Banff International Research Station

for Mathematical Innovation and Discovery

Geometric and Nonlinear Analysis
August 12th-17th 2006

MEALS

Breakfast (Continental): 7:00–9:00 am, 2nd floor lounge, Corbett Hall, Sunday–Thursday
*Lunch (Buffet): 11:30 am–1:30 pm, Donald Cameron Hall, Sunday–Thursday
*Dinner (Buffet): 5:30–7:30 pm, Donald Cameron Hall, Saturday–Wednesday
Coffee Breaks: As per daily schedule, 2nd floor lounge, Corbett Hall
***Please remember to scan your meal card at the host/hostess station in the dining room for each lunch and dinner.**

MEETING ROOMS

All lectures will be held in Max Bell 159 (Max Bell Building accessible by bridge on 2nd floor of Corbett Hall). Hours: 6 am–12 midnight. LCD projector, overhead projectors and blackboards are available for presentations. *Please note that the meeting space designated for BIRS is the lower level of Max Bell, Rooms 155–159. Please respect that all other space has been contracted to other Banff Centre guests, including any Food and Beverage in those areas.*

SCHEDULE

Saturday

16:00	Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
17:30–19:30	Buffet Dinner, Donald Cameron Hall
20:00	Informal gathering in 2nd floor lounge, Corbett Hall (if desired) Beverages and small assortment of snacks available on a cash honour-system.

Sunday

7:00–8:45	Breakfast
8:45–9:00	Introduction and Welcome to BIRS by BIRS Station Manager, Max Bell 159
9:00–9:45	F.Pacard
9:45–10:30	Z.-C. Han
10:30–11:00	Coffee Break, 2nd floor lounge, Corbett Hall.
11:00–11:45	A. Malchiodi
11:30–13:30	Lunch
14:30–15:15	J.Qing
15:15–15:45	Coffee Break, 2nd floor lounge, Corbett Hall.
15:45–16:30	S.-Y.A.Chang
17:30–19:30	Dinner

Monday

7:00–9:00	Breakfast
9:00–9:45	Y.Brenier
9:45–10:30	R. McCann
10:30–11:00	Coffee Break, 2nd floor lounge, Corbett Hall.
11:00–11:45	R. Van der Vorst
11:30–13:30	Lunch
13:00–14:00	Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
14:20–14:30	Group Photo; meet on the front steps of Corbett hall.
14:30–15:15	N.Ghoussoub
15:15–15:45	Coffee Break, 2nd floor lounge, Corbett Hall.
15:45–16:30	P.Yang
17:30–19:30	Dinner

Tuesday

7:00–9:00	Breakfast
9:00–9:45	F. Coda Marques
9:45–10:30	E.Humbert
10:30–11:00	Coffee Break, 2nd floor lounge, Corbett Hall.
11:00–11:45	R.Mazzeo
11:30–13:30	Lunch
	Free afternoon
17:30–19:30	Dinner

Wednesday

7:00–9:00	Breakfast
9:00–9:45	Y.Ge
9:45–10:30	O.Druet
10:30–11:00	Coffee Break, 2nd floor lounge, Corbett Hall.
11:00–11:45	M.Dafermos
11:30–13:30	Lunch
14:30–15:15	J.Shatah
15:15–15:45	Coffee Break, 2nd floor lounge, Corbett Hall.
15:45–16:30	M.Struwe
17:30–19:30	Dinner

Thursday

7:00–9:00	Breakfast
9:00–9:45	C.Guillarmou
9:45–10:30	S.Kim
10:30–11:00	Coffee Break, 2nd floor lounge, Corbett Hall.
11:00–11:45	H.-C.Grunau
11:30–13:30	Lunch
Checkout by 12 noon.	

** 5-day workshops are welcome to use the BIRS facilities (2nd Floor Lounge, Max Bell Meeting Rooms, Reading Room) until 3 pm on Thursday, although participants are still required to checkout of the guest rooms by 12 noon. **

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ABSTRACTS
(in alphabetic order by speaker surname)

Speaker: **Yann Brenier** (Nice, France)
Title: *L^2 formulation of some hyperbolic conservation laws.*
Abstract: It is customary to adress hyperbolic conservation laws (or Hamilton- Jacobi equations) in functional spaces that are neither hilbertian nor reflexive (typically L1, BV, C0, Lip etc...). We show that, in some simple but significative cases (multidimensional scalar conservation laws, Chaplygin gas etc...), a straighforward L2 formulation can be introduced, leading to simple well posedness and stability results. This approach can be extended to some degenerate parabolic equations too.

Speaker: **Sun-Yung Alice Chang** (Princeton, USA)
Title: *Regularity on a boundary value problem of generalized Yamabe type.*
Abstract: In this talk, I will report some recent joint work of Sophie Chen, Paul Yang and myself on a regularity problem of Bach flat metrics on 4-manifolds with boundary. I will discuss both the set up and some ϵ regularity result of the problem.

