Topological and Geometric Rigidity

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1 Introduction

This conference discussed a wide variety of problems motivated directly and indirectly by Mostow rigidity and a wide variety of approaches to these problems using tools from geometry, algebra, quantitative topology, and index theory. We shall give here a brief survey of these problems by mentioning at the appropriate places where talks in this conference fit in providing context for the work of the conference and an indication of that work. In particular, these notes are variants of the talks that Davis and Weinberger gave at the meeting.

At the end of each section we will give a few references for additional information. We will often reference a survey rather than an original source. Also, our mention of the topic discussed by a speaker is not intended to indicate that it was not an exposition of joint work.

2 Mostow rigidity (an example of geometric rigidity)

It appropriate to begin our survey with Mostow Rigidity:

Theorem 1 *Suppose M and N are closed hyperbolic manifolds of dimension $n > 2$, and $f : \pi_1 M \to \pi_1 N$ is an isomorphism, then there is a unique isometry $F : M \to N$, inducing f.*

For $n = 2$ this is false, with there being a contractible space of hyperbolic structures on any surface, Teichmuller space. The theorem is true for noncompact manifolds with finite volume (Prasad) and for irreducible lattices in semisimple Lie groups (Mostow). Later we will discuss Margulis's superrigidity which is a far-reaching extension of Mostow's theorem.

Without semisimplicity, isometry is too much to hope for: indeed even the torus has a large, yet contractible, space of flat structures SL_n/SO_n. Nevertheless, even this can be thought of as a form of rigidity (and the nonpositive curvature of that space does indeed give rise to a number of important applications.)

Belegradek's talk at the conference, among other things, give more examples of these kinds of classical rigidity phenomena.

References for this section include: [31] and [34].

3 Borel conjecture (an example of topological rigidity)

Borel, in a letter to Serre, after hearing Mostow talk about rigidity (actually in the solvable setting where a smooth, rather than geometric rigidity, occurred) made the following far-reaching conjecture:

Conjecture 2 (Borel conjecture) *Suppose M and N are closed aspherical manifolds, and $f : \pi_1 M \to \pi_1 N$ is an isomorphism, then there is a homeomorphism $F : M \to N$, inducing f.*

Borel singled out asphericality, that is, having contractible universal cover, as the appropriate topological analog of being hyperbolic (or a lattice). Being aspherical already leads to a homotopy rigidity: any two aspherical CW complexes with isomorphic fundamental groups are homotopy equivalent.

At the time, the difference between the smooth and topological categories was not yet understood, but this conjecture is definitely at the core of topological topology rather than differential topology, despite its smooth origin.

We will only discuss the situation of manifolds of dimension > 4; the low dimensional problem is closely tied to the geometrization conjecture in dimension 3, and topological surgery in dimension 4: subjects which we not the focus of this workshop.

The first progress on this conjecture was its proof by Hsiang–Shaneson and Wall for tori using Farrell's thesis[1]. Wall pointed out that essentially the same argument proves it for Poly-**Z** groups. Using Cappell's splitting theorem in place of Farrell's thesis one can handle many other manifolds inductively (such as surfaces, and many Haken 3-manifolds) but this approach definitely fails for any group with property T (explained below) and therefore, in particular, any higher rank lattice in a simple Lie group.

A real change in perspective came with the work of Farrell and Hsiang on flat manifolds that used a mix of algebraic techniques with ideas of controlled topology, most notably the α–approximation theorem of Chapman and Ferry. The subsequent revolutionary work of Farrell and Jones introduced foliated control in place of metric control and used dynamics to prove many results related to the Borel conjecture, for example proving the Borel Conjecture for a class of groups which includes more groups that the original work of Mostow. (Note that the Borel Conjecture applies in the more general context of topological rigidity while Mostow rigidity gives strongly results in a more restricted setting and is geometric rigiditiy.)

The talks by Bartels and Reich at the conference described excellent recent progress on the Borel conjecture for hyperbolic groups.

References for this section include: [12], [13], [15], [16], [25], and [43].

4 Novikov's conjecture (and theorem)

Another important problem that enters this mix is a conjecture of Novikov's. The original statement of the problem is quite simple. (See the references for several surveys; there are related surveys of the Baum–Connes conjecture.)

Recall first the Hirzebruch signature theorem:

Theorem 3 *Let M^{4k} be a closed oriented smooth manifold, and let $\mathrm{sign}(M)$ denote the signature of its cup product form on H^{2k}. There are explicit polynomials in the Pontrjagin classes of M, denoted by $L_k(M)$, so that*

$$\mathrm{sign}(M) = \langle L_k(M), [M] \rangle .$$

Noticing that the Hirzebruch formula gives an example of a homotopy invariant combination of Pontrjagin classes, Novikov suggested that for nonsimply connected manifolds there might be additional homotopy invariant classes.

