Banff International Research Station
for Mathematical Innovation and Discovery

Locally Symmetric Spaces
Sunday, May 18, 2008 to Friday, May 23, 2008

Organizers: Stephen Kudla (University of Toronto), Juergen Rohlfs (Katholische Universitaet Eichstaett), Leslie Saper (Duke University), Birgit Speh (Cornell University).

MEALS

◦ Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday
◦ Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday
◦ Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday
Coffee Breaks: As per daily schedule, 2nd floor lounge, Corbett Hall

Please remember to scan your meal card at the host/hostess station in the dining room for each meal.

MEETING ROOMS

All lectures will be held in Max Bell 159 (Max Bell Building accessible by walkway on 2nd floor of Corbett Hall). LCD projector, overhead projectors and blackboards are available for presentations.

Please note that the meeting space designated for BIRS is the lower level of Max Bell, Rooms 155–159. Please respect that all other space has been contracted to other Banff Centre guests, including any Food and Beverage in those areas.

SCHEDULE

Sunday, May 18, 2008

16:00	Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
	Lecture rooms available after 16:00
17:30–19:30	Buffet Dinner, Sally Borden Building
20:00	Informal gathering in 2nd floor lounge, Corbett Hall
	Beverages and small assortment of snacks available on a cash honour-system.

Monday, May 19, 2008 (Session Chair: Leslie Saper)

7:00–8:45	Breakfast
8:45–9:00	Introduction and Welcome to BIRS by BIRS Station Manager, Max Bell 159
9:00–10:00	**Werner Müller** (University of Bonn)
	The Arthur trace formula and spectral theory on locally symmetric spaces
10:00–10:30	Coffee Break, 2nd floor lounge, Corbett Hall
10:30–11:30	**Joachim Mahnkopf** (University of Vienna)
	Traces on Hecke algebras and p-adic families of modular forms
11:30–13:30	Lunch
13:45–14:00	Group Photo; meet on the front steps of Corbett Hall
14:00–15:00	**Gopal Prasad** (University of Michigan)
	Lengths of closed geodesics and isospectral locally symmetric spaces
15:00–15:30	Coffee Break, 2nd floor lounge, Corbett Hall
15:30–16:30	**Andras Vasy** (Stanford University)
	Scattering theory on symmetric spaces
17:30–19:30	Dinner

Tuesday, May 20, 2008 (Session Chair: Birgit Speh)

7:00–9:00	Breakfast
9:00–10:00	**Yves Benoist** (University of Paris, South)
	Effective equidistribution of S-integral points on symmetric varieties
10:00–10:30	Coffee Break, 2nd floor lounge, Corbett Hall
10:30–11:30	**Anton Deitmar** (University of Tübingen)
	Automorphic forms of higher order
11:30–13:30	Lunch
14:00–15:00	**Mathieu Cossutta** (University of Paris, 7)
	Asymptotics of L^2 Betti numbers in congruence coverings of some arithmetically defined locally symmetric varieties
15:00–15:30	Coffee Break, 2nd floor lounge, Corbett Hall
15:30–16:30	**Dan Yasaki** (University of Massachusetts)
	Spines for $\mathbb{Q}$-rank 1 groups
17:30–19:30	Dinner

Wednesday, May 21, 2008 (Session Chair: Jürgen Rohlfs)

7:00–8:30 Breakfast

8:30–9:30 **Jayce Getz** (Princeton University)
Twisted relative trace formulae with a view towards unitary groups

9:30–10:30 **Toshiyuki Kobayashi** (University of Tokyo)
Restriction of unitary representations of real reductive groups

10:30–11:00 Coffee Break, 2nd floor lounge, Corbett Hall

11:00–12:00 **Lizhen Ji** (University of Michigan)
Borel extension theorem and Mostow strong rigidity

12:00–13:30 Lunch

Free Afternoon

17:30–19:30 Dinner

Thursday, May 22, 2008 (Session Chair: Joachim Hilgart)

7:00–9:00 Breakfast

9:00–10:00 **Enrico Leuzinger** (University of Karlsruhe)
Reduction theory for arithmetic and mapping class groups

10:00–10:30 Coffee Break, 2nd floor lounge, Corbett Hall

10:30–11:30 **Nicholas Bergeron** (Institut de Mathématiques de Jussieu)
Automorphic Lefschetz Property of Locally Symmetric Spaces

11:30–13:30 Lunch

14:00–15:00 **Bernhard Krötz** (Max Planck Institute for Mathematics, Bonn)
Globalization of Harish-Chandra modules

15:00–15:30 Coffee Break, 2nd floor lounge, Corbett Hall

15:30–16:30 **Martin Olbrich** (University of Luxembourg)
Extending the realm of Patterson's conjectures

17:30–19:30 Dinner

Friday, May 23, 2008 (Session Chair: Werner Müller)

7:00–8:30 Breakfast

8:30–9:30 **Harald Grobner** (University of Vienna)
Regular and residual Eisenstein cohomology classes for inner forms of symplectic groups

9:30–10:30 **Jens Funke** (New Mexico State University)
Special cohomology classes arising from the Weil representation

10:30–11:00 Coffee Break, 2nd floor lounge, Corbett Hall

11:00–12:00 **Christian Kaiser** (Max Planck Institute for Mathematics, Bonn)
Irreducibility of Galois representations associated to automorphic forms of multiplicative groups of skew fields over function fields

12:00–13:30 Lunch

13:30–15:00 Informal discussions / Departures

Please remember to checkout by 12 noon.

5-day workshops are welcome to use the BIRS facilities (2nd Floor Lounge, Max Bell Meeting Rooms, Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon.

Locally Symmetric Spaces
Sunday, May 18, 2008 to Friday, May 23, 2008

ABSTRACTS
(in alphabetic order by speaker surname)

Speaker: **Yves Benoist** (University of Paris, South)
Title: *Effective equidistribution of S-integral points on symmetric varieties*
Abstract: Let S be a finite set of places of a global field K. We describe counting and equidistribution results for the S-integral points on a symmetric variety defined over K. We give also an upper bound for the error term in characteristic 0. This joint work with Hee Oh is based on a polar decomposition of p-adic symmetric spaces.

Speaker: **Mathieu Cossutta** (University of Paris, 7)
Title: *Asymptotics of L^2 Betti numbers in congruence coverings of some arithmetically defined locally symmetric varieties*
Abstract: Let $G/\mathbb{Q}$ be an algebraic group and $X = G(\mathbb{R})/K$ the associated symmetric space. The aim of my talk is to give some new informations on $b_{i,2}(\Gamma(p^n)\backslash X)$ when n goes to infinity for a large family of groups G, in the direction of conjectures formulated by Xue and Sarnak. Our proof is based on theta correspondence. For example when $G = \mathrm{Sp}_{2g}$, the primitive holomorphic cohomology appears only in degree $r(2gr)$ for r an integer between $1, \dots, g$. We then obtain that for $g \geq \frac{5}{4}p + 1$:

$$\mathrm{Vol}(\Gamma(p^n)\backslash X)^{\frac{2r}{g+1}\left(1-\frac{r(r+1)}{g(g+1)}\right)-\epsilon} \ll_\epsilon \dim H^{r(2g-r),0}_{\mathrm{prim},2}(\Gamma(p^n)\backslash X, \mathbb{C}) \ll_\epsilon \mathrm{Vol}(\Gamma(p^n)\backslash X)^{\frac{2r}{g+1}\left(1+\frac{r(r-1)}{g(g+1)}\right)+\epsilon}$$

Speaker: **Anton Deitmar** (University of Tübingen)
Title: *Automorphic forms of higher order*
Abstract: The present talk presents an attempt to study them in the general context of higher order invariants and cohomology. It is shown that for arithmetic groups, higher order cohomology can be computed as $(\mathfrak{g}, K)$-cohomology, even using functions of moderate growth. It is, however, an open question whether the higher order analogue of Franke's theorem holds, which states that the cohomology can be computed using automorphic forms. An action of the Hecke algebra is introduced in which the Hecke operators are bounded operators. Questions about their spectral decomposition are raised.

Speaker: **Jens Funke** (New Mexico State University)
Title: *Special cohomology classes arising from the Weil representation*
Abstract: The Weil representation is a well-known tool to study arithmetic and cohomological aspects of orthogonal groups. We construct certain, "special", cohomology classes for orthogonal groups $\mathrm{O}(p, q)$ with coefficients in a finite dimensional representation and discuss their automorphic and geometric properties. In particular, these classes are generalizations of previous work of Kudla and Millson and give rise to

Poincare dual forms for certain, "special", cycles with non-trivial coefficients in arithmetic quotients of the associated symmetric space for the orthogonal group. Furthermore, we determine the behavior of these classes at the boundary of the Borel-Serre compactification of the associated locally symmetric space. As a consequence we are able to obtain new non-vanishing results for the special cycles. This is joint work with John Millson.

Speaker: **Jayce Getz** (Princeton University)
Title: *Twisted relative trace formulae with a view towards unitary groups*
Abstract: (joint work with E. Wambach) We introduce twisted relative trace formulae and provide tools to relate them to relative trace formulae modeled on the relative trace formula introduced by Jacquet and Lai. As an application, we consider the analogue for odd rank unitary groups of the work of Harder, Langlands and Rapoport on modular curves embedded in Hilbert modular surfaces.

Speaker: **Harald Grobner** (University of Vienna)
Title: *Regular and residual Eisenstein cohomology classes for inner forms of symplectic groups*
Abstract: Let G be a semisimple Lie group with maximal compact subgroup K and arithmetic congruence subgroup Γ. The locally symmetric space $\Gamma\backslash G/K$ carries interesting arithmetic information encoded in its cohomology groups $H^*(\Gamma\backslash G/K)$. One of the major tasks in understanding this cohomology is to understand a certain subspace $H^*_{Eis}(\Gamma\backslash G/K)$, called Eisenstein cohomology. We will focus on the case of inner forms of the symplectic group Sp_n and try to give an overview of interesting phenomena which one encounters in the search of a description of $H^*_{Eis}(\Gamma\backslash G/K)$. If time permits, we will present completely new results.

Speaker: **Lizhen Ji** (University of Michigan)
Title: *Borel extension theorem and Mostow strong rigidity*
Abstract: An important result for the Baily-Borel compactification of Hermitian locally symmetric spaces is the Borel extension theorem for holomorphic maps from the punctured disk into the Hermitian locally symmetric spaces. We discuss an analogue for the Deligne-Mumford compactification for the moduli space of curves.

Then we will discuss the Mostow strong rigidity for mapping class groups of surfaces (or equivalently the moduli spaces of curves) and outer automorphism groups of free groups.

We will also discuss non-isomorphism results between three closely related groups: lattice subgroups of Lie groups, mapping class groups, and outer automorphisms of free groups.

Speaker: **Christian Kaiser** (Max Planck Institute for Mathematics, Bonn)
Title: *Irreducibility of Galois representations associated to automorphic forms of multiplicative groups of skew fields over function fields*
Abstract: This is work in progress. Not all details have been checked yet.

Let D be a skew field of degree d over some global field of characteristic p. Under some ramification constraint on D, Lafforgue and Lau associated to an automorphic representation π of $D^\times$ a d-dimensional Galois representation $\sigma(\pi)$ with the same L-functions as π. If π is (almost everywhere) tempered it is conjectured that $\sigma(\pi)$ is irreducible. We prove this conjecture in some cases.

The main tool, which may be of independent interest, is an uniformization theorem for the moduli space of $\mathcal{D}$-shtukas of rank one at a point $(0,\infty) \in X \times X$ with $\mathrm{inv}_0 D = \frac{1}{d}$ and $\mathrm{inv}_\infty D = -\frac{1}{d}$ by a product of two copies of Drinfeld's upper half space Ω^d. Using results of Boyer and Dat on the cohomology of Ω^d and its coverings one concludes for skew fields D which have invariant $\frac{1}{d}$ and $-\frac{1}{d}$ at some different places (and under some more constraints). Using simple cases of the Jacquet-Langlands conjecture one can reduce many cases to this special situation.

Speaker: **Toshiyuki Kobayashi** (University of Tokyo)
Title: *Restriction of unitary representations of real reductive groups*

Abstract: Branching problems ask how an irreducible representation of a group decomposes when restricted to its subgroup. Having an observation of bad features of branching problems in a general non-compact setting even for real reductive symmetric pairs, I will discuss how to find a nice setting for branching problems, and give an upper estimate on multiplicities. If time allows, some applications are also presented.

Speaker: **Bernhard Krötz** (Max Planck Institute for Mathematics, Bonn)
Title: *Globalization of Harish-Chandra modules*
Abstract: We will explain short and simple proofs of the globalization theorems of Casselman-Wallach and Kashiwara-Schmid. The work is partly joined with Joseph Bernstein and partly with Henrik Schlichtkrull.

Speaker: **Enrico Leuzinger** (University of Karlsruhe)
Title: *Reduction theory for arithmetic and mapping class groups*
Abstract: Reduction theory is concerned with the construction of (coarse) fundamental domains for groups acting properly discontinuously. I will describe a remarkable analogy between (a version of) reduction theory for arithmetic groups and corresponding results for mapping class groups. As an application I will discuss asymptotic cones for locally symmetric spaces and moduli spaces.

Speaker: **Joachim Mahnkopf** (University of Vienna)
Title: *Traces on Hecke algebras and p-adic families of modular forms*
Abstract: We prove that any modular eigenform f of level $\Gamma_1(Np)$, finite slope α and weight k_0 can be placed into a p-adic family of modular eigenforms f_k of the same level and slope and weight k varying over all natural numbers which are sufficiently close to k_0 in the p-adic sense. Here, the term p-adic family means that a p-adic congruence between two weights k and k' entails a certain p-adic congruence between the corresponding eigenforms f_k and $f_{k'}$. We also prove that the dimension of the slope α subspace of the space of modular forms of weight k does not depend on the weight as long as we consider weights k which are sufficiently close to each other in the p-adic sense. Both these statements are predicted by the Mazur-Gouvea conjecture, which has been proven by Coleman. Our proof of these statements, which is completely different from Coleman's proof, is based on a comparison of (topological) trace formulas.

Speaker: **Werner Müller** (University of Bonn)
Title: *The Arthur trace formula and spectral theory on locally symmetric spaces*
Abstract: In this talk I will discuss various applications of the Arthur trace formula to spectral theory on locally symmetric spaces. I will also discuss analytic problems related to the trace formula which need to be settled for further applications.

Speaker: **Martin Olbrich** (University of Luxembourg)
Title: *Extending the realm of Patterson's conjectures*
Abstract: This talk is concerned with analysis and spectral geometry of certain rank 1 locally symmetric spaces of typically infinite volume, namely geometrically finite ones. Geometrically finite spaces without cusps correspond to convex co-compact discrete groups. In 1993, Patterson formulated two conjectures concerning convex co-compact groups acting on real hyperbolic spaces. The first conjecture relates invariant currents supported on the limit set to the cohomology of the locally symmetric space, while the second gives a description of the singularities of Selberg's zeta function in terms of objects supported on the limit set. Both conjectures are now proved for many cases. We shall discuss how the conjectures should be modified in order to make sense for arbitrary geometrically finite locally symmetric spaces of rank 1. Already the inclusion of noncompact spaces of finite volume into the picture is an interesting and nontrivial task.

Speaker: **Gopal Prasad** (University of Michigan)
Title: *Lengths of closed geodesics and isospectral locally symmetric spaces*
Abstract: I will give an exposition of my recent work with Andrei Rapinchuk in which we have introduced a new notion of "weak commensurability" of Zariski-dense subgroups. Weak commensurability of arithmetic

subgroups of semi-simple Lie groups turns out to have very strong consequences. Weak commensurability is intimately related to the commensurability of the set of lengths of closed geodesics on, and isospectrality of, locally symmetric spaces of finite volume (and with nonpositive sectional curvatures). Using our results we are able to answer Marc Kac's famous question "Can one hear the shape of a drum?" for compact arithmetic locally symmetric spaces. Our proofs use algebraic number theory, class field theory, and also some results and conjectures from transcendental number theory.

Speaker: **Andras Vasy** (Stanford University)
Title: *Scattering theory on symmetric spaces*
Abstract: I will explain how methods from N-body scattering can be used to analyze the resolvent of the Laplacian and spherical functions on globally symmetric spaces (joint work with Rafe Mazzeo), and ongoing work with Rafe Mazzeo and Werner Müller to extend the framework to locally symmetric spaces.

Speaker: **Dan Yasaki** (University of Massachusetts)
Title: *Spines for $\mathbb{Q}$-rank 1 groups*
Abstract: Let $X = \Gamma\backslash D$ be an arithmetic quotient of a symmetric space associated to a semisimple algebraic group defined over $\mathbb{Q}$ with $\mathbb{Q}$-rank n. A result of Borel and Serre gives the vanishing of the cohomology of X in the top n degrees. Thus one can hope to find a Γ-equivariant deformation retraction of D onto a set D_0 having codimension n. When such a set exists, it is called a spine and is useful for computing the cohomology of X. Spines have been explicitly computed in many examples, and the general existence is known for groups associated to self-adjoint homogeneous cones. I will describe my work on the existence of spines for groups of $\mathbb{Q}$-rank 1.