Speaker: **Fernando Coda Marques** (IMPA, Brasil)
Title: *Blow-up analysis for the Yamabe equation in high dimensions.*
Abstract: In this talk we will discuss recent progress in understanding how solutions to the Yamabe equation can blow-up in high dimensions. We will describe how to use the Pohozaev identities, under a nondegeneracy condition, to get sufficient vanishing of the Weyl tensor at a blowup point and derive compactness results. This is joint work with Marcus Khuri and Richard Schoen.

Speaker: **Mihalis Dafermos** (Cambridge, UK)
Title: *The redshift effect and radiation decay for black hole spacetimes.*

Speaker: **Olivier Druet** (ENS Lyon, France)
Title: *Quantification of blow-up levels for a 2d elliptic equation with critical exponential growth.*
Abstract: We consider sequences of solutions of some 2d PDE with critical Trudinger-Moser growth and we show that they split into their weak solution plus a sum of standard bubbles.

Speaker: **Yuxin Ge** (Paris 12, France)
Title: *On a conformal quotient equation.*
Abstract: In this talk, we consider a conformal quotient equation $\frac{\sigma_2(g)}{\sigma_1(g)} = 1$ in a given conformal class and prove the existence for $n > 4$ and prove a related Sobolev type inequality.

Speaker: **Nassif Ghoussoub** (UBC, Canada)
Title: *On PDEs arising from Electrostatic Micro-Electromechanical Systems*
Abstract: We analyze the nonlinear parabolic problem $u_t = \Delta u - \frac{\lambda f(x)}{(1+u)^2}$ on a bounded domain Ω of R^N with Dirichlet boundary conditions. This equation models a simple electrostatic Micro-Electromechanical

System (MEMS) device consisting of a thin dielectric elastic membrane with boundary supported at 0 above a rigid ground plate located at -1. When a voltage –represented here by λ– is applied, the membrane deflects towards the ground plate and a snap-through may occur when it exceeds a certain critical value λ^* (pull-in voltage). This creates a so-called "pull-in instability" which greatly affects the design of many devices. The challenge is to estimate λ^* in terms of material properties of the membrane, which can be fabricated with a spatially varying dielectric permittivity profile f. Applying analytical and numerical techniques, the existence of λ^* is established together with rigorous bounds. We show the existence of at least one steady-state when $\lambda = \lambda^*$. More refined properties of steady states –such as regularity, stability, uniqueness, multiplicity, energy estimates and comparison results– are shown to depend on the dimension of the ambient space and on the permittivity profile. As to the dynamic case, the membrane globally converges to its unique maximal negative steady-state when $\lambda \leq \lambda^*$, with a possibility of touchdown at infinite time when $\lambda = \lambda^*$. On the other hand, if $\lambda > \lambda^*$ the membrane must touchdown at finite time T, and touchdown cannot take place at the location where the permittivity profile vanishes. Both larger pull-in distance and larger pull-in voltage can be achieved by properly tailoring the permittivity profile. We analyze and compare finite touchdown times by applying various analytical and numerical techniques. This is joint work with Yujin Guo.

Speaker: **Hans-Christoph Grunau** (Magdeburg, Germany)
Title: *The Paneitz equation in the hyperbolic ball.*
Abstract: Existence of a continuum of conformal radially symmetric complete metrics in the hyperbolic ball in $\mathbb{R}^n$, $n > 4$, is shown, all having the same constant Q-curvature. Moreover, similar results can be shown also for suitable non-constant prescribed Q-curvature functions. Joint work with M. Ould Ahmedou (Tuebingen), W. Reichel (Aachen).

Speaker: **Colin Guillarmou** (Nice, France)
Title: *Generalized Krein formula for Poincaré-Einstein manifolds.*
Abstract: We prove a kind of Birman-Krein formula for the Laplacian on even dimensional asymptotically hyperbolic manifolds that have an asymptotic evenness for the metric expansion at infinity. This relates a renormalized trace of the spectral projector of the Laplacian to the phase of the Ray-Singer determinant of the Scattering operator. This is used for instance to obtain a functionnal equation for Selberg's zeta function on convex co-compact hyperbolic manifolds and to compute the determinant of the GJMS conformal Laplacians in terms of this Selberg's zeta function.

Speaker: **Zheng-Chao Han** (Rutgers, USA)
Title: *On the prescribing σ_k curvature equations.*
Abstract: Let A_g denote the Schouten-Weyl tensor of a Riemannian metric g on M^n, for $1 \leq k \leq n$, $\sigma_k(g^{-1} \circ A_g)$ denote the k^{th} elementary symmetric function of the eigenvalues of the $1-1$ tensor $g^{-1} \circ A_g$. For a given function $K(x)$ on M^n, we address several issues in the question of when there exists an admissible conformal metric $g_w = e^{2w(x)}g$ such that

$$\sigma_k(g^{-1} \circ A_g) = K(x) \text{ on } M^n \tag{1}$$

First we will give an elementary and unified discussion to the Kazdan-Warner type necessary conditions for the solvability of (1). Then we will discuss the potential loss of compactness to the solutions of (1), and show that under appropriate non-degeneracy conditions on $K(x)$, so such loss of compactness can happen. This latter part is part of joint work with S.-Y.A. Chang and P. