Banff International Research Station
for Mathematical Innovation and Discovery

10w5119 Randomization, Relaxation, and Complexity
Arriving Sunday, February 28 and departing Friday, March 5, 2010

ABSTRACTS

##

J. Maurice Rojas: Simple Homotopies for Just Real Roots

We begin with a discussion of Smale's 17th Problem, it's recent near-solution, and a new extension in the direction of sparsity and complete fields. We then show how the theory of Chamber Cones enables an approach to our new conjecture. Many useful corollaries on estimating the number of real roots of polynomial systems ensue. Furthermore, we describe efficiently computable regions on which most polynomial systems admit simple real homotopies preserving the number of real roots. Numerous illustrated examples will be presented. The results are joint (in various combinations) with Martin Avendano, Ashraf Ibrahim, Philippe P. Pebay, and David C. Thompson.

##

Tien-Yien Li: The mixed volume computation: MixedVol-2.0 vs. DEMiCs

The idea of the "dynamic enumeration" of all mixed cells for the mixed volume computation was introduced by T. Mizutani et al., and the resulting software package DEMiCs exhibited its superiority in speed over the fastest mixed volume computation code MixedVol by T.Gao, T.Y.Li and M.Wu. While the fundamental approaches for the mixed volume computation in DEMiCs and MixedVol are very different, we developed, in this talk, the version of the dynamic enumeration for the algorithm in MixedVol. Illustrated by the numerical results, our resulting new code MixedVol-2.0 has reached the speed range of DEMiCs. But, more importantly, results show that MixedVol-2.0 appears to be much more reliable for accurate mixed volume computations.

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Zhonggang Zeng: Solving Ill-posed Algebraic Problems: A Geometric Perspective

One of the challenges in computational algebra is solving ill-posed problemswhose condition numbers are infinity, e.g., multiple roots of polynomials and the matrix Jordan Canonical Forms. Such a hypersensitivity forms a prohibitive barrier on accuracy, making it difficult to solve those problems in numerical computation. In this talk we present a geometric perspective on the nature of ill-posed algebraic problems: They form Riemannian manifolds of positive codimensions and those manifolds entangle in certain stratification structures. As a result, the hypersensitivity can be completely removed by transforming the problem into a combination of optimizations: Finding the maximum codimension manifold and minimizing the distance to it. Based on such a two-staged computing strategy, a series of robust numerical algorithms have been developed for solving ill-posed algebraic problems with remarkable accuracy.

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Pascal Koiran: Shallow circuits with high-powered inputs

A polynomial identity testing algorithm must determine whether an input polynomial (given for instance by an arithmetic circuit) is identically equal to 0. Following Kabanets and Implagliazzo (2004), it has become increasingly clear in recent years that efficient deterministic algorithms for polynomial identity testing would imply strong lower bounds (the connection between arithmetic circuit lower bounds and derandomization of polynomial identity testing was foreshadowed in a 30 years old paper by Heintz and Schnorr). This approach to lower bounds was advocated in particular by Agrawal (2005). In my talk I will present some further results on univariate polynomial identity testing. I will highlight three open problems which can be viewed as refinements of the tau-conjecture of Shub and Smale on integer roots of univariate polynomials. In particular, I will propose a real version of the tau conjecture. A positive answer to any of these three problems would imply a superpolynomial lower bound on the arithmetic complexity of the permanent polynomial.

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Mounir Nisse: Complex and Non-Archimedean (Co)Amoebas, and Phase Limit Sets

Amoebas (resp. Coamoebas) are the link between the classical complex geometry and the tropical (resp. complex tropical) geometry. I will start by briefly introducing these objects, and I will give their complete description for complex linear spaces. Moreover, some applications for n=2 related to Descartes' rule of signs for polynomial of two variables will be outlined. Many examples, with pictures, will be given. The first part of this talk is a joint work with P. Johansson and M. Passare, and the second part is a joint work with F. Sottile.

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Chris Hillar: Do rational certificates always exists for sum of squares problems?

