Dedekind sums in geometry, topology, and arithmetic
October 11–16, 2009

MEALS

*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday
*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday
*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday
Coffee Breaks: As per daily schedule, 2nd floor lounge, Corbett Hall
***Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

MEETING ROOMS

All lectures will be held in Max Bell 159 (Max Bell Building accessible by walkway on 2nd floor of Corbett Hall). LCD projector, overhead projectors and blackboards are available for presentations. Please note that the meeting space designated for BIRS is the lower level of Max Bell, Rooms 155–159. Please respect that all other space has been contracted to other Banff Centre guests, including any Food and Beverage in those areas.

SCHEDULE

Sunday

16:00	Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
17:30–19:30	Buffet Dinner, Sally Borden Building
20:00	Informal gathering in 2nd floor lounge, Corbett Hall Beverages and small assortment of snacks available on a cash honour-system.

Monday

7:00–8:45	Breakfast
8:45–9:00	Introduction and Welcome to BIRS by BIRS Station Manager, Max Bell 159
9:00–10:00	Karl Dilcher (Dalhousie University), *Reciprocity relations for Bernoulli numbers*
10:00–10:30	Coffee Break, 2nd floor lounge, Corbett Hall
10:30–11:30	Ruth Lawrence (Hebrew University of Jerusalem), *On quantum knot and 3-manifold invariants*
11:30–13:00	Lunch
13:00–14:00	Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
14:00–15:00	Todor Milev (Jacobs University Bremen), *Partial fraction decompositions and an algorithm for computing the vector partition function. The (quasi)periods of the Kostant partition function of the exceptional Lie algebras*
15:00–15:30	Coffee Break, 2nd floor lounge, Corbett Hall
15:30–16:00	Petr Lisonek (Simon Fraser University), *Quasi-polynomials in Combinatorics*
16:15–17:15	Giancarlo Urzua (University of Massachusetts Amherst), *Dedekind sums in Abelian covers and applications*
17:30–19:30	Dinner

Tuesday

7:00–9:00	Breakfast
9:00–10:00	Veli Kurt (Akdeniz University), *Higher Dimensional Dedekind Sums*
10:00–10:30	Coffee Break, 2nd floor lounge, Corbett Hall
10:30–11:00	Yoshinori Hamahata (Tokyo University) *Reciprocity laws of Dedekind sums in characteristic p*
11:10–11:40	Jeffrey Meyer (Syracuse University), *Symmetric Arguments in Dedekind Sums*
11:30–13:00	Lunch
14:00–15:00	Abdelmejid Bayad (Université d'Evry Val d'Essonne), *Some facets of multiple Dedekind-Rademacher sums*
15:00–15:30	Coffee Break, 2nd floor lounge, Corbett Hall
15:30–16:00	Kevin Woods (Oberlin College), *Counting With Rational Generating Functions*
16:10–16:40	Ricardo Diaz (University of Northern Colorado), *A Solid Angle Algorithm for Spherical Polytopes*
17:30–19:30	Dinner

Wednesday

7:00–9:00	Breakfast
9:00–10:00	Yilmaz Simsek (Akdeniz University), *Dedekind-type Sums*
10:00–10:30	Coffee Break, 2nd floor lounge, Corbett Hall
10:30–11:00	Sinai Robins (Nanyang Technological University), *Some Polyhedral Dedekind sums*
11:30–13:30	Lunch
	Free Afternoon
17:30–19:30	Dinner
20:00–?	Problem Session

Thursday

7:00–9:00	Breakfast
9:00–10:00	Robert Sczech (Rutgers University Newark), *Dedekind sums and derivatives of partial zeta functions in real quadratic fields at* $s = 0$
10:00–10:30	Coffee Break, 2nd floor lounge, Corbett Hall
10:30–11:30	Samit Dasgupta (University of California Santa Cruz), *Dedekind Sums and Gross–Stark Units*
11:30–13:00	Lunch
14:00–15:00	Glenn Stevens (Boston University), *Milnor Algebras, Modular Symbols, and Values of L-functions*
15:00–15:30	Coffee Break, 2nd floor lounge, Corbett Hall
15:30–16:00	Pierre Charollois (Institut de Mathématiques de Jussieu (Paris 6)), *Integral Dedekind sums for* $GL_n(\mathbb{Q})$
16:10–16:40	Richard Hill (University College London), *Shintani cocycles*
17:30–19:30	Dinner

Friday

7:00–9:00	Breakfast
9:00–?	Informal Discussions
10:00–11:00	Coffee Break, 2nd floor lounge, Corbett Hall
11:30–13:30	Lunch

Checkout by 12 noon.

