Topics in von Neumann algebras

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March 23–28, 2008

The emphasis of the workshop was given to active areas in the theory of von Neumann algebras with connections to other fields as well as to these other fields themselves. Among the participants there were many leading researchers in the theory of von Neumann algebras, but also representatives from group theory, quantum computing and conformal field theory. In particular, a plenary speaker of the ICM, U. Haagerup, and several other invited speakers at various ICM conferences, including D. Bisch, N. Ozawa and R. Longo were among the participants. There were also many younger participants (including recent Ph.D.'s, graduate students, and a good representation of women) who had the opportunity to interact with these leaders. Participants expressed in numerous occasions that the workshop was very stimulating and allowed for fruitful discussions of joint projects. As perhaps the most striking example N. Ozawa was stimulated by the logician A. Tornquist to write a new paper which has already been posted. So we feel that the objectives for this workshop were fulfilled.

1 Short overview of the field and topics targeted in the workshop

Von Neumann algebras are algebras of bounded linear operators on a Hilbert space which are closed under the topology of pointwise convergence. If their center only consists of multiples of the identity, they are called factors. Von Neumann algebras were first studied in a series of papers by Murray and von Neumann in the 1930's, such as in [MvN]. Their motivation was to have a tool for studying quantum mechanics and representations of infinite groups. As it will be seen below, these are still some of the major driving forces in research related to von Neumann algebras, with exciting recent developments.

In order to better describe the structure of the workshop, we will roughly divide the recent activities in von Neumann algebras into the study of subfactors, the interplay between groups and factors, and other developments.

1. *Subfactors.* The study of subfactors was initiated by V. Jones in the 1980's by introducing an important invariant for them called the index, [J1]. Moreover, he proved a surprising and fundamental theorem on the set of possible index values and he produced an important class of examples called the Jones subfactors. This class of examples carried a representation of braid groups, and was later used to define link invariants, [J2]. This in turn led to invariants of 3-manifolds and to connections to conformal field theories, fusion categories and quantum computing (see e.g. [Wi], [RT], [Wa], [We] and [F]). Some of these connections will be discussed below.

An important classification result for amenable subfactors of the hyperfinite II_1 factor was proved by S. Popa, [P1]. He showed that they can be reconstructed by what he calls the standard invariant; it is, however, still a wide open problem what values this standard invariant can take in general. The following topics

are still very active areas in connection with subfactors, which were addressed in the workshop.

(a) *Conformal Field Theory and Subfactors*. Von Neumann algebras have appeared in algebraic quantum field theory for a long time, e.g. in the works of R. Haag, S. Doplicher, J. Roberts, R. Longo and others. More recently, a connection has also been established between Jones' subfactor theory and conformal field theory in the works by D. Evans, Y. Kawahigashi, R. Longo, A. Wassermann [Wa] and and others. Research in this direction has continued and has been reflected in talks by R. Longo on superconformal quantum field theory and by Y. Kawahigashi on super moonshine and operator algebras. Another talk by F. Xu dealt with mirror extensions, by which one can obtain a new net as 'mirror' from a given net.

(b) *Other constructions.* There are also constructions of subfactors via other methods. One of these yields the famous Haagerup subfactor, the irreducible subfactor of the hyperfinite II_1 factor of smallest known index > 4, [AH]. This subfactor was obtained from a list of possible standard invariants provided by U. Haagerup, [H]. It has been shown recently by M. Asaeda and S. Yasuda that only two standard invariants in these list actually do produce subfactors (which had been previously constructed in a joint paper of her with Haagerup). This result was presented at the Banff workshop.