Is every rational polynomial that is a sum of real squares also a rational sum of squares (Sturmfels)? This is an especially important question given the rise of numerical and seminumerical algorithms to *prove* inequalities and optimization bounds. We first discuss recent results with an emphasis on applications to the BMV trace conjecture. We then outline some open problems and conjectures that arise naturally in the context of semidefinite programming and sums of squares.

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Greg Blekherman: Volume of the Cone of Convex Forms and new Faces of the Cone of Sums of Squares

We will describe a simple proof based on volumes that there exist convex forms that are not sums of squares. A convex form has to be nonnegative and there are currently no explicit examples of such forms. A prerequisite for constructing such an example would be building strictly positive forms that are not sums of squares. We will describe the facial structure of the cone of nonnegative forms and build explicit faces of the cone of sums of squares that contain strictly positive forms, which leads to strictly positive polynomials that are not sums of squares.

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Levant Tuncel: Local Quadratic Convergence of Polynomial-Time Interior-Point Methods for Nonlinear Convex Optimization Problems

We propose new path-following predictor-corrector schemes for solving convex optimization problems in conic form. The main structural property used in our analysis is the logarithmic homogeneity of self-concordant barrier functions. Even though our analysis has primal and dual components, our algorithms work with the dual iterates only, in the dual space. Our algorithms converge globally as the current best polynomial-time interior-point methods. In addition, our algorithms have the local quadratic convergence property under some mild assumptions. The algorithms are based on an easily computable gradient proximity measure, which ensures an automatic transformation of the global linear rate of convergence to the local quadratic one under some mild assumptions. Our step-size procedure for the predictor step is related to the maximum step size (the one that takes us to the boundary). It appears that in order to obtain local superlinear convergence, we need to tighten the neighborhood of the central path proportionally to the current duality gap.

This talk is based on joint work with Yu. Nesterov.

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Mihai Putinar: Discretization of Shapes via Orthogonal Polynomials

TBA

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Martin Harrison: Minimal Sums of Squares in a Free-* Algebra

I discuss the reduction of the number of squares needed to express a sum of squares in the free *-algebra R<X,X*>. I will give examples of sums which are irreducible in this sense, and prove bounds on the minimal number of terms needed to express an arbitrary sum of squares of a given degree in a given number of variables.

##

Susan Margulies: Vizing's Conjecture and Techniques from Computer Algebra

Given a graph G, a dominating set is a subset of vertices such that every vertex in the graph is in or adjacent to a vertex in the dominating set. The size of a minimum cardinality dominating set is denoted by Gamma(G). Given two graphs, G and H, and the cartesian product graph GxH, V. Vizing conjectured in 1968 that Gamma(G) Gamma(H) <= Gamma(GxH). We represent the problem of finding a dominating set of size k in an arbitrary graph G as a system of polynomial equations, and show that Vizing's conjecture is equivalent to a conjecture about the equality of two particular ideals. We discuss techniques from computer algebra that can aid in the proof of Vizing's conjecture, or in the search for a counter-example. Additionally, we conjecture a very specific graph-theoretic interpretation of the unique, reduced Gr\"obner basis of these ideals.

##

Brendan Ames: Convex relaxation for the clique, biclique and clustering problems

We consider the clique, biclique, and clustering problems in the case that the problem instance consists of a clique, biclique, or perfectly clustered data plus some noisy data. The noisy data may be inserted either by an adversary or at random. We show that instances constructed in this manner may be solved by convex relaxation even though clique, biclique, and clustering are all NP-hard. In the case of clique and biclique, our convex relaxation uses the nuclear norm, which has recently been proved in a series of papers to exactly solve the NP-hard matrix completion problem for instances that are constructed in a similar manner.

This talk represents joint work with S. Vavavis of University of Waterloo.

