Banff International Research Station
for Mathematical Innovation and Discovery

Modular Forms: Arithmetic and Computation
June 3-8, 2007

MEALS

*Breakfast (Buffet): 7:00 – 9:00 am, Donald Cameron Hall, Monday – Friday
*Lunch (Buffet): 11:30 am – 1:30 pm, Donald Cameron Hall, Monday – Friday
*Dinner (Buffet): 5:30 – 7:30 pm, Donald Cameron Hall, Sunday – Thursday
Coffee breaks: As per daily schedule, 2nd floor lounge, Corbett Hall
***Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

MEETING ROOMS

All lectures will be held in Max Bell 159 (Max Bell Building accessible by bridge on 2nd floor of Corbett Hall). Hours: 6 am – 12 midnight. LCD projector, overhead projectors and blackboards are available for presentations. *Please note that the meeting space designated for BIRS is the lower level of Max Bell, Rooms 155-159. Please respect that all other space has been contracted to other Banff Centre guests, including any Food and Beverage in those areas.*

SCHEDULE

Sunday

16:00   Check-in begins (Front Desk – Professional Development Centre - open 24 hours)
17:30-19:30 Buffet Dinner, Donald Cameron Hall
20:00   Informal gathering in 2nd floor lounge, Corbett Hall
             Beverages and small assortment of snacks available on a cash honour-system.

Monday

7:00-9:00   Breakfast
8:45-9:00   Introduction and welcome to BIRS by BIRS Station Manager, Max Bell 159
9:00-9:50   John Cremona, Modular forms and Hecke operators over number fields
9:50-10:20   Coffee break
10:20-11:10 Frank Calegari, Counting automorphic forms
11:20-12:10 Matthew Emerton, *p*-adic mod *p* aspects of arithmetic cohomology
12:10-13:30 Lunch
13:30-14:30 Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
14:30-15:20 Aaron Greicius, Global Galois representations associated to elliptic curves
15:20-15:40 Coffee break
15:40-16:30 Dimitar Jetchev, Global divisibility of Heegner points and Tamagawa numbers
16:30-17:30 Informal discussion
17:30-19:30 Dinner
20:00-21:00 William Stein, Computing with modular forms, part 1: MAGMA (optional tutorial)

Tuesday

7:00-9:00   Breakfast
9:00-9:50   Samit Dasgupta, Computations of Gross-Stark units via Shintani zeta-functions
9:50-10:00   Group photo
10:00-10:20 Coffee break

10:20-11:10 Matthew Greenberg, Heegner point and cohomological modular forms
11:20-12:10 Pierre Charollois, Computing Stark units in Hilbert class fields using periods of Hilbert modular forms
12:10-13:30 Lunch
13:30-14:30 Informal discussion
14:30-15:20 Christian Wuthrich, Computations about the Tate-Shafarevich group using Iwasawa theory
15:20-15:40 Coffee break
15:40-16:30 Neil Dummigan, Symmetric square L-functions and Shafarevich-Tate groups, II
16:40-17:30 Amod Agashe, Visibility of Shafarevich-Tate group for analytic rank one
17:30-19:30 Dinner
20:00-21:00 William Stein, Computing with modular forms, part 2: SAGE (optional tutorial)

Wednesday

7:00-8:45 Breakfast
8:45-9:35 Noam Elkies, Parametrizing genus-2 Jacobians with exotic real multiplication over **Q**
9:45-10:35 Henri Darmon, Chow groups of threefolds and rational points on elliptic curves
10:35-10:55 Coffee break
10:55-11:45 Lassina Dembele, Hilbert-Siegel modular forms and modular threefolds
11:45-13:30 Lunch
13:30-17:30 Free afternoon
17:30-19:30 Dinner

Thursday

7:00-9:00 Breakfast
9:00-9:50 Kenneth Ribet, Modular degrees and congruences among modular forms
9:50-10:20 Coffee break
10:20-11:10 Gabor Wiese, Multiplicities of Galois representations of weight one
11:20-12:10 Lloyd Kilford, On the failure of the Gorenstein property for Hecke algebras of prime weight
12:10-13:30 Lunch
13:30-14:30 Informal discussion
14:30-15:20 Victor Rotger, Modular endomorphism algebras
15:20-15:40 Coffee break
15:40-16:30 Ulf Kühn, Computation of scattering constants
16:40-17:30 Armand Brumer, Semistable abelian varieties and Siegel modular forms
17:30-19:30 Dinner
20:00-21:00 John Cremona and William Stein, Computing with elliptic curves (optional tutorial)

Friday

7:00-9:00 Breakfast
9:00-9:50 Bjorn Poonen, $x^2 + y^3 = z^7$
9:50-10:10 Coffee break
10:10-11:00 Gonzalo Tornaria, Brandt matrices and ternary quadratic forms
11:30-13:30 Lunch

Checkout by 12 noon.

