Geometric Analysis and General Relativity
June 20-25, 2010

MEALS

*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday
*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday
*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday
Coffee Breaks: As per daily schedule, 2nd floor lounge, Corbett Hall
***Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

MEETING ROOMS

All lectures will be held in Max Bell 159 (Max Bell Building accessible by walkway on 2nd floor of Corbett Hall). LCD projector, overhead projectors and blackboards are available for presentations. Note that the meeting space designated for BIRS is the lower level of Max Bell, Rooms 155–159. Please respect that all other space has been contracted to other Banff Centre guests, including any Food and Beverage in those areas.

SCHEDULE

Sunday

16:00	Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
17:30–19:30	Buffet Dinner, Sally Borden Building
20:00	Informal gathering in 2nd floor lounge, Corbett Hall Beverages and a small assortment of snacks are available on a cash honor system.

Monday

7:00–8:45	Breakfast
8:45–9:00	Introduction and Welcome by BIRS Station Manager, Max Bell 159
9:00–9:50	**Mihalis Dafermos** (University of Cambridge) *Superradiance, trapping, and decay for waves on Kerr spacetimes in the general subextremal case $\|a\| < M$*
10:00–10:30	Coffee Break, 2nd floor lounge, Corbett Hall
10:40–11:30	**Gustav Holzegel** (Princeton University) *Asymptotic behavior of spacetimes approaching a Schwarzschild solution*
11:50–13:30	Lunch
13:00–14:00	Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall (optional)
14:00	Group Photo; meet on the front steps of Corbett Hall
14:10–15:00	**Alan Rendall** (Max-Planck-Institute for Gravitational Physics) *Higher dimensional cosmological models*
15:10–15:30	Coffee Break, 2nd floor lounge, Corbett Hall.
15:40–16:30	**Hans Ringström** (Institutionen för Matematik - KTH) *Models of the universe with arbitrary compact spatial topology*
17:30–19:30	Dinner
20:00–21:00	**Shing-Tung Yau** (Harvard University) *Quasilocal mass in general relativity*

Tuesday

7:00–9:00	Breakfast
9:00–9:50	**Mu-Tao Wang** (Columbia University) *Limit of quasilocal mass and isometric embeddings into Minkowski space*
10:00–10:30	Coffee Break, 2nd floor lounge, Corbett Hall
10:40–11:30	**Marcus Khuri** (SUNY-Stony Brook) *The Static Metric Extension Conjecture*
11:50–13:30	Lunch
14:00–14:50	**Marc Mars** (Universidad de Salamanca) *The Bray and Khuri approach to the general Penrose inequality in two particularly simple cases*
15:00–15:30	Coffee Break, 2nd floor lounge, Corbett Hall.
15:40–16:30	**Frans Pretorius** (Princeton University) *The instability of 5-dimensional black strings*
16:40–17:30	**Piotr Chruściel** (University of Vienna) *5 dimensional black holes*
17:30–19:30	Dinner
20:00–21:00	**Gerhard Huisken** (Max-Planck-Institute for Gravitational Physics) *Foliations, flows and rigidity in asymptotically flat 3-manifolds*

Wednesday

7:00–9:00	Breakfast
9:00–9:50	**David Maxwell** (University of Alaska Fairbanks) *The Conformal Method and Concrete Examples*
10:00–10:30	Coffee Break, 2nd floor lounge, Corbett Hall
10:40–11:30	**Hubert Bray** (Duke University) *On Dark Matter, Spiral Galaxies, and the Axioms of General Relativity*
11:50–13:30	Lunch Free Afternoon
17:30–19:30	Dinner
20:00–21:00	**Richard Schoen** (Stanford University) *Singularities in positive mass arguments*