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Leonard Gurvits:

TBA

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Bernard Mourrain: Moment matrices and border basis

Moment matrices appear as new interesting tools to analyse and solve effectively some interesting problems in real effective algebraic geometry such as polynomial optimisation. In this talk, we will show some connections with border basis techniques and how this can improve the efficiency of these techniques. We will illustrate it with applications to symmetric tensor decomposition problems and to real radical computation.

##

Laura Matusevich: Monomial ideals and hypergeometric equations

Interesting insight on the commutative algebra of monomial ideals results from interpreting these objects as systems of partial differential equations. In this setting, they are special cases of binomial D-modules, and also degenerations of systems of hypergeometric equations. I will describe how this point of view produces new results in combinatorial commutative algebra, and how further study of these degenerations is crucial in the understanding of important questions from the hypergeometric world. This is ongoing joint work with Christine Berkesch.

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Jim Renegar: Numerics of algorithms for optimizing over hyperbolicity cones

TBA

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Dan Bates: Khovanskii-Rolle continuation for finding real solutions of polynomial systems

Homotopy continuation-based methods have grown to be very efficient and useful tools for solving polynomial systems. Two of the few drawbacks of these sorts of methods are that (a) their complexity is based on the number of complex solutions of the system and (b) to find the real solutions, one must first find all complex solutions. For the engineer or scientist looking for real (or positive) solutions, this can be frustrating. There are several non-homotopy continuation methods for computing real solutions, but none are good as general methods for all systems.

In this talk, I will introduce a numerical method - Khovanskii-Rolle continuation - for finding real solutions whose complexity is based on the number of real solutions rather than the number of complex solutions. This method will produce all isolated real solutions (isolated over the complex numbers). The basic idea is to transform a polynomial system into another polynomial system via Gale duality (which I will demonstrate), solve the new polynomial system by tracing some curves in real space (Khovanskii-Rolle continuation), and then transform the solutions back to the original variables by unwrapping Gale duality. On one example, the total computation time using this method was around 1% of the time that is required for basic homotopy methods. I will also demonstrate that, for other examples, this method will clearly not be the method of choice. The pairing of polynomial systems to appropriate methods is an open and interesting problem.

##

Andrew Sommese: Recent work in Numerical Algebraic Geometry

Some recent advances regeneration (with J. Hauenstein and C. Wampler) local dimension testing (with D. Bates, J. Hauenstein, and C. Peterson) will be discussed.

Also our ongoing work (with W. Hao, J. Hauenstein, B. Hu, Y. Liu, and Y. Zhang) to the solution of differential equations (zebra fish and different tumor growth models) will be disucssed.

##

Anton Leykin: Certified numerical homotopy continuation

Most algorithms used in the software for numerical algebraic geometry are based on numerical heuristics: in general their results are not certified. Using recent developments in complexity analysis rooted in the alpha theory of Smale, we have implemented a homotopy tracking algorithm that provides the status of a ``mathematical proof'' to its approximate numerical output.

A demo of NumericalAlgebraicGeometry package of Macaulay 2 will be a part of the talk. (Joint work with Carlos Beltran)

##

Martin Avendano: Descartes' Rule of Signs is exact!

We show that for any polynomial f with real coefficients there exists a polynomial g with non-negative coeffcients such that the number of positive real roots of f is exactly the number of changes of signs in the vector of coefficients of fg. We also show that g can also be chosen as a power of (x+1). A possible extension to the multivariate case, that gives a bound for the number of connected components of the complement of the zero-set, will also be discussed.

##

Victor Vinnikov: Constructing determinantal representations via noncommutative techniques

I will discuss a simple method of constructing determinantal representations of polynomials using the lifting of the polynomial to the free algebra and the state-space realization theory for noncommutative rational functions. This is joint work with Bill Helton and Scott McCullough.

##

Korben Rusek: On Certain Structured Fewnomials

We present new upper bounds on the numbers of real solutions of sparse systems of polynomials with additional polyhedral structure. To derive our bounds, we use a modified version of Gale duality. As a corollary, we can obtain new polynomial upper bounds for the number of discriminant chambers for n-variate (n+4)-nomials. The best previous bounds were exponential in n.