09w5071 Invariants of Incidence Matrices
Arriving March 29 and departing April 3, 2009

MEALS

*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday
*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday
*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday
Coffee Breaks: As per daily schedule, 2nd floor lounge, Corbett Hall
***Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

MEETING ROOMS

All lectures will be held in Max Bell 159 (Max Bell Building accessible by walkway on 2nd floor of Corbett Hall). LCD projector, overhead projectors and blackboards are available for presentations. Please note that the meeting space designated for BIRS is the lower level of Max Bell, Rooms 155–159. Please respect that all other space has been contracted to other Banff Centre guests, including any Food and Beverage in those areas.

SCHEDULE

Sunday	
16:00	Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
17:30–19:30	Buffet Dinner, Sally Borden Building
20:00	Informal gathering in 2nd floor lounge, Corbett Hall
	Beverages and small assortment of snacks available on a cash honour-system.
Monday	
7:00–8:45	Breakfast
8:45–9:00	Introduction and Welcome to BIRS by BIRS Station Manager, Max Bell 159
9:00–10:00	Eric Moorhouse (This talk will be videotaped.)
10:00–10:30	Coffee Break
10:30–11:30	Vladimir Tonchev
11:30–13:00	Lunch
13:00–14:00	Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
14:00	Group Photo; meet on the front steps of Corbett Hall
14:20–14:45	Coffee
14:45–15:45	Peter Sin (This talk will be videotaped.)
16:00–17:00	Ogul Arslan
17:30–19:30	Dinner
Tuesday	
7:00–9:00	Breakfast
9:00–10:00	Andries Brouwer
10:00–10:30	Coffee Break
10:30–11:30	Dave Saunders
11:30–13:30	Lunch
13:30–14:30	Willem Haemers
14:30–15:00	Coffee
15:00–16:00	David Chandler
16:15–17:15	Leo Storme
17:30–19:30	Dinner

Wednesday

7:00–9:00	Breakfast
9:00–10:00	Aart Blokhuis
10:00–10:30	Coffee Break
10:30–11:30	Yaokun Wu
11:30–13:30	Lunch
	Free Afternoon
17:30–19:30	Dinner

Thursday

7:00–9:00	Breakfast
9:00–10:00	Rick Wilson
10:00–10:30	Coffee Break
10:30–11:30	Reza Khosrovshahi
11:30–13:30	Lunch
13:30–14:30	Navin Singhi
14:30–15:00	Coffee Break
15:00–16:00	Ulrich Dempwolff
16:15–17:15	Behruz Tayfeh-Rezaie
17:30–19:30	Dinner

Friday

7:00–9:00	Breakfast
9:00–10:00	Aiden Bruen
10:00–10:15	Short Coffee Break
10:15–11:15	Richard Anstee
11:30–13:30	Lunch

Checkout by 12 noon.

** 5-day workshops are welcome to use the BIRS facilities (2nd Floor Lounge, Max Bell Meeting Rooms, Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. **

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ABSTRACTS

(in alphabetic order by speaker surname)

Speaker: Richard Anstee (University of British Columbia)
Title: Forbidden configurations and indicator polynomials
Abstract: Let F be a $k \times l$ (0,1)-matrix. We say that a (0,1)-matrix A has F as a *configuration* if some row and column permutation of F is a submatrix of A.

We are interested in *simple* matrices, namely (0,1)-matrices with no repeated columns. If A is a simple matrix and has no configuration F then what can we deduce about A? Our extremal problem is given m, F to determine the maximum number of columns in an m-rowed simple matrix A which has no configuration F.