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ABSTRACTS

Speaker: Amod Agashe
Title: Visibility of the Shafarevich-Tate group for analytic rank one
Abstract: Let E be an elliptic curve with associated newform f. If f is congruent to an eigenform whose associated abelian variety has Mordell-Weil rank bigger than that of E, then the theory of visibility can often be used to construct elements of the Shafarevich-Tate group of E. When E has analytic rank one, we extract a factor from the special L-value (the derivative at $s = 1$) that measures congruences as above, and hence can be related to the order of the Shafarevich-Tate group of E (as predicted by the second part of the Birch and Swinnerton-Dyer conjecture), assuming the first part of the Birch and Swinnerton-Dyer conjecture. A similar result was proven earlier when E has analytic rank zero.
In the analytic rank zero case, Stein has conjectured that the Shafarevich-Tate group can be explained by visibility at the same or higher level. We will present some computations of Cremona and Watkins which indicate that the same might be expected for the analytic rank one case.

Speaker: Armand Brumer
Title: Abelian surfaces and Siegel modular forms
Abstract: This talk will be on joint work with Kenneth Kramer. I shall report on existence of semistable abelian varieties defined over **Q** with particular emphasis on surfaces of odd conductor. The related project of Cris Poor and David Yuen on paramodular forms will be discussed briefly.

Speaker: Frank Calegari
Title: Counting automorphic forms
Abstract: We present a new method for bounding the multiplicities of automorphic representations of "cohomological type", i.e., those whose multiplicities can be computed via the cohomology of arithmetic quotients of symmetric spaces, including situations in which the arithmetic quotients are not algebraic varieties. We pay particular attention to the special case of automorphic forms over an imaginary quadratic field, where algorithms of Cremona may be applied to explicitly compute spaces of automorphic forms, and computations could shed light on the extent to which our bounds are optimal. (Joint work with Matthew Emerton).

Speaker: Pierre Charollois
Title: Computing Stark units in Hilbert class fields using periods of Hilbert modular forms
Abstract: Let F be a real quadratic field, and K a quadratic extension of F having only one complex embedding. In a common work with Henri Darmon, we propose a construction leading to a conjectural formula for the complex images of Stark units of K in terms of periods of Eisenstein series over F. We will present an algorithm for producing such periods of Hilbert modular forms, therefore providing evidences supporting our conjecture.

Speaker: John Cremona
Title: Modular forms and Hecke operators over number fields
Abstract: We develop the foundations of an explicit theory of "formal" modular forms over arbitrary

number fields K, including discussion of cusps, modular points, Hecke operators and generalized Atkin-Lehner involutions. This description incorporates the classical theory for $K = \mathbf{Q}$, and also extends earlier work of the author and his students for imaginary quadratic fields, and should be useful more generally in the computation of spaces of cusp forms for GL(2,K) via modular symbols.

Speaker: Henri Darmon
Title: Chow groups of threefolds and rational points on elliptic curves
Abstract: I will discuss calculations of algebraic points on elliptic curves in terms of higher-dimensional cycles on certain Shimura varieties. This is a report on joint work (in progress) with Massimo Bertolini and Kartik Prasanna.

Speaker: Samit Dasgupta
Title: Computations of Gross-Stark units via Shintani zeta-functions
Abstract: I recently stated a refinement of the p-adic Gross-Stark conjecture that proposes an explicit formula for the unit that is conjectured to exist by Gross and Stark. The formula is given as an integral over a p-adic space of a measure defined using Shintani zeta-functions. In this talk I will report on work of my student Kaloyan Slavov, who provided a conceptual framework by which my conjecture can be used explicitly to compute Gross-Stark units and thereby generate class fields of totally real fields. Slavov has succesfully implemented his algorithm in certain examples with quadratic and cubic ground fields. I believe that the formulas arising from Shintani's work can be related to the theory of modular symbols, but I can only make this connection explicit in the case where the ground field is a quadratic field; we present the general case as a tractable open problem.

Speaker: Lassina Dembele
Title: Hilbert-Siegel modular forms and modular threefolds
Abstract: This is a report on some work in progress. We present an algorithm for the computations of Hilbert-Siegel modular forms over real quadratic fields. The modular forms we obtain from our computations are related to the modularity of certain threefolds, thus providing new numerical evidence for the Langlands correspondence.

