



# Banff International Research Station

for Mathematical Innovation and Discovery

## Random Matrices, Inverse Spectral Methods and Asymptotics

October 5 - 10, 2008

### MEALS

\*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday

\*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday

\*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday

Coffee Breaks: As per daily schedule, 2nd floor lounge, Corbett Hall

\*Please remember to scan your meal card at the host/hostess station in the dining room for each meal.

### MEETING ROOMS

All lectures will be held in Max Bell 159 (Max Bell Building accessible by walkway on 2nd floor of Corbett Hall). LCD projector, overhead projectors and blackboards are available for presentations. Please note that the meeting space designated for BIRS is the lower level of Max Bell, Rooms 155–159. Please respect that all other space has been contracted to other Banff Centre guests, including any Food and Beverage in those areas.

### SCHEDULE

When your schedule is finalized, please e-mail it to the BIRS Station Manager [birmsmgr@birs.ca](mailto:birmsmgr@birs.ca) by 12 noon on the Thursday before your arrival. You are also encouraged to e-mail the schedule to all of your participants at:  [<birs event id>-par@lists.pims.math.ca](mailto:<birs event id>-par@lists.pims.math.ca)

#### Sunday

**16:00** Check-in begins (Front Desk - Professional Development Centre - open 24 hours)

**17:30–19:30** Buffet Dinner, Sally Borden Building

**20:00** Informal gathering in 2nd floor lounge, Corbett Hall

Beverages and small assortment of snacks available on a cash honour-system.

#### Monday

**7:00–8:45** Breakfast

**8:45–9:00** Introduction and Welcome to BIRS by BIRS Station Manager, Max Bell 159

**9:00–9:50** Christopher Sinclair, C.U. Boulder, “Real Ginibre: Correlations, Kernels and the Largest Point”

**10:00–10:45** Coffee Break, 2nd floor lounge, Corbett Hall

**10:45–11:35** Robie Buckingham, C.R.M., “Total Integrals of Painleve Functions”

**11:45–13:00** Lunch

**13:00–14:00** Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall

**15:00–15:30** Coffee Break, 2nd floor lounge, Corbett Hall

**17:30–19:30** Dinner

**19:00–19:50** Mihai Putinar, UCSB, “Bergman orthogonal polynomials on an archipelago”

**20:00–20:50** Ferenc Balogh, Concordia University, “On the asymptotics of some planar orthogonal polynomials”

## Tuesday

- 7:00–9:00** Breakfast  
**9:00–9:50** Jeffery DiFranco, Univ. of Seattle, “Asymptotics of Tracy-Widom distributions”  
**10:00–10:45** Coffee Break, 2nd floor lounge, Corbett Hall  
**10:45–11:35** Peter Miller, Univ. of Michigan, “On the Semiclassical Limit for the Sine-Gordon Equation”  
**11:45–14:00** Lunch  
**14:00–14:50** Bob Jenkins, University of Arizona, “Semi-classical focusing NLS with rough initial data.”  
**15:00–15:30** Coffee Break, 2nd floor lounge, Corbett Hall  
**15:30–16:20** Virgil Pierce, Univ. Texas Pan American, “The dispersionless Toda and Pfaff hierarchies: reduction and universality”  
**16:35** Group Photo; meet on the front steps of Corbett Hall  
**16:40–17:30** John Harnad, C.R.M. and Concordia University, “And Now for Something Completely Different”  
**17:40–19:30** Dinner

## Wednesday

- 7:00–9:00** Breakfast  
**9:00–9:50** Nicholas Ercolani, Univ. of Arizona, “Vanishing of Resonances in the Continuum Toda Hierarchy”  
**10:00–10:45** Coffee Break, 2nd floor lounge, Corbett Hall  
**10:45–11:35** Jacek Szmigielski, Univ. of Saskatchewan, “Bi-orthogonal Cauchy polynomials: peakons and simultaneous Pade approximation problems”  
**11:45–14:00** Lunch  
**14:00–14:50** Misha Gekhtman, Notre Dame, “Bi-orthogonal Cauchy polynomials: total positivity and RH problems”  
**15:00–15:30** Coffee Break, 2nd floor lounge, Corbett Hall  
**15:30–16:20** Sasha Soshnikov, U.C. Davis, “Local Linear Statistics in Large Random Matrices”  
**17:30–19:30** Dinner

## Thursday

- 7:00–9:00** Breakfast  
**9:00–9:50** Seung-Yeop Lee, CRM, “Dyson process by isomonodromic deformation”  
**10:00–10:45** Coffee Break, 2nd floor lounge, Corbett Hall  
**10:45–11:35** Aleix Prats-Ferrer, CRM, “Complete topological expansion for formal matrix models: Chain of Matrices & Cauchy two-matrix model”  
**11:45–14:00** Lunch  
**14:00–14:50** David Niles, UIPUI, “The Riemann-Hilbert-Birkhoff inverse monodromy and the asymptotic analysis of the third Painleve transcendents.”  
**15:00–15:30** Coffee Break, 2nd floor lounge, Corbett Hall  
**15:30–16:20** Dong Wang, CRM, “A new relation between complex Wishart ensembles and KP tau functions”  
**17:30–19:30** Dinner

## Friday

- 7:00–9:00** Breakfast  
**9:00** Informal Discussions  
**10:00–10:45** Coffee Break, 2nd floor lounge, Corbett Hall  
**10:45–11:30** Informal discussions  
**11:30–13:30** Lunch  
**Checkout by 12 noon.**

\*\* 5-day workshops are welcome to use the BIRS facilities (2nd Floor Lounge, Max Bell Meeting Rooms, Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. \*\*



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## Random Matrices, Inverse Spectral Methods and Asymptotics

October 5 - October 10, 2008

### ABSTRACTS

(in alphabetic order by speaker surname)

Speaker: **Ferenc Balogh** (Concordia University)

Title: *On the asymptotics of some planar orthogonal polynomials*

Speaker: **Robbie Buckingham** (C.R.M.)

Title: *Total Integrals of Painleve Functions*

Abstract: We present a method for computing the total integral (the definite integral over the entire real line) of solutions to certain integrable ordinary and partial differential equations. Our method takes advantage of the structure of the Lax pair and applies the steepest-descent analysis of Riemann-Hilbert problems. We calculate the total integral for global solutions to the Painleve II equation. Using these total integrals we obtain a straightforward solution to the constant problem in the asymptotic expansion of the GOE and GSE Tracy-Widom distributions. We then use an integral of a function related to the Painleve V equation to give a new proof of the constant problem for the GOE and GSE sine kernels. This is joint work with Jinho Baik, Jeffery DiFranco, and Alexander Its.

Speaker: **Jeffery DiFranco** (Seattle University)

Title: *Asymptotics of Tracy-Widom distributions*

Abstract: The Tracy-Widom distribution functions appear in numerous areas of combinatorics and probability. In particular these functions are the limiting distributions of the largest eigenvalues of the GUE, GOE and GSE random matrix ensembles. These functions can be expressed in terms of an integrals starting at positive infinity of a Painleve II function. Using the steepest descent method for Riemann-Hilbert problems we are able to represent these functions, in terms of integrals starting at negative infinity of the same Painleve functions. This new representation will be suitable for calculating the asymptotics of the Tracy-Widom functions near negative infinity. Additionally, I will discuss on-going work to study similar problems for the Tracy-Widom distributions for the largest eigenvalue when each eigenvalue is detected with a certain probability  $p$ .

Speaker: **Nicholas Ercolani** (University of Arizona)

Title: *Vanishing of Resonances in the Continuum Toda Hierarchy for Hermitian Random Matrices*

Abstract: We will report on progress in establishing the "rationality" (and form) of the generating functions for  $g$ -maps (graphical enumeration).

Speaker: **Misha Gekhtman** (Notre Dame)

Title: *Bi-orthogonal Cauchy polynomials: total positivity and RH problems*

Abstract: Motivation behind the introduction of bi-orthogonal Cauchy polynomials (Cauchy BOPs) stems from the cubic peakon equation and novel 2-matrix models. We will discuss the general framework for Cauchy BOPs and their fundamental properties. This includes: total positivity of recursion operators,

interlacing of zeroes, generalized Christoffel-Darboux identities and a characterization in terms of a 3 by 3 matrix Riemann-Hilbert problem. This is a joint project with M. Bertola and J. Szmigielski.

Speaker: **John Harnad** (C.R.M. and Concordia University)

Title: *And Now for Something Completely Different ... Tau functions, exclusion processes and equilibrium models of 1-D fermi lattice gases*

Abstract: Tau functions expressed as fermionic expectation values are shown to provide a natural description of a number of random exclusion processes and statistical models involving configurations of identical fermi particles on the integer lattice. These include a discrete version of simple exclusion processes (ASEP), nonintersecting random walkers, lattice Coulomb gas models and others. This also provides a powerful tool for combinatorial calculations involving paths between pairs of partitions. (Based on joint work with Alexander Yu. Orlov.)

Speaker: **Bob Jenkins** (University of Arizona)

Title: *Semi-classical focusing NLS with rough initial data.*

Speaker: **Seung-Yeop Lee** (CRM)

Title: *Dyson process by isomonodromic deformation*

Abstract: We study the Dyson process which is equally described by coupled matrix model. We look at the joint probability where the largest eigenvalue of one matrix is less than  $A$  and the largest eigenvalue of the other matrix is less than  $B$ . This probability satisfies a closed system of equations (by Tracy-Widom) or a set of PDE (by Adler-Moerbeke) in  $A$  and  $B$ . Using the isomonodromic deformation approach, we present another representation of equations that determines the joint probability. We show the relation of the joint probability to the isomonodromic tau function. Asymptotic solution may be obtained by applying the steepest descent method on our Riemann-Hilbert problem, which is a work in progress. This is a joint work with Marco Bertola.

Speaker: **Peter Miller** (University of Michigan)

Title: *On the Semiclassical Limit for the Sine-Gordon Equation*

Abstract: I will discuss some aspects of recent work on the Cauchy problem in laboratory coordinates for the sine-Gordon equation, subject to a semiclassical scaling that introduces an essential separation of scales into the dynamics. This type of scaling naturally occurs in the modeling of superconducting Josephson tunneling junctions in which macroscopic (laboratory scale) excitations create a large number of quanta of magnetic flux whose nonlinear interactions can become complicated. This is joint work with Robert Buckingham.

Speaker: **Irina Nenciu** (Univ. of Ill. at Chicago)

Title: *On confining potentials and essential self-adjointness for Schroedinger operators on bounded domains*

Speaker: **David Niles** (IUPUI)

Title: *The Riemann-Hilbert-Birkhoff inverse monodromy and the asymptotic analysis of the third Painleve transcendents.*

Abstract: I will discuss the inverse monodromy problem for a  $2 \times 2$  linear system of ordinary differential equations with rational coefficients having two irregular singular points of Poincare index 1. The meromorphic (with respect to the deformation parameter) solvability of the problem will be proven for an arbitrary set of the relevant monodromy data. The approach is based on the Birkhoff-Grothendieck-Malgrange factorization theorem, and it represents a generalization of the method previously applied by A. Bolibruch, A. Its, and A. Kapaev to the Riemann-Hilbert-Birkhoff problem related to the second Painleve equation. Applications to the derivations of the explicit connection formulae for the third Painleve equation will be outlined as well. The talk is based on the joint work with A. Its.

Speaker: **Virgil Pierce** (University of Texas Pan American)

Title: *The dispersionless Toda and Pfaff hierarchies: reduction and universality*

Abstract: The partition function of the unitary ensembles of random matrices is given by a tau-function of the Toda lattice hierarchy and those of the orthogonal and symplectic ensembles are tau-functions of the Pfaff lattice hierarchy. In these cases the asymptotic expansions of the free energies given by the logarithm of the partition functions lead to the dispersionless limits for the Toda and Pfaff hierarchies. In the Toda case we will show that one consequence of this dispersionless hierarchy is a closed form expression for the two point function,  $F_{nm}$ , which gives the number of connected ribbon graphs with two vertices of degree  $n$  and  $m$  on a sphere. We will show two different relationships between the dispersionless Toda and Pfaff hierarchies: one can be characterized as a reduction, while the other is a manifestation of the universality between the GOE-GSE and GUE ensembles. This is joint work with Yuji Kodama.

Speaker: **Aleix Prats-Ferrer** (CRM)

Title: *Complete topological expansion for formal matrix models: Chain of Matrices & Cauchy two matrix model.*

Abstract: In the last few years, a new technique has been developed to describe the topological expansion of the so called "formal" version of several matrix models: One and two matrix models, Kontsevich model. This technique relies in an underlying algebraic curve to all these models and can be applied to arbitrary algebraic curves (not necessarily related to any matrix model). Here I will present two more formal matrix models to which this technique can be applied: the formal Itzykson-Zuber type matrix chain, and the recently introduced Cauchy two-matrix model. In the former, some very preliminary analysis shows some very simple formulas for the infinite matrix chain, also known as Matrix Quantum Mechanics.

Speaker: **Mihai Putinar** (UCSB)

Title: *Bergman orthogonal polynomials on an archipelago*

Abstract: Fine asymptotics for the Bergman orthogonal polynomials on a disjoint union of planar Jordan domains, as well as a study of the asymptotics of their zero location will be derived from potential theoretic techniques. A couple of shape reconstruction from moments algorithms will be analyzed in parallel. Based on joint work with B. Gustafsson, E. Saff and N. Stylianopoulos.

Speaker: **Chris Sinclair** (Colorado University, Boulder)

Title: *Real Ginibre: Correlations, Kernels and the Largest Point*

Abstract: I will summarize recent work on the integrable structure of Ginibre's ensemble of real asymmetric matrices. In particular, I will talk about the scaled kernel in various regimes (real bulk, complex bulk, real edge, complex edge). I will also report on ongoing work on the limit law for the largest eigenvalue in the scaling limit. (This talk includes joint work with (Alexei Borodin and Brian Rider).

Speaker: **Alexander Soshnikov** (U.C. Davis)

Title: *Local Linear Statistics in Large Random Matrices*

Abstract:

Speaker: **Jacek Szmigielski** (University of Saskatchewan)

Title: *Bi-orthogonal Cauchy polynomials: peakons and simultaneous Pade approximation problems*

Abstract: This is the first of the two talks highlighting the emerging theory of bi-orthogonal polynomials (Cauchy BOPs) associated with the Cauchy kernel  $1/(x+y)$  and two positive measures supported on positive reals. One motivation behind this class of bi-orthogonal polynomials came from studying non-smooth soliton-like solutions to nonlinear wave equations of the Camassa-Holm type, in particular, the Degasperis-Processi (DP) equation. These special solutions, called peakons, are fundamentally related to the origins of the theory of orthogonal polynomials and the historic work of M.G. Krein on the spectral theory of an inhomogeneous string. In the case of the DP equation the relevant theory is a non-self adjoint third order boundary value problem called the cubic string to emphasize its affinity with the classical

inhomogeneous string problem. I will explain the role of peakons in the DP equation and how some features of their dynamics are related to the spectral problem for the cubic string and, subsequently, how all the information about the dynamics of peakons is tied to the inverse spectral problem for the cubic string. I will introduce certain Pade-type approximation problem which naturally leads to a new class of bi-orthogonal polynomials, the subject of this talk. I will mention also other boundary problems, not related to peakons, which also lead to Cauchy BOPs. Finally, I will explain how the theory of Cauchy BOPs can be used to solve other peakon equations. This talk is based on joint work with Marco Bertola and Misha Gekhtman as well as an earlier work with Hans Lundmark.

Speaker: **Dong Wang** (CRM)

Title: *A new relation between complex Wishart ensembles and KP tau functions*

Abstract: A new KP tau function structure of the matrix integrals of complex Wishart ensemble is discovered by the Boson-Fermion correspondence.