** 5-day workshops are welcome to use the BIRS facilities (2nd Floor Lounge, Max Bell Meeting Rooms, Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. **

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ABSTRACTS
(in alphabetic order by speaker surname)

Speaker: **Abdelmejid Bayad** (Université d'Evry Val d'Essonne)
Title: *Some facets of multiple Dedekind-Rademacher sums*
Abstract: We introduce two kind of multiple Dedekind-Rademacher sums, in terms of Bernoulli and Jacobi modular forms. We prove their reciprocity Laws, we establish the Hecke action on these sums and we obtain new Knopp–Petersson identies. We show how to deduce Dedekind's, Rademacher's, Sczech's reciprocity formulas from our main results. Some applications in number theory (special values of some L-functions, Periods, etc.) will be discussed.

Speaker: **Pierre Charollois** (Institut de Mathématiques de Jussieu (Paris 6))
Title: *Integral Dedekind sums for* $GL_n(\mathbb{Q})$
Abstract: This is a report on a joint work with Samit Dasgupta, based on the construction by R. Sczech of a rational valued cocycle for $GL_n(\mathbb{Q})$. Our refinement now provides an integral valued cocycle, that can be expressed by a simpler fomula in terms of Bernoulli numbers.

Speaker: **Samit Dasgupta** (University of California Santa Cruz)
Title: *Dedekind Sums and Gross–Stark Units*
Abstract: In 2006 I stated a conjectural formula for Gross-Stark units over number fields. In this talk I will discuss the role played by Dedekind sums in this formula. I will also look towards the function field setting for inspiration, where the conjectural formula may be proven following work of Hayes and using the theory of Drinfeld modules.

Speaker: **Ricardo Diaz** (University of Northern Colorado)
Title: *A Solid Angle Algorithm for Spherical Polytopes*
Abstract: An algorithmic procedure for computing the spherical measure of spherical polytopes is outlined, based upon downward induction on dimension, via the Divergence Theorem. Potential applications include the determination of canonical solid-angle weights that behave additively under decomposition of lattice polytopes.

Speaker: **Karl Dilcher** (Dalhousie University)
Title: *Reciprocity relations for Bernoulli numbers*
Abstract: We start with the observation that several classical identities for Bernoulli numbers can be written as reciprocity relations, and then prove a new type of three-part reciprocity relation for Bernoulli numbers. As a consequence we obtain a quadratic recurrence for these numbers. This recurrence requires, surprisingly, the knowledge of only one third of the previous numbers.

In a second part of this talk I will give a new elementary proof of the reciprocity law for the classical Dedekind sums, based on uniform distributions of integers in subintervals of the real line. (Joint work with T. Agoh and K. Girstmair).

Speaker: **Yoshinori Hamahata** (Tokyo University)
Title: *Reciprocity laws of Dedekind sums in characteristic p*
Abstract: We introduce Dedekind sums for lattices defined over finite fields and show the reciprocity law for them. Also, we establish the similar thing over function fields.

Speaker: **Richard Hill** (University College London)
Title: *Shintani cocycles*
Abstract: I'll describe an $(n-1)$-cocycle on the group $GL_n(\mathbb{Q})$, taking values in a space of power series called the Shintani functions. The coefficients of these power series are quite general Dedekind sums, and special values of the power series may be used to express special values of L-functions. The cocycle relation encodes the reciprocity laws for the Dedekind sums.

Speaker: **Veli Kurt** (Akdeniz University)
Title: *Higher Dimensional Dedekind Sums*
Abstract: The aim of this work is to construct new Dedekind type sums. We construct generating functions of Barnei Type multiple Frobenius-Euler numbers and polynomials. By applying Mellin transformation to these functions. we define Barnes type multiple l-functions which interpolate Frobenius-Euler numbers at negative integers. By using generalization of the Frobenius-Euler functions. We define generalized Dedekind type sums and prove corresponding reciprocity law.

Speaker: **Ruth Lawrence** (Hebrew University of Jerusalem)
Title: *On quantum knot and 3-manifold invariants*
Abstract: We will survey some of the many known results on the structure of quantum invariants of knots and 3-manifolds, in particular properties of integrability, p-adic convergence and existence of cyclotomic expansions.

Speaker: **Petr Lisonek** (Simon Fraser University)
Title: *Quasi-polynomials in Combinatorics*
Abstract: We present several families of combinatorial structures with the following properties: Each family of structures depends on two or more parameters, and the number of isomorphism types of structures is quasi-polynomial in one of the parameters whenever the values of the remaining parameters are fixed to arbitrary constants. The proofs are based on translating the problem of counting isomorphism types to the problem of counting integer points in a union of parametrized rational polytopes. We discuss the relation of these counting problems with Dedekind sums.