If A has no configuration F then each k-set S of rows, the matrix $A|_S$ will have some columns only appearing a limited number of times else A will have F. If for each k-set S of rows, the matrix $A|_S$ is missing at least one column then A has at most $O(m^{k-1})$ columns by the result of Sauer, Perles and Shelah and Vapnik and Chervonenkis. Assume A has m rows and let $x_1, x_2, \ldots, x_m$ be m variables. For set of k rows S for which the matrix $A|_S$ has certain columns that appear a bounded number of times we can readily find a degree k multilinear polynomial (in m variables) that when evaluated at an m-rowed (0,1)-column $(a_1, a_2, \ldots, a_m)$ by setting $x_i = a_i$, we have that the polynomial is non zero if the column restricted to the rows S is equal to a certain k-rowed column. It is not hard to find a degree $k-1$ multilinear polynomial that when evaluated at an m-rowed (0,1)-column $(a_1, a_2, \ldots, a_m)$ by setting $x_i = a_i$, we have that the polynomial is non zero if the column restricted to the rows S is equal to one of two columns. This idea was used by the authors and Füredi and Sali to establish one forbidden configuration bound. We extend this idea to establish what was a harder forbidden configuration bound.

Speaker: Ogul Arslan (University of Florida)
Title: Weyl modules and p-ranks
Abstract: We use filtration of Weyl modules and the Jantzen Sum formula to find the dimensions of some simple submodules for algebraic groups. This has an application to p-rank problems. In particular, we can get the p-ranks of the point-hyperplane incidence matrices of the Hermitian quadrangles $H(3, q)$, and $H(4, q)$ and its dual and point-polar hyperplane incidence matrix of orthogonal projective spaces.

Speaker: Aart Blokhuis (Eindhoven University of Technology)
Title: Covering all points but one, variations on a theme by Jamison and Brouwer-Schrijver.
Abstract: In many point-line geometries (or more generally point-hyperplane geometries) more lines are needed (much more hyperplanes are needed) to cover all points except one, than to cover all points. Bounds can be given by looking at the dimension of the space of functions induced by polynomials of bounded degree.

Speaker: Andries Brouwer (Eindhoven University of Technology)
Title: On the p-rank of the adjacency matrices of strongly regular graphs
Abstract: Let G be a strongly regular graph with adjacency matrix A. Let I be the identity matrix, and J the all-1 matrix. Let p be a prime. Our aim is to study the p-rank (that is, the rank over $\mathbb{F}_p$, the finite field with p elements) of the matrices $M = aA + bJ + cI$ for integral a, b, c.

Speaker: Aiden Bruen (University of Calgary)

Title: On the ranks of incidence matrices–algebra, geometry and combinatorics

Abstract: We discuss general bounds on the ranks of incidence matrices over various fields. These bounds, developed by algebraic means, are due variously to Lander, Bruen and Ott, de Caen, Godsil, Royle, and Hillebrandt. Improvements are offered in some special cases. A side-issue on when linear spaces have full rank is discussed.

In the second part of the lecture we impose a constraint involving certain "forbidden configurations" on the incidence matrix as well as some regularity conditions. Examples are presented. Finally, a new bound on the rank and structure of such matrices, which turns out to be best possible, is described.

Speaker: David B. Chandler (University of Delaware)

Title: Incidence matrices, p-adic representations, and Smith normal forms, with some applications

Abstract: Let $\mathbb{F}_q$ be a finite field of characteristic p. While the p-ranks of the incidence matrices of points versus r-flats in projective n-space over $\mathbb{F}_q$ were given by Hamada's formula, the Smith normal form was not in general known, except the p'-part, until 2003. In joint work with Sin and Xiang, we consider the permutation action of the general linear group over a p-adic ring, and extend it to the action on the characteristic function of an r-flat. The result is a filtration of finite-field modules by powers of p. The dimensions of these modules give the number of invariants of the incidence matrix whose p-part is a specified power. The bases for these modules turn out to be monomials in the coordinate functions of the underlying vector space. The corresponding power of p is given by a simple formula on the p-adic expansion of the exponents in the monomial.

Applying this point of view to unitals embedded in a finite desarguesian plane of order $q = p^t$, we find gievn a Hermitian unital $\mathcal{H}$ and an arbitrary unital $\mathcal{U}$ embedded in $PG(2, q)$, one has $|\mathcal{H} \cap \mathcal{U}| \equiv 1 \pmod{p^{\lceil t/2 \rceil}}$. In the case when $\mathcal{U}$ is of Buekenhout-Metz type, the above result can be strengthened to $|\mathcal{H} \cap \mathcal{U}| \equiv 1 \pmod{q}$.