(c) *Planar Algebras*. It is difficult to work directly with the standard invariant of a subfactor. A useful algebraic/combinatorial description of these invariants has been given by V. Jones in the context of planar algebras. New results in this context were presented in the workshop by D. Bisch about planar algebras of group-type subfactors and by V. Sunder. We also mention two talks here primarily dealing with the algebraic structure of subfactors, even though planar algebras do not explicitly appear in these results: In joint work with M. Izumi, P. Grossman gave a complete classification of quadrilaterals (i.e of subfactors with two intermediate subfactors whose intersection is the given subfactor). R. Burstein talked about a certain class of subfactors, constructed via Hadamard matrices, which turn out to be isomorphic to another class of subfactors constructed by D. Bisch and U. Haagerup by a completely different method.

(d) *Quantum Computing*. An approach for building quantum computers, going back to M. Freedman, is based on the theory of anyons; these are quasi-particles whose exchange statistics lies between bosons (phase $= 1$) and fermions (phase $= -1$). They can be studied in the framework of unitary topological quantum field theory and braid statistics, which are intimately related to the theory of subfactors. This approach and its connections to operator algebras were explained in the talk by Z. Wang. Moreover, E. Rowell talked about the role of unitary braid representations and of certain tensor categories in this field.

2. *Factors and groups.* One of the main problems in von Neumann algebras is the classification of II_1 factors. One can define an important class of examples of such factors from discrete groups acting on a measure space; this includes as a special case the group von Neumann algebra. One of the big questions in this context is how much the von Neumann algebra still remembers of the group from which it was constructed. Various invariants for II_1 factors have been introduced by A. Connes, e.g. [Co], and several deep results have been proved by him. He showed all the factors obtained from amenable groups are isomorphic to the hyperfinite II_1 factor. It is known that this factor is not isomorphic to the one obtained from a free group with n generators. It has been a longstanding unsolved problem to decide whether the factors obtained from the free groups with n and m generators respectively are isomorphic if n is not equal to m with both $n, m > 1$. This was one of the motivations for the creation of D. Voiculescu's free probability. There was another Banff workshop on free probability a few months prior to this one. So we shall concentrate on other developments in this context, primarily based on work by S. Popa and his collaborators.

The most exciting developments in the theory of von Neumann algebras in the last few years undoubtedly took place in connection with group theory. D. Gaboriau defined a notion of ℓ^2 Betti numbers for countable measure preserving equivalence relations in a Borel space, [Ga]. This proved a crucial tool in S. Popa's solution of a long-standing problem in von Neumann algebras, the construction of a II_1 factor with trivial fundamental group, [P2]. In addition Popa has continued proving exciting (super)rigidity results concerning group actions on probability spaces. More precisely, he shows for certain groups acting on probability spaces that an equivalence between their orbits already induces an equivalence between the groups them-

selves, e.g. [P3]. Popa's techniques have led to many new recent results in connections with von Neumann algebras coming from groups.

In particular, there are several new constructions of type II_1 factors with prescribed fundamental group. Talks about this were given by C. Houdayer and by S. Vaes (joint work with S. Popa), who use quite different approaches to tackle this problem.

N. Ozawa gave talks on his recent work [OP2] with Popa which contains several deep results on Cartan subalgebras for certain von Neumann factors. For instance, they show the nonexistence of Cartan subalgebras for the tensor product of a free group factor with a subfactor of any tensor product of free group factors, and they also show the uniqueness of the Cartan subalgebra (up to conjugacy) for certain measure space subfactors with respect to free group actions. These are consequences of a general result about the(non)existence of diffuse AFD sub von Neumann algebras in factors satisfying Haagerup complete metric approximation property. Further results in connection with discrete groups acting on measure spaces were presented in the talk by A. Ioana.

J. Peterson talked about von Neumann algebras closed under (Γ)-extensions, which leads to results on maximally injective subalgebras of von Neumann algebras. J. Asher gave a talk on an analog of the Kurosh subgroup theorem in the context of type III_1 factors.

There have been similarly exciting and related results within geometric group theory, e.g by Y. Shalom, N. Monod and A. Furman. We also had some interesting talks in this field:

R. Sauer talked about his joint work with U. Bader and A. Furman on l^1-orbit equivalence rigidity for hyperbolic lattices. Moreover, M. Pichot talked about Wise's non-Hopfian group and showed, among other things, that it has polynomial growth.