Speaker: **Jeffrey Meyer** (Syracuse University)
Title: *Symmetric Arguments in Dedekind Sums*
Abstract: I investigate the question of which pairs of integers (a, b) have the property that $s(a, b) = s(b, a)$ where $s(a, b)$ is the regular Dedekind sum. There is a strong connection to the Fibonacci sequence. I present some partial results and conjectures on the corresponding questions about some generalized Dedekind sums.

Speaker: **Todor Milev** (Jacobs University Bremen)
Title: *Partial fraction decompositions and an algorithm for computing the vector partition function. The (quasi)periods of the Kostant partition function of the exceptional Lie algebras*
Abstract: Given a finite set I of non-zero integral vectors with nonnegative coordinates, and a vector γ with coordinates $(\gamma^1, \ldots, \gamma^n)$, the vector partition function $P_I(\gamma)$ is by definition the number of ways we can split γ as an integral sum with non-negative coefficients of the vectors in I. There exist finitely many polyhedra such that over each, P_I is a quasipolynomial as a function of the coordinates $(\gamma^1, \ldots, \gamma^n)$.

I will present a definition of a (non-unique) partial fraction decomposition in more than one variable for a special type of rational functions that include the generating functions of arbitrary vector partition functions. Using the Szenes-Vergne formula I will present an algorithm for computing partial fraction decompositions and with it an algorithm for computing the vector partition function as a quasipolynomial. In this way I will algorithmically reprove the well-known fact that the vector partition function is a quasipolynomial over finitely many polyhedra.

The partial fraction decomposition algorithm relies on an arbitrary choice of a "preferred linear dependence" between the vectors in I. Different strategies for making such a choice can be used both mathematically and for making explicit computer computations. I will apply one strategy for a choice of "preferred linear dependence" for the specific case when the finite set I is a positive root system of a simple Lie algebra and will obtain upper bounds for the (quasi)periods of the Kostant partition functions of exceptional Lie algebras E_6, E_7, E_8, F_4, G_2 (the periods are divisors of respectively 6, 12, 60, 12, 6).

A version of the algorithm with two different strategies for "preferred linear dependence" choice has been realized and is publicly available under the Library General Public License v3.0 at

`http://vectorpartition.sourceforge.net/`.

The first strategy works for arbitrary sets I. The second is applicable only to subsets of the positive root systems of the classical Lie algebras A_n, B_n, C_n, D_n.

If time permits I will make an overview of the "vector partition function" program and its C++ realization.

Speaker: **Sinai Robins** (Nanyang Technological University)
Title: *Some Polyhedral Dedekind sums*
Abstract: Generalizations of Dedekind sums that we call polyhedral Dedekind sums occur when we use the Poisson summation formula to analyze various sums over the integer points in a polytope. Similar sums occur in the work of Sczech and Gunnells, but from a different perspective. It is often useful to deal directly with the infinite lattice sums, (as Sczech and Gunnells also do) that have the finite Dedekind sum properties, and here we find a similar phenomenon.

Speaker: **Robert Sczech** (Rutgers University Newark)
Title: *Dedekind sums and derivatives of partial zeta functions in real quadratic fields at* $s = 0$
Abstract: The classical Dedekind sums arise in formulas for special values of partial zeta values in real quadratic fields at $s = 0$. In my talk, I will introduce a new type of Dedekind sum and show how they can be used to calculate derivatives of partial zeta functions at $s = 0$ which show up in Stark's conjecture.

Speaker: **Yilmaz Simsek** (Akdeniz University)
Title: *Dedekind-type Sums*
Abstract: In many applications of **Elliptic Modular Functions** to Number Theory the eta function plays a central role. It was introduced by Dedekind in 1877 and is defined by the upper helf-plane $\mathbb{H} = \{\tau \in \mathbb{C} : \Im(z) > 0\}$ by the equation

$$\eta(\tau) = e^{\frac{\pi i \tau}{12}} \prod_{n=1}^{\infty} (1 - e^{2\pi i n \tau}).$$

The infinite product has the form $\prod_{n=1}^{\infty}(1 - x^n)$ where $x = e^{2\pi i \tau}$. If $\tau \in \mathbb{H}$, then $\mid x \mid < 1$ so the product converges absolutely and is nonzero. Furthermore, since the convergence is uniform on compact subsets of $\mathbb{H}$, $\eta(\tau)$ is analytic on $\mathbb{H}$.