Speaker: Ulrich Dempwolff

Title: Invariants of semilinear transformations

Abstract: The topic of this talk are normal forms of semilinear operators. We report of some old and some recent results. Invariants characterizing semilinear transformations are presented. We discuss concrete constructions and examples.

Speaker: Willem Haemers (Tilburg University)

Title: On the p-ranks of the adjacency matrices of distance-regular graphs

Abstract: We will survey results on the topic in the title, mainly based on a paper with the same title by (my former student) René Peeters, which appeared in J. Alg. Combinatorics 15 (2002). The abstract of that paper reads: Let G be a distance-regular graph with adjacency matrix A. Let I be the identity matrix and J the all-one matrix. Let p be a prime. We study the p-rank of the matrices $A + bJ - cI$ for integral b, c and the p-rank of corresponding matrices of graphs cospectral with G. Using the minimal polynomial of A and the theory of Smith normal forms we first determine which p-ranks of A follow directly from the spectrum and which, in general, do not. For the p-ranks that are not determined by the spectrum (the so- called relevant p-ranks) of A the actual structure of the graph can play a role, which means that these p-ranks can be used to distinguish between cospectral graphs. We study the relevant p-ranks for some classes of distance-regular graphs and try to characterize distance-regular graphs by their spectrum and some relevant p-rank.

Speaker: G.B. Khosrovshahi (Institute for Research in Fundamental Sciences (IPM), Iran)

Title: Some remarks on (t, k)-inclusion matrices

Abstract: In this talk I intend to review the approach we have taken to study the $W^v_{t,k}$ matrices. When we started to work on the subject, we devised a very easy and good looking basis for the $\ker(W^v_{t,k})$, then we used this basis to produce t-designs and halving designs in particular. But later on as Wilson's $M^v_{t,k}$

matrix, appeared in the literature we focused on studying different aspects of $W^v_{t,k}$ and $M^v_{t,k}$. I will describe the notions of rank, poset, the new $\overline{W}^v_{t,k}$ matrix and some difficulties that we are facing.

Speaker: Eric Moorhouse (University of Wyoming)
Title: p-ranks of nets
Abstract: A k-net of order n is a partial linear space having n^2 points and nk lines of size n, in k parallel classes of n lines each. We are interested in knowing the possible values for the p-rank of the incidence matrix of such a net, for primes p dividing n. As motivation for considering this problem, I will describe the application to certain basic questions of existence of projective planes of given order n. I will then proceed to outline recent limited progress in the case of a 4-net of prime order p.

Speaker: David B. Saunders (University of Delaware)
Title: Computation of rank and Smith normal form.
Abstract: We'll give an overview of LinBox capabilities applicable to the matrix equivalence invariant, rank, over a finite field and the matrix equivalence invariant, Smith normal form, over the integers. LinBox is a C++ library of functions for linear algebra over the integers and over finite fields. In computation over the integers there can be considerable problems of intermediate expression swell. For instance, Gaussian elimination produces the LU decomposition whose entries normally require many times more memory space than the original matrix. LinBox vastly reduces the intermediate expression swell problem by computing modulo word sized primes and then using either the Chinese Remainder Theorem or Hensel lifting to produce integer answers. For computation over finite fields, we make a distinction between sparse and dense matrices. For sparse matrices the primary tools are the so called blackbox methods which avoid the problems of fill-in encountered by elimination techniques. For dense matrices the primary tool is the use of the highly tuned numeric BLAS library, used in a way that exactness of arithmetic is preserved.

The talk begins and ends with open problems. The first of these concerns 3-ranks of the adjacency matrices of a family of strongly regular graphs defined by using Dickson semifields, a computational problem posed to us by Xiang. We have computed the 3-ranks for orders $3^2, 3^4, \ldots, 3^{14}$. These ranks are 20, 85, 376, 1654, 7283, and 32064 respectively. Observe that these numbers satisfy a 4-term linear recurrence sequence with minimal polynomial $x^3-4x-2x+1$. We conjecture that the sequence given by this recurrence relation is in fact the sequence of ranks of Dickson's family. We hope soon to strengthen or disprove the conjecture by computing the 3-rank of the next matrix, order 3^{16}. The computational tools needed for this will be discussed.

