including, if time permits, computations of homological link invariants.

Speaker: **Fernando Muro** (U. de Sevilla)

Title: *Obstructions to Adams representability*

Abstract: There are two representability theorems for homological functors that a triangulated category may or may not satisfy: Brown's theorem is about functors defined on the whole category, while Adams's theorem refers to functors defined in a small subcategory spanned by objects with a certain compactness degree. The former is weaker than the latter and is therefore more widely satisfied. These theorems were first proved for the stable homotopy category and later generalized by Neeman, Krause, Franke... to triangulated categories under suitable assumptions. The classical Adams representability theorem is seldom satisfied. For instance, the derived category of a finite-dimensional hereditary algebra over an uncountable algebraically closed field satisfies Adams's theorem if and only if it is of finite representation type. A failed result of Rosicky claimed that any well generated triangulated category with a model would asymptotically satisfy a transfinite version of Adams's theorem. Such a result for a given triangulated category would still have amazing consequences, therefore this problem deserves to be studied in specific cases. In this talk I will present an approach to this problem from the classical point of view of obstruction theory. This provides a better understanding of the situation even in the classical case. Towards the end of the talk, I will advertise the following open problem: is there any well generated triangulated category such that for any regular cardinal there is a non-representable homological functor defined on objects with that degree of compactness?

Speaker: **Julia Pevtsova** (U. of Washington)

Title: *Representations of elementary abelian  $p$ -groups and bundles on Grassmannians*

Abstract: The study of representations of a finite group (scheme) over a field of characteristic  $p$  via their restrictions to the subalgebras of the form  $k[t]/t^p$ , known as cyclic shifted subgroups or  $\pi$ -points, goes back to the work of J. Carlson in the 80s and has an extensive literature base. In this talk, I'll report on the ongoing project joint with J. Carlson and E. Friedlander in which we initiate the study of representations of an elementary abelian  $p$ -group  $E$  via their restrictions to " $r$ -parameter shifted subgroups", that is, subalgebras of  $kE$  of the form  $k[t_1, \dots, t_r]/(t_1^p, \dots, t_r^p)$ . In this framework, we introduce various geometric invariants that generalize familiar concepts in the special and well studied case of  $r = 1$ . In particular, we'll quickly look at a somewhat surprising behavior of " $r$ -support varieties" and "nonmaximal  $r$ -support varieties" for  $r > 1$ . I shall also describe the generalization of the notion of a module of constant Jordan type and show how such a generalization leads to constructing algebraic vector bundles on Grassmannians.

Speaker: **Brooke Shipley** (U. of Illinois, Chicago)

Title: *Rigidity and algebraic models for rational equivariant stable homotopy theory*

Abstract: We begin with a discussion of realizing triangulated equivalences by underlying equivalences of model categories. We then discuss how we use rigidity to show that the category of rational  $G$ -spectra for a torus  $G$  is Quillen equivalent to an explicit small and practical algebraic model. This is joint work with John Greenlees.

Speaker: **Don Stanley** (U. of Regina)

Title:  *$t$ -structures on the derived category of a hereditary category*

Abstract: For commutative rings  $R$  with a dualizing complex,  $t$ -structures on the derived category of modules over  $R$  can be classified in terms of support data. Even for hereditary abelian categories support data is not enough to determine the  $t$ -structures. However in the hereditary case we describe a classification in terms of wide subcategories and torsion classes that works in many cases.

Speaker: **Greg Stevenson** (U. of Bielefeld)

Title: *Actions by tensor triangulated categories and applications*

Abstract: We will discuss a framework for proving classification theorems for localizing subcategories of compactly generated triangulated categories using actions of tensor triangulated categories. As both motivation and an application we will discuss the classification of localizing subcategories of the stable derived category of a complete intersection over a field.

Speaker: **Neil Strickland** (U. of Sheffield)

Title: *Interaction between tensor structure and triangulation*

Abstract:

Speaker: **Gonçalo Tabuada** (U. Nova de Lisboa)

Title: *Noncommutative motives*

Abstract: I will introduce the category of noncommutative motives and describe joint work with Ivo Dell'Ambrogio, resp. with Paul Balmer, on the tensor triangular geometry of noncommutative motives, resp. on the assembly isomorphism conjectures.

Speaker: **Amnon Yekutieli** (Ben Gurion University)

Title: *Cohomologically complete complexes*

Abstract: Let  $A$  be a noetherian commutative ring, and  $\mathfrak{a}$  an ideal in it. In this lecture I will talk about several properties of the derived  $\mathfrak{a}$ -adic completion functor and the derived  $\mathfrak{a}$ -torsion functor.

In the first half of the talk I will discuss  $\mathfrak{a}$ -adically projective modules, GM Duality (first proved by Alonso, Jeremias and Lipman), and the closely related MGM Equivalence. The latter is an equivalence between the category of cohomologically  $\mathfrak{a}$ -adically complete complexes and the category of cohomologically  $\mathfrak{a}$ -torsion complexes. These are triangulated subcategories of the derived category  $D(\text{Mod } A)$ .

In the second half of the talk I will discuss new results: (1) A characterization of the category of cohomologically  $\mathfrak{a}$ -adically complete complexes as the right perpendicular to the derived localization of  $A$  at  $\mathfrak{a}$ . This shows that our definition of cohomologically  $\mathfrak{a}$ -adically complete complexes coincides with the original definition of Kashiwara and Schapira. (2) The Cohomologically Complete Nakayama Theorem. (3) A characterization of cohomologically cofinite complexes. (4) A theorem on completion by derived double centralizer.

This is joint work with Marco Porta and Liran Shaul.

For full details see the lecture notes

<http://www.math.bgu.ac.il/~amyekut/lectures/cohom-complete/notes.pdf>

or the paper [arxiv:1010.4386](https://arxiv.org/abs/1010.4386) .