3. Other developments in von Neumann algebras. U. Haagerup gave a talk on the solution of the Effros-Ruan conjecture for bilinear forms on C^*-algebras. The methods in his joint work with M. Musat rely on classical type III von Neumann algebra techniques such as Tomita-Takesaki theory and properties of Powers factors, completely different to previous related results by Pisier and Shlyakhtenko. In another talk, Ken Dykema discussed a characterization of Connes' embedding problem in terms of a question of spectral distributions of a sum of certain self-adjoint elements in a II_1 factor.

We finally also mention two talks related to general classifications of von Neumann algebras. R. Sasyk shows (joint work with Tornquist) that von Neumann factors can not be classified using countable structure. G. Elliott presented his approach of studying pointed von Neumann algebras.

2 Presentation Highlights (following same thematic order as in the workshop schedule)

Day 1.

Speaker: **Narutaka Ozawa** (University of Tokyo)
Title: *On a class of II_1 factors with at most one Cartan subalgebra II*
Abstract: (Joint work with S. Popa.) We extend some of our previous result to a large class of discrete groups. This includes proving the non-existence of Cartan subalgebras for tensor products of a free group subfactor with a subfactor Q of a tensor product of free group subfactors. We also show that several von Neumann algebras have at most one Cartan subalgebra. Here Haagerup's complete metric approximation property is still required.

Speaker: **Jesse Peterson** (University of California, Berkeley)
Title: *von Neumann subalgebras closed under (Γ)-extensions*
Abstract: Given a finite von Neumann algebra N, we will say that a diffuse subalgebra B is closed under (Γ)-extensions in N if whenever $P \subset N$ is a subalgebra with $P \cap B$ diffuse and $P' \cap N^\omega$ diffuse for some free ultrafilter ω then we have $P \subset B$. We show that if δ is a densely defined closable derivation into the Hilbert-Schmidt operators which is of the form $\delta(x) = [D, x]$, for some $D \in \mathcal{B}(L^2N)$ then $\overline{\ker(\delta)}$ is closed under (Γ)-extensions in N. In particular if $\overline{\ker(\delta)}$ is injective then it is maximal injective and we obtain generalizations of results of Popa and Ge on maximal injective subalgebras. Also by applying this result to derivations coming from group cocycles we show that if G is a countable discrete group with a proper ℓ^2-cocycle and if $H \subset G$ is an infinite maximal amenable subgroup then LH is maximal injective in LG.

Speaker: **Adrian Ioana** (University of California, Los Angeles)
Title: *On the subequivalence relations induced by a Bernoulli action*
Abstract: Let Γ be a countable group and denote by $\mathcal{S}$ the equivalence relation induced by the Bernoulli action $\Gamma \curvearrowright [0,1]^\Gamma$, where $[0,1]^\Gamma$ is endowed with the product Lebesgue measure. I will prove that for any subequivalence relation $\mathcal{R}$ of $\mathcal{S}$, there exists a partition $\{X_i\}_{i\geq 0}$ of $[0,1]^\Gamma$ with $\mathcal{R}$-invariant measurable sets such that $\mathcal{R}_{|X_0}$ is hyperfinite and $\mathcal{R}_{|X_i}$ is strongly ergodic (hence ergodic), for every $i \geq 1$. This is joint work with Ionut Chifan.

Speaker: **Jason Asher** (University of California, Los Angeles)
Title: *A Kurosh-Type Theorem for Type III Factors*
Abstract: We will present an extension of the Kurosh-Type Theorem of N. Ozawa to the case of the reduced free product of II_1 factors with non-tracial states. The argument will proceed via a generalization of S. Popa's intertwining-by-bimodules technique.

Speaker: **Zhenghan Wang**
Title: *Topological phases of matter: classification and application*
Abstract: Topological phases of matter are exotic states of matter with anyonic excitations such as the fractional quantum Hall (FQH) liquids. FQH liquids are described effectively by Witten's Chern-Simons theories. More general topological phases of matter are described by unitary TQFTs or unitary braided tensor categories. I will discuss the mathematical models for topological phases of matter, their emergence from electrons, classification and application to quantum computing.