Conjecture 4 (Novikov conjecture) *If $f : M \to B\pi$ is a map and $\alpha \in H^r(B\pi; \mathbf{Q})$, then $\langle f^*(\alpha) \cup L(M), [M] \rangle$ is a homotopy invariant.*

Here $L = 1 + L_1 + L_2 + L_3 + \cdots$.

Nowadays it is common to Poincaré dualize and phrase the problem in terms of other homology theories. This will be discussed in the next section.

Novikov himself used the homotopy invariance properties for simply connected manifolds to lay the foundations of the topological category, by showing the Pontrjagin classes are rationally independent of the smooth structure of the defining manifold. From a modern point of view, Novikov's theorem can be viewed as a metric analogue of his conjecture (see [19]).

References for this section include: [9], [17], [19] and [28].

[1] Khan's talk at this meeting explained some low dimensional variants of Farrell's fibering theorem.

5 Reformulation

Surgery theory enables one to reformulate the Borel conjecture algebraically and connect it to the Novikov conjecture. If one believes the Borel conjecture in full generality, then it actually follows that one is committed to believing it for aspherical manifolds with boundary, if one deals with maps of pairs that are already homeomorphisms on the boundary. (The most direct way to see this uses M. Davis's reflection group method.) That version then allows us, by thickening any finite-dimensional $B\pi$ in a high dimensional Euclidean space, enables us to discuss "the Borel conjecture for the group π" – that a more general class of groups than the fundamental groups of closed aspherical manifolds. In fact, below, we extend the Borel conjecture to a more general class yet, that of torsion-free groups.

Note for example that if the Borel conjecture were to be true for an aspherical manifold V, and for $V \times [0,1]$, then the Whitehead group of $\pi_1(V)$ would have to vanish.

Conjecture 5 *If π is torsion free, then $Wh(\pi) = 0$.*

This relates to the geometric problem only for countable groups of finite cohomological dimension (or, if we insist compactness, of type FP). Nevertheless, the above is compatible with all of the evidence and with the conjectures of Farrell and Jones discussed in section 10.

Conjecture 6 *If π is torsion free, then the assembly map*

$$A : H_n(B\pi; \mathbf{L}(e)) \longrightarrow L_n(\pi)$$

is an isomorphism.

This map arises in the modern (post–Wall) formulation of surgery theory as the critical ingredient arising in the classification of manifolds simple homotopy equivalent to a given one. If one tensors with $\mathbf{Q}$, then the left hand side becomes $\oplus H_{n-4i}(B\pi; \mathbf{Q})$. The injectivity of this map then is a restatement of the Novikov conjecture for manifolds:

Conjecture 7 *If π is a group, then the assembly map*

$$A : H_n(B\pi; \mathbf{L}(e)) \longrightarrow L_n(\pi)$$

is an injection after tensoring with $\mathbf{Q}$.

Below we will discuss statements for all groups that include both the conjectures of Novikov and Borel. However, for the meantime, we shall also state an analogue of this conjecture in the setting of C^*-algebras, that was initially introduced by Mischenko and Kasparov as a tool for the Novikov conjecture. It is sometimes called the (strong) Novikov conjecture (although its integral form does not imply the integral form of the Novikov conjecture, e.g. that for torsion free groups, the assembly map is injective).

Conjecture 8 *If π is a group, then the assembly map*

$$A : K_n(B\pi) \longrightarrow K_n(C^*\pi)$$

is an injection after tensoring with $\mathbf{Q}$..

(If π is torsion free, then this map is conjectured to be an isomorphism, if one is careful to use the reduced C^* algebra. This is the torsion free case of the Baum–Connes conjecture.)

References for this section include: [9], [11], [22], [35], [36], and [44].

6 Other operators

The main advantage of topological formulations of the problem is that the problem (and the variants that it generates) are well adapted to a range of topological problems. The analytic formulation has the advantage that it is well adapted to other elliptic operators.

One of these is the following:

Conjecture 9 (Gromov-Lawson Conjecture) [2] *If M is a Riemannian manifold with positive scalar curvature whose universal cover is spin, then for any map $f : M \to B\pi$ and $\alpha \in H^r(B\pi; \mathbf{Q})$, one has $\langle f^*(\alpha) \cup \hat{A}(M), [M]\rangle = 0$.*

In particular, no aspherical manifold should admit a metric of positive scalar curvature.