The behavior of this function under the modular group ($= \Gamma(1)$) is given by the following functional equation:

Theorem. *Let* $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in \Gamma(1)$. *Then*

$$\log \eta(Az) = \log \eta(z) + \frac{\pi i (a + d)}{12c} - \pi i (S(d, c) - \frac{1}{4}) + \frac{1}{2} \log(cz + d),$$

where $S(d, c)$ *is called the (classical) Dedekind sum, which is defined by*

$$S(d, c) = \sum_{\mu \bmod c} ((\frac{\mu}{c}))((\frac{d\mu}{c})),$$

where d and c are coprime integers with $c > 0$, and $((x))$ is defined by

$$((x)) = \begin{cases} x - [x]_G - \frac{1}{2}, & x \text{ is not an integer} \\ 0, & \text{otherwise} \end{cases}$$

where $[x]_G$ is the largest integer $\leq x$.

The above theorem is very important in the theory of the Dedekind sums and in Analytic Number Theory. Therefore, this theorem have been studied by many Mathematicians. In 1950, Apostol gneralized Dedekind sums $S_p(h,k)$ as follows:

$$S_p(h,k) = \sum_{a \bmod k} \frac{a}{k} \overline{B}_p(\frac{ah}{k}),$$

where h and k are coprime positive integers and $\overline{B}_p(x)$ is the p-th Bernoulli function. Apostol also proved the following theorem, which is very important to define new generating functions of the Dedekind type sums:

Theorem. *Let $(h,k) = 1$. For odd $p \geq 1$, we have*

$$S_p(h,k) = \frac{p!}{(2\pi i)^p} \sum_{\substack{m=1 \\ m \not\equiv 0(k)}}^{\infty} m^{-p}(\frac{e^{2\pi imh/k}}{1-e^{2\pi imh/k}} - \frac{e^{-2\pi imh/k}}{1-e^{-2\pi imh/k}}).$$

In this talk, we also give some fundamental properties of Dedekind sums. By p-adic q-Volkenborn integral and generating functions of Bernoulli numbers, we give q-analogue of family of the zeta functions. We give relation between p-adic q-Volkenborn integral and p-adic q-Dedekind type sums. Furthermore, p-adic interpolation function of the q-Dedekind type sums are given. We also give the others p-adic q-Dedekind type sums.

Speaker: **Glenn Stevens** (Boston University)
Title: *Milnor Algebras, Modular Symbols, and Values of L-functions*
Abstract: In this talk we construct a family of *Eisenstein modular symbols* over $GL_n(\mathbb{Q})$ taking values in the the Milnor algebra of a certain ring of trigonometric functions. Higher Dedekind sums arise as certain coefficients of this modular symbol and homological relations correspond to standard Dedekind reciprocity laws. A curious feature of the Eisenstein symbol (for $n = 2$) is that it contributes to both $\pm$-eigenspaces for complex conjugation acting on the cohomology of modular curves. This contrasts with periods of Eisenstein series, which contribute only to the *odd*-eignenspace. We illustrate this phenomena with a theorem of Cecelia Busuioc, proving congruence formulas for *even* critical values of L-functions of cusp forms satisfying Eisenstein congruences, as conjectured by Romyar Sharifi.

Speaker: **Giancarlo Urzua** (University of Massachusetts Amherst)
Title: *Dedekind sums in Abelian covers and applications*
Abstract: Dedekind sums appear naturally in the computation of Chern numbers of Abelian covers of algebraic varieties. I will show how, and some applications of this to Dedekind sums and to simply connected smooth projective surfaces of general type. For surfaces, our method to obtain high Chern ratios involves a large scale behavior of Dedekind sums and continued fractions described recently by K. Girstmair. Thanks to this, everything is encoded in certain invariants of the branch divisor. I will describe them and a bit of their geography.

Speaker: **Kevin Woods** (Oberlin College)
Title: *Counting With Rational Generating Functions*
Abstract: A step-polynomial is created by taking sums and products of the floor functions of degree

one polynomials (in one or more variables). Like Dedekind sums, step-polynomials are closely tied to lattice point enumeration and related generating function methods. As an example, consider the Ehrhart quasi-polynomial, $f(t)$, of a rational polytope, P (that is, $f(t)$ counts the number of integer points in tP). One often considers the Hilbert series which is the generating function obtained by summing $f(t)x^t$ over all nonnegative t. This generating function has the advantage that it can be manipulated through algebraic means. On the other hand, we will see that $f(t)$ can be written as a step-polynomial (and the algorithm to find it is efficient). This representation has the advantage that it is an explicit function that can immediately be evaluated for any t. Fortunately, we do not have to choose between nimble generating functions and concrete step-polynomials, as one can convert back and forth between them in polynomial time (in fixed dimension). This is joint work with Sven Verdoolaege.