Banff International Research Station
for Mathematical Innovation and Discovery

Loss of compactness in nonlinear PDE: Recent trends
August 26-31, 2007

MEALS

*Breakfast (Buffet): 7:00–9:00 am, Sally Borden Building, Monday–Friday
*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday
*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday
Coffee Breaks: As per daily schedule, 2nd floor lounge, Corbett Hall
***Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

MEETING ROOMS

All lectures will be held in Max Bell 159 (Max Bell Building accessible by bridge on 2nd floor of Corbett Hall). Hours: 6 am–12 midnight. LCD projector, overhead projectors and blackboards are available for presentations. *Please note that the meeting space designated for BIRS is the lower level of Max Bell, Rooms 155–159. Please respect that all other space has been contracted to other Banff Centre guests, including any Food and Beverage in those areas.*

SCHEDULE

Sunday

16:00	Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
17:30–19:30	Buffet Dinner, Sally Borden Building
20:00	Informal gathering in 2nd floor lounge, Corbett Hall Beverages and small assortment of snacks available on a cash honour-system.

Monday

7:00–8:45 Breakfast

8:45–9:00 Introduction and Welcome to BIRS by BIRS Station Manager, Max Bell 159

Lectures

9:00–9:50 **M. Struwe** (ETH Zentrum)
Quantization for fourth order pde with critical exponential growth

10:00–10:50 **O. Druet** (École Normale Supérieure de Lyon)
The critical Lazer-McKenna conjecture in low dimensions

10:50–11:10 Coffee Break, 2nd floor lounge, Corbett Hall

11:10–12:00 **A. Malchiodi** (SISSA)
Morse theory and scalar field equations on compact surfaces

12:00–13:00 Lunch

13:00–14:00 Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall

14:00 Group Photo; meet on the front steps of Corbett Hall

Lectures

14:40–15:30 **C.-S. Lin** (National Taiwan University)
Computing the topological degree of the mean field equation with singularity and its application

15:30–16:00 Coffee Break, 2nd floor lounge, Corbett Hall

16:00–16:50 **Y. Ge** (Université Paris XII-Val de Marne)
On the σ_2-scalar curvature and its application

17:00–17:50 **L. Mazzieri** (Scuola Normale Pisa)
Generalized gluing for Einstein constraint equations

18:00–19:30 Dinner

Tuesday

7:00–9:00 Breakfast

Lectures

9:00–9:50 **Y. Li** (Rutgers University)
Extension to a theorem of Liouville and applications

10:00–10:50 **M. del Pino** (Universidad de Chile)
Multiple bump-lines and transition layers for planar autonomous elliptic problems

10:50–11:20 Coffee Break, 2nd floor lounge, Corbett Hall

11:20–12:10 **F. Robert** (Université Nice-Sophia Antipolis)
Quantization issues for fourth order elliptic equations in dimension four

12:30–13:30 Lunch

18:00–19:30 Dinner

Wednesday

7:00–9:00 Breakfast

Lectures

9:00–9:50 **J. Wei** (Chinese University of Hong Kong)
On Fourth Order Mean Field Equations

10:00–10:50 **N. Ghoussoub** (University of British Columbia)
Bessel potentials and optimal Hardy and Hardy-Rellich inequalities

10:50–11:20 Coffee Break, 2nd floor lounge, Corbett Hall

11:20–12:10 **M. Lucia** (Universitat zu Koln)
A Deformation Lemma applied to some mean field equations

12:30–13:30 Lunch

18:00–19:30 Dinner

Thursday

7:00–9:00 Breakfast

Lectures

9:00–9:50 **S. Alama** (McMaster University)
Ginzburg–Landau vortices concentrating on curves

10:00–10:50 **J. Davila** (CMM & DIM University of Chile)
Critical points of the regular part of the Green's function with Robin boundary condition

10:50–11:20 Coffee Break, 2nd floor lounge, Corbett Hall

11:20–12:10 **I. Shafrir** (Technion-Israel Institute of Technology)
Global minimizers for a p-Ginzburg-Landau energy

12:30–13:30 Lunch

Lectures

16:00–16:50 **E. Sandier** (Université Paris XII-Val de Marne)
Gamma convergence of anisotropic models in superconductivity

17:00–17:50 **M. Musso** (Universidad Catolica de Chile)
Bubbling near boundary geodesics at the second critical exponent

18:00–19:30 Dinner

Friday

7:00–9:00 Breakfast

Lectures

9:00–9:50 **A. Pistoia** (Universitá di Roma "La Sapienza")
On the existence of solutions to some critical problems

10:00–10:50 **F. C. Marques** (IMPA)
Compactness and Noncompactness Theorems for the Yamabe Problem

10:50–11:20 Coffee Break, 2nd floor lounge, Corbett Hall

11:20–12:10 **F. Gladiali** (Universitá di Sassari)
Singular limit of radial solutions in an annulus

12:30–13:30 Lunch

Checkout by 12 noon.

