Report on the BIRS workshop

Operator Algebras and Representation Theory: Frames, Wavelets and Fractals

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For the co-organizers:

Brenken, Berndt (University of Calgary),
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Because of an unexpected last minute illness Dr Silvestrov was not able to attend.

October 8–10, 2010

This weekend workshop offered a great opportunity for the participant researchers to further the subject, the ongoing collaborations, and to make headway on a list of problems. It helped us to gain insight into the open questions. The subject is at the cross roads of Operator Theory/Algebras, harmonic analysis, and applied mathematics.

Our workshop has been especially important for the young participants, postdocs and recent Ph.D students. Since many of us have met and collaborated at other conferences, we were able to make headway in the relatively short time. This includes research collaborations, planning, and exchange of ideas. This is vital for research advances, and headway was made with several research papers that had been in the planning stage for some time.

Presentation Highlights

The list of speakers with titles is as follows.

1. Marcin Bownik, Existence of frames with prescribed norms and frame operator
2. Peter Casazza, Kadison-Singer: A few results and a lot of questions
3. Jens Gerlach Christensen, Sampling and representations of Lie groups
4. Dorin Dutkay, Fourier bases on fractal measures
5. Jean Pierre Gabardo, Convolution inequalities and Beurling density of wavelet systems
6. Bin Han, Some results and open problems on nonhomogeneous wavelet systems
7. Deguang Han, Group representation frames: questions and partial results

8. Keri Kornelson, Operators on Bernoulli measures spaces
9. Michael Lamoureux, Generalized frames in seismic imaging
10. Shidong Li, Sparse dual frames and the most compact dual Gabor function
11. Peter Massopust, Exponential B-splines and the partition of unity property
12. Judith Packer , Operators arising from generalized multiresolution analyses
13. Qiyu Sun, Nonlinear Wiener's lemma and numerical implementation

Scientific Progress Made

This weekend workshop has allowed the participants to collaborate and to prepare for future longer workshops with time for in-depth follow-up. While the subject has always been closely related to quantum physics, very recently, other connections to more applied sciences, in particular to engineering, have emerged and stimulated research in mathematics which in turn has led to such interdisciplinary work as: wavelet theory, frame theory, fractals, function spaces related to representations, analysis on loop groups, the geometry of geometric tilings, approximation theory, numerical mathematics, and microlocal analysis; all topics covered in the workshop.

The Kadison-Singer conjecture

Overview of the Field

A main focus was an early problem in operator theory and quantum physics is the Kadison-Singer problem or conjecture on pure states, originating from the work of Paul Dirac: Does every pure state on the algebra of bounded diagonal operators on l^2 have an unique extension to a pure state on the von Neumann algebra of all bounded operators on l^2? In the past few years, it has been shown that this problem in operator theory and physics is equivalent to fundamental unsolved problems in a dozen areas of pure and applied mathematics and engineering. These equivalent problems includes the Paving conjecture, the Feichtinger conjecture in frame theory, and the Bourgain-Tzafriri conjecture in operator theory. This problem also has a strong connection to number theory (frames of exponentials) and to the theory of Toeplitz operators on reproducing kernel Hilbert spaces.

Recent Developments and Open Problems

While the subjects in the workshop are diverse, they were focused on a single conjecture; in turn known to be equivalent to fundamental unsolved problems in a dozen areas of pure and applied mathematics, and even in engineering. In mathematics this includes operator theory, Banach space, harmonic analysis, frame theory, including the theory of fusion frames, and signal/image processing. The first speaker was P. Casazza, who has been a pioneer, and recently, together with co-authors, has made great strides; see e.g., [1]. Renewed interest in these problems has also stimulated new advances in these other diverse areas of mathematics and applications.

Outcome of the Meeting

Frames, and their refinement, fusion frames, like the notion of bases, offer numerical representations of vectors. While the representations are stable, they are typically non-unique, hence their use in applications with intrinsic redundancies: filter bank theory, sigma-delta quantization, image processing, and wireless communications. Other applications to distributed processing and sensor networks in the human brain require clever splitting of large frame systems into sets of (overlapping) smaller systems.

The organizers and the participants have a long history of significant contribution to the field and collaboration, both in research and organization of workshops, special sessions and conferences. This includes

a mini-workshop in Oberwolfach, two workshops on the Kadison-Singer problem at the American Institute of Mathematics, a week-long workshop at Banff International Research Station (BIRS, 2006), and several special sessions at AMS meetings.

The organizers.

References

[1] Casazza, Peter G., Fickus, Mathew, and Tremain, Janet C., The Kadison-Singer problem in mathematics and engineering: a detailed account, Operator theory, operator algebras, and applications, *Contemp. Math.* **414** (2006), 299-355.