Our primary tool for Smith form of dense matrices is an "engineered" combination of several techniques. The major components of this will be discussed and experience with it reported. A similar approach applies to sparse matrices, where, however, we have a less than optimal solution for an important subproblem. It is our second open problem, to compute the Smith form over the PIR Z_{p^e} in a memory efficient way.

Speaker: Peter Sin (University of Florida)
Title: p-ranks of incidence matrices and modular representations of classical groups.
Abstract: Many incidence relations arising in classical projective geometry, such as the incidence between points an isotropic flats of a given dimension, are invariant under the action of the corresponding classical group. The invariants of the relation then have representation-theoretic significance. For example, the p-ranks of incidences of the type "points versus polar hyperplanes" can be shown to be of the form $1+d^t$, where t is the exponent in the field order $q = p^t$, and d is the dimension of an irreducible group representation. Other examples of recent progress using this approach include the p-ranks of generalized quadrangles and the p-ranks of points versus symplectic flats of a symplectic vector space. In the first half of the talk I will describe the formulation of p-rank problems in terms of representation theory of finite and algebraic classical groups and explain some of the tools from this theory which have been used. The second half will discuss recent joint work with Chandler and Xiang on subspaces of a symplectic space. By studying the action of the symplectic group we obtained a symplectic analogue of Hamada's sum formula for the p-rank

of points versus sympectic flats of a given dimension. In the special case where the vector space dimension is 4, our results include closed formulae for the p-ranks of the symplectic generalized quadrangles.

Speaker: Navin Singhi (Tata Institute of Fundamental Research, India)
Title: Tags on subsets and functions
Abstract: Let $X = \{x_1, x_2, \ldots, x_v\}$, be a finite totally ordered set, $x_1 < x_2 < \cdots < x_v$. An ℓ-tag, $v \geq 2\ell$, is an ℓ-subset $\{y_1, y_2, \ldots y_\ell\}$, $y_1 < y_2 < \cdots < y_\ell$, of X such that $y_i \geq x_{2i}$, $1 \leq i \leq \ell$. A 0-tag is the empty set. The set of all ℓ-subsets, $\binom{X}{\ell}$ is totally ordered under the lexicographic ordering. An ℓ-tag is the largest element under the lexicographic ordering in the support of a signed t-(v, k, λ) design on X with $t = \ell$ and $\lambda = 0$.

Each ℓ-tag has a natural dual, which is an ℓ-tag in the reverse ordering. Also with each k-subset of X, a unique ℓ-tag, $0 \leq \ell \leq k \leq v - \ell$, is associated such that the k-subset contains the ℓ-tag and is disjoint with its dual. This gives a natural partition of the set $\binom{X}{k}$ into subsets of sizes $\begin{pmatrix} v \\ \ell \end{pmatrix} - \begin{pmatrix} v \\ \ell - 1 \end{pmatrix}$, $0 \leq \ell \leq k$.

This decomposition is similar to a canonical decomposition of the $\mathbb{Z}$-module of all integral valued functions on $\binom{X}{k}$, studied by Wilson, Graver, Jukert, Graham, Li, Li, Shrikhande and Ray-Chaudhuri etc., to study the existence conjecture for $t-$designs and other similar problems. Just like the concept of the tag on a k-subset, one can define a tag on a function on $\binom{X}{k}$. Tags on subsets and such functions provide a strong tool to study various existence problems like existence of t-designs or characterization of degree sequences of k-uniform hypergraphs etc. These aspects will be discussed in the lecture.

Speaker: Leo Storme (Ghent University)
Title: Small weight codewords in the linear codes arising from incidence matrices from finite projective spaces.
Abstract: The p-ary linear codes $C_k(n, q)$ defined by the incidence matrices of points and k-dimensional spaces of the Desarguesian projective spaces $\mathrm{PG}(n, q)$, $q = p^h$, p prime, $h \geq 1$, have received great attention.

It is known that the minimum weight codewords of $C_k(n, q)$ have weight $q^k + q^{k-1} + \cdots + q + 1$ and are equal to the scalar multiples of the incidence vectors of the k-dimensional subspaces of $\mathrm{PG}(n, q)$ [1]. Regarding the second weight of these linear codes $C_k(n, q)$, the difference of the incidence vectors of two k-dimensional subspaces intersecting in a $(k-1)$-dimensional subspace is a codeword of weight $2q^k$. The question arises whether this number $2q^k$ effectively is the second non-zero weight of the linear codes $C_k(n, q)$.

Chouinard proved in his PhD thesis that this indeed is true for the linear codes $C_1(2, p)$, defined by the incidence matrices of the projective planes $\mathrm{PG}(2, p)$, p prime [2, 3].

We obtained the following extensions to the results of Chouinard [4]:

(1) the second non-zero weight of the linear codes $C_{n-1}(n, q)$, $q = p^h$, p prime, $h \geq 1$, $p > 5$, is equal to $2q^{n-1}$,

(2) the second non-zero weight of the linear codes $C_k(n, p)$, p prime, $p > 5$, $1 \leq k \leq n - 1$, is equal to $2p^k$.

References

[1] E.F. Assmus, Jr. and J.D. Key, Designs and their codes. *Cambridge University Press*, 1992.

[2] K. Chouinard, Weight distributions of codes from planes (PhD Thesis, University of Virginia) (August 1998).

[3] K. Chouinard, On weight distributions of codes of planes of order 9. *Ars Combin.* **63** (2002), 3–13.

[4] M. Lavrauw, L. Storme, P. Sziklai, and G. Van de Voorde, An empty interval in the spectrum of small weight codewords in the code from points and k-spaces of $PG(n,q)$. *J. Combin. Theory, Ser. A*, to appear.

Speaker: Behruz Tayfeh-Rezaie (Institute for Studies in Theoretical Physics and Mathematics (IPM), Iran)
Title: Spectral characterization of some families of graphs
Abstract: A graph is said to be determined by the spectrum (DS for short) if there is no other non-isomorphic graph with the same adjacency spectrum. In this talk, we consider the problem of spectral characterization of some families of graphs. We show that the so called θ-graph is always DS except possibly if it contains a unique 4-cycle. Some other classes of graphs like starlike trees, graphs with small spectral radius and Hamming graphs are also discussed.

Speaker: Vladimir D. Tonchev (Michigan Tech. University)
Title: Polarities, Quasi-symmetric designs, and Hamada's conjecture
Abstract: We prove that every polarity of $PG(2k-1,q)$, where $k \geq 2$, gives rise to a design with the same parameters and the same intersection numbers as, but not isomorphic to $PG_k(2k,q)$. In particular, the case $k=2$ yields a new family of quasi-symmetric designs. We also show that our construction provides an infinite family of counterexamples to Hamada's conjecture, for any field of prime order p.

Speaker: Richard M. Wilson (Caltech)
Title: Set inclusion matrices, diagonal forms, and some applications
Abstract: We review some history of the subset inclusion matrices W_{tk} (with rows indexed by the t-subsets and columns by the k-subsets of a v-set). Results about the p-rank and diagonal forms of these matrices can be applied to a certain zero-sum Ramsey-type problem about uniform hypergraphs, and to construct hypergraphs with evenly distributed subhypergraphs.

Speaker: Yaokun Wu (Shanghai Jiao Tong University, P.R. China)
Title: Incidence matrix and cover matrix of a relative simplicial complex over the binary field
Abstract: Let $(P,<)$ be a poset. The *incidence matrix* of P takes value 1 at its (x,y)-entry if $x<y$ and takes value 0 otherwise; The *cover matrix* of P takes value 1 at its (x,y)-entry if x is covered by y in P and takes value 0 otherwise.

A *relative simplicial complex* P is a set system on a finite ground set such that $A,B \in P$ with $A \subseteq B$ implies that $C \in P$ for any C satisfying $A \subseteq C \subseteq B$. We view a relative simplicial complex as a poset by choosing the usual set inclusion relation as the partial order. We prove that the incidence matrix and the cover matrix of a relative simplicial complex are similar over binary field. This is joint work with Andreas Dress.