** 5-day workshops are welcome to use the BIRS facilities (2nd Floor Lounge, Max Bell Meeting Rooms, Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. **

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ABSTRACTS
(in alphabetic order by speaker surname)

Speaker: **Stanley Alama** (McMaster University)
Title: *Ginzburg–Landau vortices concentrating on curves*
Abstract: We study a two-dimensional Ginzburg-Landau functional, which describes superconductors in an externally applied magnetic field. We are interested in describing the energy minimizers at the critical value of the magnetic field for which vortices first appear in the superconductor (the "lower critical field"). The vortices are quantized singularities, and we are interested in their number and their distribution in the sample nearby the critical field. I will describe recent results with L. Bronsard and V. Millot in which we study the number and distribution of these vortices which concentrate along a curve. Their distribution is determined by a classical problem from potential theory.

Speaker: **Juan Davila** (CMM & DIM University of Chile)
Title: *Critical points of the regular part of the Green's function with Robin boundary condition*
Abstract: We consider the Green's function for the Laplacian in a smooth bounded domain Ω of $\mathbb{R}^N$ with Robin boundary condition

$$\frac{\partial G_\lambda}{\partial \nu} + \lambda b(x) G_\lambda = 0, \quad \text{on } \partial\Omega,$$

where $b > 0$ is smooth and $\lambda > 0$. Let $S_\lambda(x, y)$ be the regular part of G_λ and $R_\lambda(x) = S_\lambda(x, x)$ be the Robin function. We show that as $\lambda \to \infty$, the Robin function has at least 3 critical points. Moreover, in the case $b \equiv const$ we prove that R_λ has critical points near non-degenerate critical points of the mean curvature of the boundary, and when $b \not\equiv const$ there are critical points of R_λ near non-degenerate critical points of b.
This work is in collaboration with Michal Kowalczyk (Universidad de Chile) and Marcelo Montenegro (Unicamp).

Speaker: **Manuel del Pino** (Universidad de Chile)
Title: *Multiple bump-lines and transition layers for planar autonomous elliptic problems*
Abstract: In this talk I will describe a construction of new solutions to some classical autonomous semilinear elliptic equations in the plane. These solutions constitute a "gluing" of one-dimensional profiles with a single transition, located very far apart one to each other. In the case of the Allen-Cahn equation, solutions with a finite number of nearly parallel transition layers are built, while for the stationary nonlinear Schrodinger equation multiple bump-line patterns are found. The Toda system is shown to rule the asymptotic shape of these transition lines.
This is joint work with Michal Kowalczyk, Frank Pacard and Juncheng Wei.

Speaker: **Olivier Druet** (École Normale Supérieure de Lyon)
Title: *The critical Lazer-McKenna conjecture in low dimensions*
Abstract: We try to explain why some methods used to solve the critical Lazer-McKenna conjecture in high dimensions fail in low dimensions by performing an asymptotic analysis of the solutions of the equation $\Delta u + \lambda u = u_+^{\frac{n+2}{n-2}} + \alpha\varphi_1$ in a domain when $\alpha \to +\infty$.

Speaker: **Yuxin Ge** (Université Paris XII-Val de Marne)
Title: *On the σ_2-scalar curvature and its application*
Abstract: In this talk, we establish an analytic foundation for a fully non-linear equation $\frac{\sigma_2}{\sigma_1} = f$ on manifolds with positive scalar curvature from conformal geometry. As application, we prove that, if a compact 3-dimensional manifold M admits a riemannian metric with positive scalar curvature and $\int \sigma_2 > 0$, then topologically M is a quotient of sphere.
This is a joint work with G. Wang et C.S. Lin.