Thursday

7:00–9:00	Breakfast
9:00–9:50	**Spyridon Alexakis** (University of Toronto) *On the black hole uniqueness problem*
10:00–10:30	Coffee Break, 2nd floor lounge, Corbett Hall
10:40–11:30	**Sergio Dain** (University of Cordoba) *Linear perturbations and mass conservation for axisymmetric Einstein equations*
11:50–13:30	Lunch
14:00–14:50	**Lydia Bieri** (Harvard University) *Null Asymptotics of Solutions of the Einstein-Maxwell Equations in General Relativity and Gravitational Radiation*
15:00–15:30	Coffee Break, 2nd floor lounge, Corbett Hall.
15:40–16:30	**Pieter Blue** (University of Edinburgh) *Hidden symmetries and decay for the wave equation outside a Kerr black hole*
17:30–19:30	Dinner
20:00–21:00	**Igor Rodnianski** (Princeton University) *On formation of trapped surfaces*

Friday

7:00–9:00	Breakfast
9:00–9:50	**Michael Eichmair** (Massachusetts Institute of Technology) *An overview of Jang's equation and its relation to MOTS*
10:00–10:30	Coffee Break, 2nd floor lounge, Corbett Hall
10:40–11:30	Talk #19 or Open Problem Session
11:30–13:30	Lunch
Checkout by 12 noon.	

** 5-day workshops are welcome to use BIRS facilities (2nd Floor Lounge, Max Bell Meeting Rooms, Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. **

Geometric Analysis and General Relativity
June 20 – 25, 2010

ABSTRACTS
(in alphabetic order by speaker surname)

Speaker: **Spyridon Alexakis** (University of Toronto)
Title: *On the black hole uniqueness problem*
Abstract: I will discuss recent progress on the black hole uniqueness problem.

Speaker: **Lydia Bieri** (Harvard University)
Title: *Null Asymptotics of Solutions of the Einstein-Maxwell Equations in General Relativity and Gravitational Radiation*
Abstract: A major goal of mathematical General Relativity (GR) and astrophysics is to precisely describe and finally observe gravitational radiation, one of the predictions of GR. In order to do so, one has to study the null asymptotical limits of the spacetimes for typical sources. Among the latter we find binary neutron stars and binary black hole mergers. In these processes typically mass and momenta are radiated away in form of gravitational waves. D. Christodoulou showed that every gravitational-wave burst has a nonlinear memory. In this talk, we discuss the null asymptotics for spacetimes solving the Einstein-Maxwell (EM) equations, compute the radiated energy and derive limits at null infinity and compare them with the Einstein vacuum (EV) case. Here, we rely on the methods introduced in the works of Christodoulou, Klainerman, Bieri and Zipser.

Speaker: **Pieter Blue** (University of Edinburgh)
Title: *Hidden symmetries and decay for the wave equation outside a Kerr black hole*
Abstract: The Kerr solutions to Einstein's equations describe rotating black holes. For the wave equation in flat-space and outside the non-rotating, Schwarzschild black holes, one method for proving decay is the vector-field method, which uses the energy-momentum tensor and vector-fields. Outside the Schwarzschild black hole, a key intermediate step in proving decay involved proving a Morawetz estimate using a vector-field which pointed away from the photon sphere, where null geodesics orbit the black hole. Outside the Kerr black hole, the photon orbits have a more complicated structure. By using the hidden symmetry of Kerr, we can replace the Morawetz vector-field by a fifth-order operator which, in an appropriate sense, points away from the photon orbits. This allows us to prove the necessary Morawetz estimate, also called a local energy decay estimate, which is a key step in proving pointwise decay estimates.

This is joint work with Lars Andersson.

Speaker: **Hubert Bray** (Duke University)
Title: *On Dark Matter, Spiral Galaxies, and the Axioms of General Relativity*
Abstract: We define geometric axioms for the metric and the connection of a spacetime where the gravitational influence of the connection may be interpreted as dark matter. We show how these axioms lead to the Einstein-Klein-Gordon equations with a cosmological constant, where the scalar field of the Klein-Gordon equation represents the deviation of the connection from the standard Levi-Civita connection on the tangent bundle and is interpreted as dark matter. This form of dark matter is compatible with the ΛCDM cosmological model of the universe. In addition, unlike the WIMP model of dark matter, this dark matter is automatically cold (as is observed) in a homogeneous, isotropic universe. Finally, we show how this scalar field dark matter, which naturally forms dark matter density waves due to its wave nature, may cause the observed barred spiral pattern density waves in many disk galaxies and triaxial shapes with plausible brightness profiles in many elliptical galaxies. If correct, this would provide a unified explanation

for spirals and bars in spiral galaxies and for the brightness profiles of elliptical galaxies. We compare the results of preliminary computer simulations with photos of actual galaxies.

Speaker: **Piotr Chrusciel** (University of Vienna)
Title: *5 dimensional black holes*
Abstract: I will discuss the global structure of Emparan-Reall black-rings, and present evidence that the Pomeransky-Senkov black rings and the Elvang-Figueras black saturns are well behaved black hole space-times.

Speaker: **Mihalis Dafermos** (University of Cambridge)
Title: *Superradiance, trapping, and decay for waves on Kerr spacetimes in the general subextremal case* $|a| < M$

Speaker: **Sergio Dain** (University of Cordoba)
Title: *Linear perturbations and mass conservation for axisymmetric Einstein equations*
Abstract: In axial symmetry, there exists a gauge for Einstein equations such that the total mass of the spacetime can be written as a conserved, positive definite, integral on the spacelike slices. This property is expected to play an important role in the global evolution. In this gauge the equations reduce to a coupled hyperbolic-elliptic system which is formally singular at the axis. Due to this singular behavior, the local in time existence of this system can not be analyzed by standard methods. To analyze the principal part of the equations, which represents the main source of the difficulties, we study linear perturbation of the flat solution in this gauge. We prove existence and uniqueness of solutions of this singular linear system. The solutions are obtained in terms of integral transformations in a remarkable simple form. This representation is suitable for proving useful estimates for the non-linear case. We believe that this result open the door to the study of the full Einstein equations in this gauge. This work is in collaboration with Martn Reiris and the related article is in arXiv:1005.5347.

Speaker: **Michael Eichmair** (Massachusetts Institute of Technology)
Title: *An overview of Jang's equation and its relation to MOTS*
Abstract:I will provide an overview of Jang's equation as a technique to study existence and properties of marginally outer trapped surfaces. The discussion will include a sketch of several recent applications (joint work with J. Metzger) regarding mixed blow up behavior of Jang's equation at outermost surfaces.

Speaker: **Gustav Holzegel** (Princeton University)
Title: *Asymptotic behavior of spacetimes approaching a Schwarzschild solution*
Abstract: Consider a spacetime which approaches a Schwarzschild solution. We will discuss the following problem: Assuming decay of appropriate norms of the Ricci rotation coefficients and their derivatives, can one prove boundedness/ decay for the curvature components and their derivatives? The talk will give a positive answer to this question and explain some of the difficulties arising from the fact that not all curvature components decay. As an important ingredient, we generalize recent work of Dafermos and Rodnianski regarding decay for the wave equation to the setting of the Bianchi equations.

Speaker: **Gerhard Huisken** (Max-Planck-Institute for Gravitational Physics)
Title: *Foliations, flows and rigidity in asymptotically flat 3-manifolds*

Speaker: **Marcus Khuri** (SUNY-Stony Brook)
Title: *The Static Metric Extension Conjecture*
Abstract: There are several competing definitions of quasi-local mass. A very promising and natural candidate, proposed by R. Bartnik, seeks to localize the ADM mass. Fundamental to understanding Bartnik's construction is the question of existence and uniqueness for a canonical geometric boundary

value problem associated with the static vacuum Einstein equations. In this talk we will report on joint work with M. Anderson, which confirms that existence holds (under a nondegeneracy condition) but also shows that uniqueness fails. The possible implications of this result will be discussed.

Speaker: **Marc Mars** (Universidad de Salamanca)
Title: *The Bray and Khuri approach to the general Penrose inequality in two particularly simple cases*
Abstract: Bray and Khuri have put forward an interesting new approach to address the Penrose inequality for arbitrary initial data sets. The two simplest possible situations where one can think of applying these results involve spherically symmetric initial data and static initial data. In the spherical case I will extend previous results by Bray and Khuri when the outermost horizon is strictly stable and non-minimal to the general case. In the static case, I will show that there are slices of the Kruskal spacetime with mass m for which the outermost generalized apparent horizon has area strictly larger that $16\pi m^2$ and will present a Penrose inequality for general static initial data sets satisfying the dominant energy condition, provided the mean curvature of the degenerate components satisfies an integral inequality.

