Banff International Research Station
for Mathematical Innovation and Discovery

10w5096: Whittaker Functions, Crystal Bases, and Quantum Groups
June 6–11, 2010

MEALS

*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday
*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday
*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday
Coffee Breaks: As per daily schedule, 2nd floor lounge, Corbett Hall
Remember to scan your meal card at the host station in the dining room for each meal.

MEETING ROOMS

All lectures will be held in Max Bell 159 (Max Bell Building accessible by walkway on 2nd floor of Corbett Hall). LCD projector, overhead projectors and blackboards are available for presentations. Note that the meeting space designated for BIRS is the lower level of Max Bell, Rooms 155–159. Please respect that all other space has been contracted to other Banff Centre guests, including any Food and Beverage in those areas.

SCHEDULE

Sunday

16:00	Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
	Lecture rooms available after 16:00 (if desired)
17:30–19:30	Buffet Dinner, Sally Borden Building
20:00	Informal gathering in 2nd floor lounge, Corbett Hall
	Beverages and a small assortment of snacks are available on a cash honor system.

Monday

7:00–8:45	Breakfast
8:45–9:00	Introduction and Welcome by BIRS Station Manager, Max Bell 159
9:00–10:15	**Ben Brubaker (MIT), Daniel Bump (Stanford)** – Modeling p-adic Whittaker functions I and II
10:15–10:45	Coffee Break, 2nd floor lounge, Corbett Hall
10:45 - 11:45	**Sergey Oblezin (ITEP)** – Whittaker functions and topological field theories
12:00–13:00	Lunch
13:00–14:00	Optional guided tour of The Banff Centre; meet in 2nd floor lounge, Corbett Hall
14:00	Group Photo; meet on the front steps of Corbett Hall
14:15–15:00	**Gordan Savin (Utah)** – Two Bernstein components for the metaplectic group
15:00–15:30	Coffee Break, 2nd floor lounge, Corbett Hall
15:30–16:15	**Angèle Hamel (Wilfrid Laurier)** – Bijective Proofs of Schur Function and Symplectic Schur Function Identities
17:30–19:30	Dinner

Tuesday

7:00–9:00	Breakfast
9:00–10:00	**Arkadiy Berenstein (Oregon)** – Geometric Crystals
10:00–10:30	Coffee Break, 2nd floor lounge, Corbett Hall
10:30–11:00	**Peter McNamara (MIT)** – Crystals and Metaplectic Whittaker Functions
11:15–12:00	**Joel Kamnitzer (Toronto)** – Mirkovic-Vilonen Cycles and MV Basis
12:00–13:00	Lunch
13:00–13:45	**Kyu-Hwan Lee (UConn)** – Representation theory of p-adic groups and canonical bases
14:00–14:45	**Maki Nakasuji (Stanford)** – Casselman's basis of Iwahori vectors and Bruhat order
14:45–15:15	Coffee Break, 2nd floor lounge, Corbett Hall
15:15–16:00	**Anne Schilling (UC-Davis)** – Combinatorics of Kirillov-Reshetikhin crystals
16:10–16:55	**Siddhartha Sahi (Rutgers)** – An introduction to Double affine Hecke algebras
17:30–19:30	Dinner

Wednesday

7:00–9:00	Breakfast
9:00–10:00	**Ivan Mirkovic (UMass)** – Lusztig's conjecture for Lie algebras in positive characteristic
10:00-10:30	Coffee Break, 2nd floor lounge, Corbett Hall
10:30-11:15	**Manish Patnaik (Harvard)** – Hecke algebras for p-adic loop groups
11:15-12:00	**Omer Offen (Technion)** – Spherical Whittaker functions on metaplectic GL(r)
12:00–13:00	Lunch
	Free Afternoon
17:30–19:30	Dinner
20:00	Open Problem Session (lead by **Paul Gunnells (UMass)**)

Thursday

7:00–9:00	Breakfast
9:00–9:45	**Solomon Friedberg (Boston College)** – Eisenstein series, Gauss sums, and crystal graphs
9:45–10:30	**Samuel Patterson (Göttingen)** – Some challenges from number theory
10:30–11:00	Coffee Break, 2nd floor lounge, Corbett Hall
11:00–11:45	**Adrian Diaconu (Durham)** – Trace formulas and multiple Dirichlet series
11:45–13:00	Lunch
13:00–13:45	**Arun Ram (Melbourne)** – Combinatorics and spherical functions
14:00–15:00	**Soichi Okada (Nagoya)** – Symmetric Functions and spinor representations
15:00–15:30	Coffee Break, 2nd floor lounge, Corbett Hall
15:30–16:00	**Thomas Bliem (SFSU)** – Demazure modules of $\widehat{sl}(2)$
16:00–16:45	**Alex Kontorovich (Brown)** – Sieving in groups
17:30–19:30	Dinner

Friday

7:00–9:00	Breakfast
9:00	Informal Discussions
10:15-10:45	Coffee Break, 2nd floor lounge, Corbett Hall
11:30–13:30	Lunch
12 noon	**Checkout**

** 5-day workshops are welcome to use BIRS facilities (2nd Floor Lounge, Max Bell Meeting Rooms,

Reading Room) until 3 pm on Friday, though check-out remains at noon. **

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TITLES AND (OPTIONAL) ABSTRACTS
(in alphabetic order by speaker surname)

Speaker: **Arkady Berenstein** (University of Oregon)
Title: *From geometric crystals to crystal bases*
Abstract: The goal of my talk is to construct crystal bases (for irreducible modules over semisimple Lie algebras) by means of geometric crystals.