Speaker: **Eric Rowell** (Purdue University)
Title:*Topological quantum computers: when universality fails*
Abstract: In the topological model for quantum computation, the computational power is controlled by the closed image of the braid group. Universal quantum computers correspond to dense images, while the weakest quantum computers correspond to finite images. We conjecture that finite images occur precisely when the categorical dimensions in the underlying braided category are square-roots of integers. I will present evidence for this conjecture and discuss its potential ramifications.

Day 2.

Speaker: **Roman Sauer** (University of Chicago)
Title:*l^1-orbit equivalence rigidity for hyperbolic lattices*
Abstract: (Joint work with Uri Bader and Alex Furman.) We say that two group actions are l^1-orbit equivalent if the corresponding cocycle satisfies a certain l^1-integrability condition. This l^p condition interpolates between the extreme cases $p = 0$ (usual orbit equivalence) and $p = \infty$ (implying quasi-isometry). We show that any group that is l^1-orbit equivalent to a lattice in $SO(n, 1)$ ($n > 2$) is also a lattice in the same Lie group, and the cocycle is cohomologous to a standard cocycle coming from this situation (i.e. it can be straightened). The methods involve a generalization of Thurston's proof of Mostow rigidity and new homological methods.

Speaker: **Mikaël Pichot** (Institut des Hautes Etudes Scientifiques (IHES))
Title: *On the Wise group*
Abstract: We will study D. Wise's non Hopfian group from an intermediate rank perspective and prove that it

satisfies the Haagerup inequality (property RD), and in fact that it is of polynomial growth rank. This is joint work with S. Barre.

Speaker: **Cyril Houdayer** (University of California, Los Angeles)
Title: *Another construction of II_1 factors with prescribed countable fundamental group.*
Abstract: I will present another construction of such II_1 factors using free products of von Neumann algebras endowed with almost periodic states.

Speaker: **Dietmar Bisch** (Vanderbilt University)
Title: *The planar algebra of group-type subfactors*
Abstract: Haagerup and I introduced some 10 years ago a class of subfactors associated to outer actions of two finite groups. These subfactors play an important role in the theory, since they provide a very simple mechanism to construct irreducible subfactors whose standard invariant has infinite depth. We will review this construction and describe the planar algebra of these subfactors. We obtain natural IRF models in this way. This is joint work with Paramita Das and Shamindra Ghosh.

Speaker: **Roman Sasyk** (University of Ottawa)
Title:*On the (non)classification of factors*
Abstract: We show that the sets of factors of types II_1, II_∞, and III_λ, $0 \leq \lambda \leq 1$ on a separable Hilbert space are not classifiable using countable structures. Joint work with A. Tornquist.

Speaker: **Pinhas Grossman** (Vanderbilt University)
Title: *Pairs of Intermediate Subfactors*
Abstract: An intermediate subfactor is an algebra P in between two factors:$N \subset P \subset M$, where $N \subset M$ is an irreducible inclusion of factors with finite Jones index. For non-commuting pairs of intermediate subfactors, there is a rigidity to the inclusions which severely limits the number of possible configurations, in terms of the indices and the standard invariant. In particular, there are exactly seven non-commuting, irreducible quadrilaterals of factors whose sides have index less than or equal to 4. This is joint work with Masaki Izumi.

Day 3.

Speaker: **Uffe Haagerup** (University of Southern Denmark)
Title: *Solution of the Effros-Ruan conjecture for bilinear forms on C*-algebras*
Abstract: (Joint work with Magdalena Musat.) In 1991 Effros and Ruan conjectured that a certain Grothendieck type inequality for a bilinear form on a pair of C*-algebras holds if (and only if) the bilinear form is jointly completely bounded. In 2002 Pisier and Shlyakhtenko proved that this inequality holds in the more general setting of operator spaces, provided that the operator spaces in question are exact, in particular they proved the Effros-Ruan conjecture for pairs of exact C*-algebras. In a recent joint work with Magdalena Musat we prove the Effros - Ruan conjecture for general C*-algebras (and with constant one), i.e. for every jointly completely bounded (jcb) bilinear form u on a pair of C*-algebras A, B there exists states f_1, f_2 on A and g_1,g_2 on B, such that

$$|u(a,b)| \leq \|u\|_{jcb}(f_1(aa^*)g_1(b^*b) + f_2(a^*a)g_2(bb^*))$$

While the approach by Pisier and Shlyahktenko relied on free probability theory, our proof uses more classical operator algebra methods, namely Tomita Takesaki theory and special properties of the Powers factors of Type III_λ, $0 < \lambda < 1$.

Speaker: **Ken Dykema** (Texas A&M University)
Title: *Connes's embedding problem and Horn's inequalities*
Abstract: Connes's embedding problem asks whether every separable II_1-factor can be embedded in the ultrapower of the hyperfinite II_1-factor; this is equivalent to asking whether every finite set in every II_1-factor has microstates. Bercovici and Li have related this to a question concerning the possible spectral distributions of $a + b$, where a and b are self-adjoint elements in a II_1-factor having given spectral distributions. We show

that Connes' embedding problem is equivalent to a version of this spectral distribution question with matrix coefficients.

Speaker: **Stefaan Vaes** (Katholieke Universiteit Leuven) Title: *An action of the free group F_∞ whose orbit equivalence relation has no outer automorphisms*
Abstract: (Joint work with Sorin Popa.) We prove that there exist uncountably many stably orbit inequivalent, essentially free, ergodic, probability measure preserving actions of the free group with infinitely many generators such that their associated orbit equivalence relations have trivial outer automorphism group and trivial fundamental group.

Day 4.

Speaker: **Roberto Longo** (University of Rome Tor Vergata)
Title: *SUSY in the Conformal World*
Abstract: The talk concerns part of a recent joint work with S. Carpi and Y. Kawahigashi on the operator algebraic analysis of Superconformal Quantum Field Theory, with an initial step in the program of constructing Noncommutative Geometrical invariants for certain representations.

Speaker: **Feng Xu** (University of California, Riverside)
Title: *An application of mirror extension*
Abstract: Mirror extension is a general result about obtaining new nets as mirror of known ones. As an application of this result, in this talk I will discuss construction of new holomorphic conformal nets of central charge 24.

Speaker: **Richard Burstein** (University of California, Berkeley)
Title: *Subfactors Obtained from Hadamard Matrices*
Abstract: A subfactor may be obtained from a commuting square via iteration of the basic construction. For certain commuting squares coming from Hadamard matrices, the resulting subfactor may be described using the group construction of Bisch and Haagerup. We will show how this description allows us to find the principal graphs of these subfactors, and how it may lead to a full classification up to subfactor isomorphism.

Speaker: **Yasuyuki Kawahigashi** (University of Tokyo)
Title: *Super moonshine and operator algebras*
Abstract: We constructed an operator algebraic counterpart of the Moonshine vertex operator algebra with Longo before. Its automorphism group is the Monster group and its character is the modular elliptic j-function without the constant term. We now construct its "super" analogue for Conway's sporadic group Co_1, following work of Duncan for an enhanced super vertex operator algebra.

Speaker: **V.S. Sunder** (IMSc, Chennai)
Title: *Non-crossing partition $\simeq$ 2-cabled Temperley-Lieb*
Abstract: For each complex number $\delta \neq 0$, we consider a planar algebra whose space $NC_n(\delta)$ of 'k-boxes' has a basis consisting of non-crossing partitions of a set of $2k$ points, (usually thought of as being arrayed on two parallel lines, with k points on each line), and with multiplication and other planar algebra structure being defined almost exactly as in the case of the Temperley-Lieb planar algebra $TL(\delta)$. We show that this planar algebra $NC(\delta)$ is a C^*-planar algebra when $\delta > 4$. We do this by showing that $NC(\delta^2)$ is isomorphic to the so-called 2-cabling of $TL(\delta)$.

Speaker: **Marta Asaeda** (University of California, Riverside)
Title: *Non-existense of finite depth subfactors with certain small indices*
Abstract: (With S.Yasuda.) In 1991 Haagerup gave the list of graphs as candidates of principal graphs of subfactors with indices within $(4, 3+\sqrt{3})$. We prove that one of the parametrized series of the graphs are not realized as principal graphs except for the first two, using algebraic number theory.

Day 5.

Speaker: **George Elliott** (University of Toronto)
Title: Pointed von Neumann algebras

Speaker: **Narutaka Ozawa** (University of Tokyo)
Title: Cocycle super-rigidity and profinite actions
Abstract: (With Popa) First, I will briefly review the history of cocycle super-rigidity. Cocycle super-rigidity per se is a subject of ergodic theory, but I talk how von Neumann algebraic perspective helps in finding new examples of cocycle super-rigid actions, and present applications of the new examples to von Neumann algebras.

3 Scientific Progress Made

The meeting provided plenty of opportunity for participants to exchange ideas, some of which is expected to influence future publications. As a very concrete result, let us just mention a paper by Ozawa, posted at http://arxiv.org/abs/0804.0288 which was directly inspired by a question asked at BIRS by one of the participants, Asger Tornquist.

4 Final comments

As mentioned at the beginning, the workshop was very successful in bringing together many leading experts in von Neumann algebras as well as researchers from related areas. We think it provided an excellent reflection of the current exciting developments in this subject and its influences on/from other areas. This should be particularly helpful for the many younger researchers which attended our workshop.

We received positive comments and feedback about the meeting from many participants. So we believe that it was indeed stimulating and did contribute to further progress in our field.

5 Participants

Argerami, Martin (University of Regina)
Asaeda, Marta (University of California, Riverside)
Asher, Jason (University of California, Los Angeles)
Belinschi, Serban (University of Saskatchewan)
Bisch, Dietmar (Vanderbilt University)
Brenken, Berdnt (University of Calgary)
Burstein, Richard (University of California, Berkeley)
Censor, Aviv (University of California, Riverside)
Dean, Andrew (Lakehead University)
Dykema, Ken (Texas A&M University)
Dynov, Ivan (Max Planck Institute for Mathematics)
Elliott, George (University of Toronto)
Erlijman, Juliana (University of Regina)
Grossman, Pinhas (Vanderbilt University)
Haagerup, Uffe (University of Southern Denmark)
Houdayer, Cyril (University of California, Los Angeles)
Ioana, Adrian (Caltech)
Ivanescu, Cristian (University of Toronto)
Jensen, Troels Steenstrup (University of Southern Denmark)
Kawahigashi, Yasuyuki (University of Tokyo)

Longo, Roberto (University of Rome Tor Vergata)
Mingo, Jamie (Queen's University)
Musat, Magdalena (University of Memphis)
Niu, Zhuang (University of Oregon)
Ozawa, Narukata (University of Tokyo)
Peters, Emily (University of California, Berkeley)
Peterson, Jesse (University of California, Berkeley)
Pichot, Mikaël (Institut des Hautes Etudes Scientifiques (IHES))
Rowell, Eric (Texas A&M University)
Sako, Hiroki (University of Tokyo)
Sasyk, Román (University of Purdue)
Sauer, Roman (University of Chicago)
Sunder, V.S. (The Institute of Mathematical Sciences, India)
Tornquist, Asger (University of Toronto)
Vaes, Stefaan (Catholic University of Leuven)
Viola, Maria Grazia (Fields Institute)
Wang, Zhenghan (Microsoft Corporation/UCSB)
Wenzl, Hans (University of California, San Diego)
Xu, Feng (University of California, Riverside)

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