Speaker: **Nassif Ghoussoub** (University of British Columbia)
Title: *Bessel potentials and optimal Hardy and Hardy-Rellich inequalities*
Abstract: We give necessary and sufficient conditions on a pair of positive radial functions V and W on a ball Ω of radius R in $\mathbb{R}^n$, $n \geq 2$, so that the following inequalities hold for all $u \in C_0^\infty(\Omega)$:

$$\int_\Omega V(x)|\nabla u|^2 dx \geq \int_\Omega W(x)u^2 dx$$

and

$$\int_B V(x)|\Delta u|^2 dx \geq \int_B W(x)|\nabla u|^2 dx + (n-1)\int_B (\tfrac{V(x)}{|x|^2} - \tfrac{V'(|x|)}{|x|})|\nabla u|^2 dx.$$

We then identify a large number of such couples (V, W) – that we call Bessel pairs – and their corresponding best constants. This will allow us to complete, improve, extend, and unify most related old and recent Hardy and Hardy-Rellich inequalities obtained by Caffarelli-Kohn-Nirenberg, Brezis-Vázquez, Adimurthi-Chaudhuri-Ramaswamy, Filippas- Tertikas, Adimurthi-Grossi-Santra, Tertikas-Zographopoulos and Liskevich-Lyachova-Moroz, among others.
This is joint work with Amir Moradifam.

Speaker: **Francesca Gladiali** (Universitá di Sassari)
Title: *Singular limit of radial solutions in an annulus*
Abstract: In this paper we study the radial solutions of the problem

$$\begin{cases} -\Delta u = \lambda e^u & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega \end{cases}$$

where Ω is an annulus of $\mathbb{R}^N$, $N \geq 2$ and λ is close to zero.
Among the other results we show the existence of a singular limit and the nondegeneracy of the solution in the space of the radial functions.

Speaker: **Yanyan Li** (Rutgers University)
Title: *Extension to a theorem of Liouville and applications*
Abstract: The classical Liouville theorem says that a positive entire harmonic function must be a constant. We give a fully nonlinear version of it. This extension enables us to establish local gradient estimates of solutions to general conformally invariant fully nonlinear elliptic equations of second order. This talk will start from a proof of the classical Liouville theorem using only the comparison principle and the invariance of harmonicity under Mobius transformations. We will then outline the proof of the comparison principle used in establishing the new Liouville theorem. Finally we outline the proof of the gradient estimates via the Liouville theorem.

Speaker: **Chang-Shou Lin** (National Taiwan University)
Title: *Computing the topological degree of the mean field equation with singularity and its application*

Speaker: **Marcello Lucia** (Universitat zu Koln)
Title: *A Deformation Lemma applied to some mean field equations*
Abstract: We will present a deformation lemma for a class of functionals that do not satisfy the Palais-Smale condition. This can fruitfully be applied to prove existence of solutions for some mean field equations.

Speaker: **Andrea Malchiodi** (SISSA)
Title: *Morse theory and scalar field equations on compact surfaces*
Abstract: We consider a nonlinear scalar equation on compact surfaces motivated by the Kazdan-Warner problem or by models in mathematical physics. We discuss a Morse theoretical approach to the equation, which yields existence results and simplifies the proof of a degree formula obtained by Chen and Lin. Similar methods apply to the problem of prescribing the Q-curvature of a compact four manifold.

Speaker: **Fernando Coda Marques** (IMPA)
Title: *Compactness and Noncompactness Theorems for the Yamabe Problem*
Abstract: It has been conjectured that the set of metrics of volume one and constant scalar curvature in a given conformal class is compact, unless the manifold is conformally equivalent to the round sphere. In a joint work with Marcus Khuri and Richard Schoen, we have verified this conjecture if the dimension is less than or equal to 24. On the other hand, Simon Brendle has recently constructed smooth counterexamples if the dimension is at least 52. In collaboration with him we have been able to extend these counterexamples to the remaining dimensions $25 \leq n \leq 51$.
In this talk we plan to give an overview of these recent results.

Speaker: **Lorenzo Mazzieri** (Scuola Normale Pisa)
Title: *Generalized gluing for Einstein constraint equations*
Abstract: We construct a family of new solutions to the Einstein constraint equations by performing the generalized connected sum of two known compact m-dimensional constant mean curvature solutions (M_1, g_1, Π_1) and (M_2, g_2, Π_2) along a common isometrically embedded k-dimensional sub-manifold (K, g_K). Away from the gluing locus the metric and the second fundamental form of the new solutions can be chosen as close as desired to the ones of the original solutions. The proof is essentially based on the conformal method and the geometric construction involves conformal transformations along the slices of the normal fiber bundle of K, for this reason the codimension $n := m - k$ of K in M_1 and M_2 is required to be ≥ 3. In this sense our result is a generalization of the Isenberg-Mazzeo-Pollack gluing, which works for connected sum at points and in dimension 3. The solution we obtain for the Einstein constraint equations can be used to produce new short time vacuum solutions of the Einstein system on a Lorentzian $(m+1)$-dimensional manifold, as guaranteed by a well known result of Choquet-Bruhat.