Geometric and unipotent crystals have been introduced a few years ago in a joint work with David Kazhdan as a useful geometric analogue of Kashiwara crystals. More recent observations (based on the recent joint paper with David Kazhdan) show that geometric crystals, in addition to providing families of piecewise-linear parametrizations of crystal bases, also reveal such hidden combinatorial structures as 'crystal multiplication' and 'central charge' on tensor products of crystal bases.

Speaker: **Thomas Bliem** (San Francisco State University)
Title: *Expected degree of weights in Demazure modules of $\widehat{\mathfrak{sl}}_2$*

Speaker: **Ben Brubaker** (MIT)
Title: *Modeling p-adic Whittaker functions I*
Abstract: We'll introduce the themes of the workshop through a discussion of p-adic Whittaker functions for unramified principal series – all terms to be defined in the talk. We mention known methods for giving explicit descriptions of these Whittaker functions, including new expressions as generating functions on crystal bases and other combinatorially defined data associated to bases for highest weight representations.

Speaker: **Daniel Bump** (Stanford University)
Title: *Modeling p-adic Whittaker functions II*
Abstract: We'll look deeper into representations of p-adic Whittaker functions. For spherical Whittaker functions on an algebraic group, these are the same as the characters of the irreducible characters of the L-group, but we are also interested in metaplectic covers of the algebraic group in which case these are similar to but different from such characters. We will look at representations coming from the six-vertex model in statistical physics and explain their relationship to the crystal base description.

Speaker: **Adrian Diaconu** (University of Minnesota)
Title: *Trace formulas and multiple Dirichlet series*

Speaker: **Solomon Friedberg** (Boston College)
Title: *Global objects attached to p adic Whittaker functions*
Abstract: In this talk I will describe some of the connections between p adic Whittaker functions and global objects and their implications for number theory.

Speaker: **Angèle Hamel** (Wilfried Laurier University)
Title: *Bijective Proofs of Schur Function and Symplectic Schur Function Identities*
Abstract: We give modified jeu de taquin proofs of two symmetric function identities. The first relates shifted gl(n)-standard tableaux to the product of a Schur function and $\prod_{i<j}(x_i + y_j)$. This result generalizes the work of Robbins and Rumsey, Tokuyama, Okada, and others. It has recently been given a new proof and set in a new context by Brubaker, Bump and Friedberg.

The second identity is a symplectic character identity relating the sum of a product of symplectic Schur functions to the product $\prod_{i=1}^{m} \prod_{j=1}^{n}(x_i + x_i^{-1} + y_j + y_j^{-1})$. This result has its origin in work of Hasegawa, King, and Jimbo and Miwa. It has previously been proved by Terada and Bump and Gamburd.

This is joint work with Ron King.

Speaker: **Joel Kamnitzer** (University of Toronto)
Title: *Mirkovic-Vilonen cycles and MV basis*
Abstract: Mirkovic-Vilonen cycles are a family of subvarieties of the affine Grassmannian, which under the geometric Satake correspondence give a basis for representations of reductive groups. In my talk, I will begin with older work giving a description of MV cycles using MV polytopes. Then I will explain more recent results, joint with Pierre Baumann, on properties of the resulting MV basis.

Speaker: **Alex Kontorovich** (Brown University)
Title: *Sieving in groups*
Abstract: We will discuss recent progress on the Affine Sieve, which aims to find primes or almost-primes in sets of integers generated by group actions. Applications include the Apollonian circle packing and prime entries in matrix groups. Portions are joint with Hee Oh, Jean Bourgain, and Peter Sarnak.

Speaker: **Kyu-Hwan Lee** (UConn)
Title: *Representation theory of p-adic groups and canonical bases*
Abstract: In this talk, we interpret Gindikin-Karpelevich formula and Casselman-Shalika formula as sums over Lusztig's canonical bases, generalizing the results of Bump-Nakasuji and Tokuyama to arbitrary split reductive groups. We also rewrite formulas for spherical vectors and zonal spherical functions in terms of canonical bases.

Speaker: **Peter McNamara** (MIT)
Title: *Crystals and Metaplectic Whittaker Functions*

Speaker: **Ivan Mirkovic** (UMass-Amherst)
Title: *Lusztig's Conjecture for Lie algebras in positive characteristic*
Abstract: This is a work with Bezrukavnikov and Rumynin. The method is to reformulate representation theoretic problem in positive characteristic in terms of D-modules on flag varieties, coherent sheaves on the cotangent bundle to the flag variety and perverse sheaves on affine flag variety. The same should apply to quantum groups at roots of unity but some parts have not been worked out.