Here $\hat{A}$ is the $\hat{A}$-genus studied by Borel and Hirzebruch, and identified with the index of the Dirac operator by Atiyah and Singer. As a conjecture, it has the same relation to the theorem of Atiyah–Lichnerowicz–Singer on positive scalar curvature, as the Novikov conjecture has to the Hirzebruch Signature theorem.

It follows, as was proved by Rosenberg, from the strong Novikov conjecture. We leave its survey to other writers. We remark, though, that the ideas involved in the study of the positive scalar curvature problem have had important influence on the study of the Novikov conjecture as well.

Rosenberg had also suggested that the classical birational invariance of the Todd class, should have a higher analogue involving the fundamental group. This is true (not merely conjectural) as was proven by several groups of researchers[3].

The algebra–geometric side of this area appeared in the talks of both Block and Cappell where ideas of non-commutative algebraic geometry were applied to complex manifolds which are not varieties, and maps between varieties that are far from birational were studied respectively. Roe's talk developed the analogy between surgery theory and invertible elliptic operators.

References for this section include: [20], [38], [39], and [40].

7 The L^2-index theorem

One of the ideas implicit in the previous section is the use of the universal cover as a tool in understanding the topology and geometry of a compact manifold. This is indeed one of the important ideas in most proofs of the Novikov conjecture.

A tool in this development is Atiyah's index theorem. It takes off from the multiplicativity of the index of an elliptic operator under finite sheeted covers. Thus, the "G-index" for a finite group acting freely on (M, D), meaning ind$(D)/|G|$, is the same as ind$(M/G, D/G)$. Atiyah defined, using von Neumann traces, the "G-index" even if G is an infinite group acting only properly discontinuously. If the action is free, he showed the expected equality. (Note that in the non-free case, even for finite groups, there are contributions from the fixed point sets of elements of G in a formula for the G-index.)

It is also possible to define purely topological invariants by this method—which gives the L^2-betti numbers. Note, for example, that M has finite fundamental group G, then its 0^{th} (and top) L^2-betti number is (are) $1/|G|$. If G is infinite then this number is 0. In any case, it is easy to give examples where these numbers have fractional part when G contains torsion[4], but as far as we know, this is impossible if G is torsion free. This impossibility is called the Atiyah conjecture—although Atiyah had only asked this as a question.

The vanishing of Whitehead groups in the torsion free case hypothesized in the previous section has a spiritual connection to the idea that $\mathbf{Z}\pi$ has only the obvious units $\pm\pi$ and no zero divisors if π is torsion free. For the precise connection between these conjectures and the Atiyah conjecture and the Baum–Connes conjecture, we recommend Lück's book.

At the conference, Linnell gave a talk showing extremely strong results on denominators present in L^2-betti numbers for linear groups. Sauer gave new methods via measurable equivalence relations and more sophisticated algebra to compute L^2-betti numbers.

References for this section include: [1], [8] and [30].

[2] There is a stronger conjecture, made by Rosenberg, that includes integral information and a converse as well.

[3] Block–Weinberger, Brasselet–Schuermann–Yokura and Borisov–Libgober.

[4] It is not known whether these numbers are always rational. There are no examples where an *index* is irrational by an orbifold index theorem.

8 Groups with torsion

It is time to discuss groups with torsion. A geometric way to generalize the Borel conjecture is to assert the rigidity of e.g. hyperbolic orbifolds. In the differential geometric setting there is no difficulty in extending the theorem from manifolds to orbifolds. (Note that the isometry in Mostow's theorem is unique, so it is automatically equivariant with respect to any group of isometries of domain and range that induce the same outer automorphisms of fundamental groups.)

However, in the topological setting the presence of singularities in the orbifold does change the situation a great deal. Indeed there are counterexamples to the Equivariant Borel conjecture from different sources (see Weinberger's book referred to above); we will discuss some presently.

Note that in the geometric setting, singularities are all aspherical manifolds, so that inductively, assuming the ordinary Borel conjecture, it is reasonable to assume that our map is a homeomorphism on the lower strata. The relative structure set is (aside from algebraic K-theoretic decoration issues) the fiber of an assembly map that has a purely algebraic topological interpretation given by [10].

Generalizing the universal cover of $B\Gamma$, that corresponds to free actions, we now need to replace the role that hyperbolic space plays for discrete subgroups of $O(n,1)$. Let $\underline{E}\Gamma$ be the universal space for proper discontinuous Γ maps, i.e. the terminal equivariant homotopy type in the category of proper discontinuous Γ actions. It is characterized by having a contractible fixed sets for each finite subgroup of Γ and empty fixed set for each infinite subgroup of Γ. (For Γ torsion free, $\underline{E}\Gamma$ is merely $E\Gamma$.)

With this notation, then rigidity would boil down to an assembly map being an isomorphism:

$$H_n^\Gamma(\underline{E}\Gamma;\mathbf{L}) = H_n\left(\underline{E}\Gamma/\Gamma;\mathbf{L}(\Gamma_x)\right) \longrightarrow L_n(\Gamma).$$

A similar description can be given for the relative equivariant Whitehead group using the assembly map in Algebraic K-theory. In that last case it has long been known (through its connection to "the fundamental theorem of algebraic K-theory") that Nil groups obstruct this being an isomorphism.

Cappell defined a similar UNil group that arises in surgery theory. He showed that the above conjecture is false even for the extraordinarily simple group $\mathbf{Z}_2 * \mathbf{Z}_2$ (this group acts properly discontinuously on $\mathbf{R}$. Of course it is too low dimensional to apply surgery to, but ultimately these examples do give rise to counterexamples to equivariant rigidity for some crystallographic orbifolds.)

At the meeting, the talks by Davis, Quinn, Ranicki, and Reich all gave new information about the calculation of Nil and UNil groups by a mixture of algebraic and geometric methods. Indeed, this area is, in our view, one of the most exciting areas of development.

If one changes coefficients, to say replace $\mathbf{Z}$ by $\mathbf{Q}$ (or another field) in the assembly maps, then the Nil and UNil terms would vanish.

The Farrell–Jones conjecture (discussed below) is an extraordinary conjecture that applies even when the Nil and UNil groups are nonzero. It (conjecturally) reduces the study of these groups to groups that act properly discontinuously on $\mathbf{R}$ — the virtually cyclic groups. This conjecture grew out of their successful attempts to prove the Borel conjecture for compact non-positively curved manifolds and invisible but crucial role that geodesics played in that work.

In operator theory, there is no analogue of these Nil and UNil groups. In that case, one simply obtains a conjectural isomorphism (the Baum-Connes conjecture),

$$K_n^\Gamma(\underline{E}\Gamma) \longrightarrow K_n(C^*\Gamma).$$

Of course, there are different subtleties here, for instance that we use the reduced C^* algebra (i.e. the completion of $\mathbf{C}\Gamma$ with respect to the action of Γ on $L^2\Gamma$ by the regular representation). Using the completion with respect to arbitrary unitary representations runs afoul of Kazhdan's property T — a subject that makes several appearances in our story and will therefore be introduced in the next section.

The talks by Ji and Rosenthal gave many cases where it is possible to prove the integral split injectivity of these assembly maps. Higson's was devoted to his deep work with Kasparov on the Baum–Connes conjecture for groups that act isometrically on Hilbert space, and the injectivity of assembly maps (due to Skandalis, Tu, and Yu) for groups that embed in Hilbert space.

References for this section include: [3],[4], [6], [10], and [42].

9 Property T (Silberman)

Kazhdan's property T is a profound representation theoretic property of groups that has had an extraordinary range of application. It asserts that the trivial representation is isolated in the space of all unitary representations of the group. Equivalently, it can be viewed as a fixed point property: any affine isometric action of the group on Hilbert space must have a fixed point (or that a unitary representation with "almost invariant" unit vectors has a fixed vector).

A quick connection between property T and the problems we are considering here is that it immediately shows that a group like $SL_n(\mathbf{Z})$ for $n > 2$ (or any lattice therein) cannot act without fixed points on any tree — and therefore the Novikov conjecture cannot be proven using "splitting" methods (i.e. the inductive method that covers the case of the torus).

Other connections are via super-rigidity (see below). Note that the theorem of Higson and Kasparov asserting the Baum–Connes conjecture requires the exact opposite of property T — the existence of a proper discontinuous action of the group on a Hilbert space. This leads to another, more recent connection.

Margulis had long ago observed that property T can be used to construct families of graphs, called expanders that are of great interest to computer scientists. Gromov observed that a sequence of expanders cannot be embedded in any Hilbert space with any reasonable distortion. (Technically, they cannot be uniformly embedded in Hilbert space.) Thus a group containing expanders is a candidate for counterexamples to the Baum–Connes conjecture, and it was indeed verified by several authors that such a group provides counterexamples to variants of the Baum-Connes conjecture. However the original Baum-Connes Conjecture still stands.