Speaker: Neil Dummigan
Title: Symmetric square L-functions and Shafarevich-Tate groups, II
Abstract: We re-examine some critical values of symmetric square L-functions for cusp forms of level one. We construct some more of the elements of large prime order in Shafarevich-Tate groups, demanded by the Bloch-Kato conjecture. For this we use the Galois interpretation of Kurokawa-style congruences between vector-valued Siegel modular forms of degree two, making further use of a construction due to Urban. Our calculations with Fourier expansions use the Eholzer-Ibukiyama generalisation of the Rankin-Cohen brackets. Further details here: http://neil-dummigan.staff.shef.ac.uk/papers.html

Speaker: Noam Elkies
Title: Parametrizing genus-2 Jacobians with exotic real multiplication over $\mathbf{Q}$
Abstract: Degree-2 factors of $J_0(N)$ correspond to abelian surfaces $A/\mathbf{Q}$ such that $\text{End}_\mathbf{Q}(A)$ contains an order O in a real quadratic field. Explicit formulas for the general genus-2 curves $C/\mathbf{Q}$ whose Jacobian satisfies this condition on A are known only for the smallest few discriminants $D = \text{disc}(O)$. We outline a new method that considerably extends the range of D for which it is feasible to exhibit the moduli surface for such curves C and the Igusa-Clebsch invariants as rational functions on that surface. For example, when $D = 41$ we birationally parametrize C by the $K3$ surface
$$y^2 = x^3 + (t-1)(t^3 + 23t^2 + 96t - 32)x^2 + 16(t-1)(4t^4 + 53t^3 - 217t^2 + 112t - 16)x.$$

Speaker: Matthew Emerton
Title: p-adic and mod p aspects of arithmetic cohomology

Abstract: As is well-known, the cohomology of arithmetic groups is closely related to the theory of automorphic forms, and often plays a useful role in that theory, since it provides an "avatar" for automorphic forms which is often more amenable to number-theoretic considerations. (For example, one can form cohomology with mod p or p-adic coefficients.) In this talk I will describe some p-adic and mod p constructions based on the (co)homology of arithmetic groups, which provide a natural framework for considering questions such as generalizations of Serre's conjecture to cases other than GL_2, and more generally, for phrasing a truly p-adic version of Langlands conjectures. Part of the talk will be devoted to raising various open problems and conjectures, and to describing some of the computations which it might be helpful to make, as a means of gathering evidence to support (or refute) these conjectures. This talk is based in part on joint work with Frank Calegari.

Speaker: Matthew Greenberg
Title: Heegner point and cohomological modular forms
Abstract: Since Eichler and Shimura, group cohomology has been an extremely important tool for the study of modular forms. In the special case of classical modular forms, the associated cohomology groups may be understood explicitly in terms of modular symbols. By intertwining the theory of modular symbols with his notion of multiplicative p-adic integration, Darmon has constructed conjecturally algebraic Stark-Heegner points on modular elliptic curves. Unlike the classical Heegner points, Darmon's Stark-Heegner points are expected to be rational over class fields of *real* quadratic fields. Recasting Darmon's theory purely in the language of group cohomology, I will describe new Stark-Heegner point constructions based on properties of cohomology groups associated to modular forms on Shimura curves.

Speaker: Aaron Greicius
Title: Global Galois representations associated to elliptic curves
Abstract: Let E/K be an elliptic curve over a number field K. Let G_K be the absolute Galois group of K. The action of G_K on the torsion points of E defines a continuous representation, rho, from G_K to $\text{Aut}(E^{tor})$, which is isomorphic to $GL_2(\hat{\mathbf{Z}})$. We refer to rho as *the global Galois representation* associated to E. Serre, in his 1972 *Inventiones* paper, showed that the image of rho is open when E/K is non-CM. In particular, it is of finite index. By examining the maximal closed subgroups of $GL_2(\hat{\mathbf{Z}})$, we come up with simple necessary and sufficient conditions for the global representation to be surjective. Our analysis allows us to easily find examples of surjective global Galois representations over a certain cubic extension $K/\mathbf{Q}$.

Speaker: Dimitar Jetchev
Title: Global divisibility of Heegner points and Tamagawa numbers
Abstract: We improve Kolyvagin's upper bound on the order of the p-primary part of the Shafarevich-Tate group of an elliptic curve of rank one over a quadratic imaginary field. In many cases, our bound is precisely the one predicted by the Birch and Swinnerton-Dyer conjectural formula.