Speaker: **Maki Nakasuji** (Stanford University)
Title: *Casselman's basis of Iwahori vectors and the Bruhat order*
Abstract: This is a joint work with Daniel Bump. Casselman defined a basis of the vectors in a spherical representation of a reductive p-adic group which is defined as being dual to the intertwining operators. We studied the explicit expression of this basis and obtained a conjecture, which is a generalization of the formula of Gindikin and Karpelevich. In this talk, I will present this conjecture and give partial results using Hecke algebra with some examples and related combinatorial conjectures.

Speaker: **Sergey Oblezin** (ITEP)
Title: *Whittaker functions and topological field theories*
Abstract: This talk is a survey of my recent results (in collaboration with A.Gerasimov and D.Lebedev) on GL(N,R)-Whittaker function and their q-deformations.

In the first part of my talk I will construct an element Q(g) of sperical Hecke algebra H(G,K) with G=GL(N,$\mathbb{R}$) and K=SO(N), acting in the space of K-invariant functions on G. Then the Whittaker function is an eigenfunction of Q(g), with the eigenvalue given by a product of Gamma-functions. Actually, the eigenvalue of the Whittaker function can be identified with the Archimedean L-function $L(s, C^N)$.

Further it will be shown that the Archimedean L-function can be interpreted as correlation functions in (a pair of mirror dual) topological sigma-models on two-dimensional disk. In particular, the Archimedean L-function is identified with an equivariant symplectic volume of the space of maps of a disk into a complex space $\mathbb{C}^N$ with certain boundary conditions.

In the second part of my talk I will define a q-deformed GL(N,ℝ)-Whittaker function and introduce a pair of its integral representations. It will be shown that the q-deformed Whittaker function coincides with a character of a Demazure module of affine Lie algbera $\hat{gl}(N)$. This result can be interpreted as an (Archimedean) q-version of the Casselman-Shalika formula for p-adic Whittaker function. Besides, I will explain an interpretation of q-deformed Whittaker functions in terms of the spaces of maps of projective line into (partial) flag varieties.

In the end of my talk I will outline directions of further research and generalizations to other Lie algebras.

Speaker: **Omer Offen** (Technion)
Title: *Spherical Whittaker functions on metaplectic GL(r)*
Abstract: We provide a formula for a basis of spherical Whittaker functions with a fixed Hecke eigenvalue of the n-fold metaplectic cover of GL(r). Our formula expresses the Whittaker function as a sum over the Weyl group. We show that the p-part of the WMDS of type A constructed by Chinta-Gunnells is expressed in terms of such a spherical Whittaker function. The computation adapts the method of Casselman and Shalika to the case that multiplicity is finite but not one. In the case of n=1 we recover the Shintani, Casselman-Shalika formula. This is joint work with G. Chinta.

Speaker: **Soichi Okada** (Nagoya University)
Title: *Symmetric functions and spinor representations*
Abstract: Symmetric functions are useful to the representation theory of classical groups. In this talk, we introduce a family of symmetric functions with coefficients in the ring of integers adjoining a new element e with the property $e^2 = 1$, and investigate their properties. These symmetric functions can be used to describe the structure of the representation ring involving spinor representations of the Pin groups.

Speaker: **Manish Patnaik** (Harvard University)
Title: *Hecke algebras for p-adic loop groups*

Speaker: **Samuel Patterson** (Göttingen)
Title: *Some challenges from number theory*

Speaker: **Arun Ram** (University of Melbourne)
Title: *Combinatorics and spherical functions*

Speaker: **Siddhartha Sahi** (Rutgers University)
Title: *An introduction to Double affine Hecke algebras* Double affine Hecke algebras were introduced by Cherednik who used them to prove the Macdonald cojectures on root systems. In this talk we will provide an introduction to double affine Hecke algebras following joint work with Bogdan Ion.

Speaker: **Gordan Savin** (University of Utah)
Title: *Two Bernstein components for the metaplectic group* Abstract: The Weil representation decomposes as a sum of two irreducible representations, odd and even Weil representations. Let M(e) and M(o) be the components, in the sense of Bernstein, of the category of smooth representations of the metaplectic group Mp$(2n)$ containing the even and the odd Weil representation, respectively. Let V^+ and V^- be two orthogonal p-adic spaces of dimension $2n+1$, with the trivial and non-trivial Hasse invariant, respectively. Let $B(+)$ and $B(-)$ be the Bernstein components of SO(V^+) and SO(V^-) containing the trivial representation. I will describe canonical equivalences of M(e) and $B(+)$ and of M(o) and $B(-)$. This is a joint work with Wee Teck Gan.

Speaker: **Anne Schilling** (UC - Davis)
Title: *Combinatorics of Kirillov-Reshetikhin crystals*

Abstract: I will review recent work with several coauthors (Masato Okado, Ghislain Fourier, Brant Jones) on combinatorial models for Kirillov-Reshetikhin crystals. These are affine finite-dimensional crystals that play an important role in mathematical physics and representation theory. The affine structure makes it possible to define an energy statistics that can be used to define partition functions. If time permits, we will also see how the Kirillov-Reshetikhin crystals can be used to find expression for fusion/quantum cohomology coefficients.