The construction of these groups is done by random methods. These groups have very strong and unusual properties; Silberman in his talk explained some of these fixed point properties and described the use of "heat flow" techniques on them. This led, subsequently to the conference, to a joint work with Fisher wherein torsion free groups that do not smoothly and volume preservingly on any compact manifold are produced.

References for this section include: [5], [21], [24] and [29].

10 Techniques

There are, by now, many approaches to the Novikov conjecture. We first mention two that apply equally well in operator theoretic and topological contexts. One is the method of descent wherein coarse properties of the group with its word metric are then coupled with a family argument (or equivalently a homotopy fixed set argument) to give a result about the group as an algebraic object. In operator theory, this often goes via a "Dirac–dual Dirac" argument, and in topology/K-theory, one uses either controlled arguments growing out of the α-approximation theorem or more categorical alternatives to that.

Rosenthal's talk explained how to do topological descent for groups with torsion in purely algebraic settings, and also how to remove compactness hypothesis on $B\Gamma$. Ferry's talk was about the relevant properties of the group with its word metric, and alternatives to boundedness, with connections to de Rham cohomology.

The other general result (true in all the settings, by several different arguments) is that for groups of finite asymptotic dimension, all of these results are correct. Dransihnikov's talk was about the asymptotic dimension of Coxeter groups.

Bestvina's talk about the boundary of Teichmüller space and of outer space was designed to help determine whether these techniques could be applied to (outer) automorphisms of free groups.

Ji's talk surveyed many results that can approached by these methods—in particular, how one deals with many specific classes of linear groups. Prassidis's talk explained how to use a criterion of Lubotzky to show linearity of various groups.

General linear groups in the C^*-algebra setting have been disposed of by the embedding method of Skandalis–Yu–Tu, mentioned above. This does not imply the integral L-theory conjecture at the prime 2.

References for this section include: [2], [7], [18], [23], [26], [27], [37], [41], and [45].

11 Superrigidity (Melnick, Monod, Fisher)

Margulis has generalized Mostow's rigidity theorem in an extraordinary way: He showed that many homomorphisms of a lattice into another Lie group can (in the higher rank situation) be classified Lie theoretically.

This immediately suggests versions of Borel for embeddings, immersions, fiberings, and so on. Some of these are implicit in "twisted" versions of the conjecture—that turn out to be important in proving the original versions in certain cases.

Zimmer has extended Margulis's work to suggest a large scale research program involving lattice actions on manifolds. This involves intimately (through work of Benveniste, and Fisher–Margulis) versions of property T for other Banach spaces.

Monod described a cohomological approach to superrigidity for irreducible lattices in products that is very different from Margulis's original approach.

Fisher, on the other hand, described some strange quasi-isometric embeddings in a bare-handed tour de force, that violates the spirit of quasi-isometric super-rigidity.

Melnick's talk was also in this area: she combined dynamic, ergodic, and algebraic arguments to give rigidity results in the setting of Cartan geometries.

The reference included in this section is [46].

12 Farrell–Jones conjecture (Mineyev, Bartels, Reich, Davis)

We have already mentioned that for groups with torsion, all of these problems, even at the level of conjecture, are much more subtle. When we move to algebraic K-theory and L-theory, there are additional difficulties due to Nil and UNil groups.

In the course of their extraordinary work proving the Borel conjecture for torsion free lattices, Farrell and Jones were taken by the special role that closed geodesics on these spaces played in the proof. Indeed, this role is invisible if one considers just usual L-theory of K-theory of integral group rings, but it was readily apparent for pseudoisotopy and for analogues involving more general group rings.

Ultimately, they realized that one can reformulate everything in terms of isolating out subgroups that are virtually cyclic, and formulating an analogue to the statement for $\underline{E}\Gamma$ to another classifying space involving virtually cyclic isotropy.

The talks of Quinn and Reich devoted controlled ideas to the analysis of Nil. David discussed the Farrell–Jones conjecture for crystallographic groups.

Mineyev's talk was on the detailed structure of the boundary of hyperbolic groups, and how that can be applied geometrically. Indeed, it was so applied in Bartels's talk which sketched the proof of the Farrell–Jones conjecture for hyperbolic groups.

References in this section include: [13], [14], [32], and [33].

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Below we attach detailed program and abstracts of the talks given. Many of these have subsequently been written up for publication.

A highlight of the conference was a two evening five hour lively problem session. The participants asked and answered many of each others' questions. Given the interdisciplinary nature of the participation, this was very valuable in breaking down the barriers between fields.

Lior Silberman took excellent notes on the problem session, and it is anticipated that these will ultimately be polished and published.