Speaker: Lloyd Kilford
Title: On the failure of the Gorenstein property for Hecke algebras of prime weight
Abstract: In this seminar we will report on extensive calculations of the Gorenstein defect for spaces of modular forms of prime weight p, presenting many new examples where mod p multiplicity one fails. This is joint work with Gabor Wiese.

Speaker: Ulf Kühn
Title: Computation of scattering constants
Abstract: Scattering constants are defined as the first non-vanishing coefficient of the L-series that contribute to the constant terms of non-holomorphic Eisenstein Series. They naturally occur as the contribution at the infinite primes in arithmetic intersection numbers or in Neron-Tate heights on Jacobians of curves defined over number fields. In this talk I'll report on the recent diploma thesis by

Anna Posingies, see http://www.math.uni-hamburg.de/home/kuehn/students.html, in which an algorithm that determines the coefficients of these *L*-series is developed. This algorithm applies in particular for not necessary congruence subgroups of the modular group $SL_2(\mathbf{Z})$ determined by Belyi morphisms.

Speaker: Bjorn Poonen
Title: $x^2 + y^3 = z^7$
Abstract: We combine a descent argument involving the simple group of order 168 with the modularity approach to find all integer solutions to the title equation satisfying gcd(*x*,*y*,*z*) = 1: one such solution is (21063928,-76271,17). This is joint work with Ed Schaefer and Michael Stoll.

Speaker: Kenneth Ribet
Title: Modular degrees and congruences among modular forms
Abstract: I will explain what is known about the relation between the congruence number and the modular degree of an elliptic curve over $\mathbf{Q}$ and then sketch the method that S. Yazdani used in his Berkeley thesis to obtain information about elliptic curves over $\mathbf{Q}$ whose modular degrees are odd. His results complement previous results of Calegari and Emerton.

Speaker: Victor Rotger
Title: Modular endomorphism algebras
Abstract: Let $f = \Sigma\, a_n\, q^n$ be a normalized newform of weight 2 and Nebentypus ε on $\Gamma_1(N)$ for some positive integer N. Assume for simplicity that it has no complex multiplication. Constructions due to Shimura and Ribet associate to the Fourier coefficients of f several algebras: $E_f = \mathbf{Q}(a_n : n > 1)$, $F_f = \mathbf{Q}(a_p^2/\varepsilon(p) : (p,N) = 1)$ and a central simple algebra B_f over F_f which ties together the inner-twists of f. All these can be geometrically interpreted as endomorphism algebras of the modular abelian variety A_f attached to f. Given $d > 0$, it is conjectured that there exist only finitely many such endomorphism algebras of degree at most d over $\mathbf{Q}$. The purpose of this talk is to report on recent progress towards this conjecture. Part of this is joint work with Bruin-Flynn-Gonzalez and Skorobogatov-Yafaev. The tools used in the proofs are both computational and theoretical, including Galois representations, Shimura varieties, computation of explicit models of Shimura curves, computation of rational points on them and descent methods.

Speaker: Gonzalo Tornaria
Title: Brandt matrices and ternary quadratic forms
Abstract: As proposed by Birch, one can construct partial Brandt matrices by the method of neighboring lattices applied to classes of ternary quadratic forms. This reduces the complexity of the problem from working with quaternary lattices (ideal classes on quaternion algebras) to working with ternary lattices. We propose a refinement to the classic notion of proper equivalence of lattices which leads to the construction of the full Brandt matrices, at least in the squarefree level case. Moreover, this refinement leads naturally to (and is motivated by!) the definition of generalized ternary theta series. The net result is a construction of modular forms of half integral weight, which preserves the action of Hecke operators (Brandt matrices) and thus makes explicit the Shimura correspondence.

Speaker: Gabor Wiese
Title: Multiplicities of Galois representations of weight one
Abstract: In the talk I will discuss the multiplicities of mod proof of my recent theorem saying that precisely in the open case of a weight one representation with a scalar Frobenius at p, the multiplicity fails to be 1. This result hence completes the study of the multiplicity one question.

Speaker: Christian Wuthrich
Title: Computations about the Tate-Shafarevich group using Iwasawa theory

Abstract: Let $E/\mathbf{Q}$ be an elliptic curve and $p > 2$ a prime where E has semi-stable reduction. The p-adic version of the conjecture of Birch and Swinnerton-Dyer tells us that there is a link between the p-adic L-function and the arithmetic invariants of E, namely the rank of the Mordell-Weil group and the order of the Tate-Shafarevich group of $E/\mathbf{Q}$. Using a combination of results in Iwasawa theory of elliptic curves, we know now enough of this conjecture to use it to compute upper bounds on the rank and the p-primary part of the Tate-Shafarevich group.