Speaker: **David Maxwell** (University of Alaska Fairbanks)
Title: *The Conformal Method and Concrete Examples*
Abstract: Recent advances by Holst, et. al and Maxwell concerning the conformal method and non-CMC initial data might have led an optimist to conjecture that the conformal method could be just as effective for constructing non-CMC data as it is in the CMC case. In this talk I'll discuss concrete examples that indicate that this is not true. For certain reasonable conformal data violating a near-CMC condition we find that there cannot be a unique solution: there will either be no solutions or there will be more than one. In many of these cases we find with definiteness that there are multiple solutions. These concrete examples are independently interesting as they exhibit a number of new phenomena for the conformal method, including existence of certain solutions under a very weak near-CMC hypotheses, explicit dependence on the choice of conformal class representative, and extreme sensitivity of the solution theory with respect to a coupling constant in the Einstein constraint equations.

Speaker: **Frans Pretorius** (Princeton University)
Title: *The instability of 5-dimensional black strings*
Abstract: 5 dimensional black strings were shown to be unstable to long-wavelength perturbations by Gregory and Laflamme. Entropy considerations imply the prefered end-state of the unstable spacetime is a sequence of black holes with spherical topology. For this to happen, the black string event horizon would have to bifurcate, accompanied by a naked singularity. This would be an example of generic violation of cosmic censorship in 5 dimensional Einstein gravity. We present results from recent numerical evolution attempting to illucidate the end-state of the Gregory-Laflamme instability.

Speaker: **Alan Rendall** (Max-Planck-Institute for Gravitational Physics)
Title: *Higher dimensional cosmological models*
Abstract: In this talk I present some results of Arne Goedeke and myself on solutions of the vacuum Einstein equations in dimensions greater than four. The central question discussed is whether spatially homogeneous models which are forever expanding are geodesically complete in the future. I recall a proof that this is true in four dimensions and note that it does not generalize directly. I then introduce a sufficient condition for completeness and show that it is satisfied in a class of models of dimension five by means of Kaluza-Klein reduction. Finally I discuss the prospects for obtaining a more global understanding of this problem.

Speaker: **Hans Ringström** (Institutionen för Matematik - KTH)
Title: *Models of the universe with arbitrary compact spatial topology*
Abstract: The current standard model of the universe is spatially homogeneous, isotropic and spatially flat. Furthermore, the matter content is described by two perfect fluids (dust and radiation) and there is

a positive cosmological constant. Such a model can be well approximated by a solution to the Einstein-Vlasov equations with a positive cosmological constant. As a consequence, it is of interest to study stability properties of solutions in the Vlasov setting. This is one of the topics of the talk. It is also of interest to analyze to what extent the assumption that the universe is very close to the standard model to our past restricts the spatial topology of the universe. This is the main subject of the talk.

Speaker: **Igor Rodnianski** (Princeton University)
Title: *On formation of trapped surfaces*
Abstract: The talk will describe the ingredients of the recent results on formation of trapped surfaces. We will review the heuristic mechanism, the short-pulse method and the evolutionary control introduced and exploited by Christodoulou in his breakthrough work. We will also explain recent extensions, amplifications and generalizations of the result as well as future directions.

Speaker: **Richard Schoen** (Stanford University)
Title: *Singularities in positive mass arguments*
Abstract: We will discuss types of singularities which allow positive mass arguments to work. There have been a few instances when singularities have been shown to be allowable, but there is no comprehensive characterization of them.

Speaker: **Mu-Tao Wang** (Columbia University)
Title: *Limit of quasilocal mass and isometric embeddings into Minkowski space*
Abstract: I shall first discuss how the limit of quasilocal mass on a family of surfaces in spacetime can be evaluated in terms of the mean curvature vectors. This gives a uniform description of ADM mass and Bond mass for asymptotically flat and asymptotically null spaces, respectively. Then I shall explain how the related variational problem for quasilocal mass anchors a "ground state" as a hypersurface in Minkowski space.

This is based on joint work with S.-T. Yau.

Speaker: **Shing-Tung Yau** (Harvard University)
Title: *Quasilocal mass in general relativity*