Speaker: **Monica Musso** (Universidad Catolica de Chile)
Title: *Bubbling near boundary geodesics at the second critical exponent*
Abstract: We consider the problem $\Delta u + u^{\frac{N+1}{N-3}-\epsilon} = 0$ in a bounded, smooth domain in $\mathbb{R}^N$, under Dirihlet boundary conditions. Given a closed geodesic Γ of the boundary which is non-degenerate and has strictly negative curvature, we find a family of solutions whose energy density concentrates near Γ in the form of a Dirac line measure as $\epsilon \to 0$ provided that ϵ avoids certain critical values where resonance occurs.
This is joint work with M. del Pino and F. Pacard.

Speaker: **Angela Pistoia** (Universitá di Roma "La Sapienza")
Title: *On the existence of solutions to some critical problems*
Abstract: We present some recent results, obtained in collaboration with Monica Clapp and Monica Musso, about existence of positive and sign changing solutions to some nonlinear elliptic problems involving the critical Sobolev exponent in pierced domains.

Speaker: **Frédéric Robert** (Université Nice-Sophia Antipolis)
Title: *Quantization issues for fourth order elliptic equations in dimension four*
Abstract: In dimension four, the fourth order elliptic nonlinear equation $\Delta^2 u = e^u$ (E) enjoys conformal invariance properties that allow blowing up of sequences of solutions to (E). This invariance correspond to a similar one for second order equations in dimension two studied, among others, by Brézis-Merle: they actually enlightened a quantization of the energy associated to the dimension two. Surprisingly, there is absolutely no such a quantization in dimension four, and the situation can become quite weird. In this talk, we will describe completely the blow-up in the general case and in interesting specific situations.

Speaker: **Étienne Sandier** (Université Paris XII-Val de Marne)
Title: *Gamma convergence of anisotropic models in superconductivity*
Abstract: A popular model for high temperature superconducters is the Lawrence Doniach model in which the material is represented by a stack of interacting superconducting layers. In certain regimes, this model is close to an anisotropic Ginzburg-Landau model when the interlayer space is small.

Both anisotropic 3D models have interesting Gamma limits when the Ginzburg-Landau parameter is large, I will describe ongoing joint work with S.Alama and L.Bronsard.

Speaker: **Itai Shafrir** (Technion-Israel Institute of Technology)
Title: *Global minimizers for a p-Ginzburg-Landau energy*
Abstract: We study the problem of existence of global minimizers for a p-Ginzburg-Landau type energy on the plane and on the half-plane, for $p > 2$, under a degree condition at infinity. We prove existence of a minimizer when the degree equals 1.
This is joint work with Yaniv Almog, Leonid Berlyand and Dmitry Golovaty.

Speaker: **Michael Struwe** (ETH Zentrum)
Title: *Quantization for fourth order pde with critical exponential growth*
Abstract: For concentrating solutions $0 < u_k \rightharpoonup 0$ weakly in $H^2(\Omega)$ to the equation $\Delta^2 u_k = \lambda_k u_k e^{2u_k^2}$ on a domain $\Omega \subset \mathbb{R}^4$ with Navier boundary conditions the concentration energy $\Lambda = \lim\limits_{k\to\infty} \int_\Omega |\Delta u_k|^2 dx$ is shown to be strictly quantized in multiples of the number $\Lambda_1 = 16\pi^2$.

Speaker: **Juncheng Wei** (Chinese University of Hong Kong)
Title: *On Fourth Order Mean Field Equations*
Abstract: We consider the following fourth order mean field equation

$$(*) \qquad \Delta^2 u = \rho \frac{h(x)e^u}{\int_\Omega h e^u} \ \text{ in } \Omega,$$

with Navier boundary condition

$$u = \Delta u = 0 \ \text{ on } \partial\Omega$$

or Dirichlet boundary condition

$$u = \frac{\partial u}{\partial \nu} = 0 \ \text{ on } \partial\Omega,$$

where h is a $C^{2,\beta}$ positive function, Ω is a bounded and smooth domain in $\mathbb{R}^4$. We establish sharp estimates and as well as degree-counting formula. In particular, we show that for $\rho \in (32m\sigma_3, 32(m+1)\sigma_3)$ the degree-counting formula for (*) is given by

$$d(\rho) = \begin{cases} \frac{1}{m!}(-\chi(\Omega)+1)\cdots(-\chi(\Omega)+m) & \text{for } m > 0, \\ 1 \ \text{ for } m = 0 \end{cases}$$

where $\chi(\Omega)$ is the Euler characteristic of Ω.
(Joint works with C.S. Lin, F. Robert and L. Wang)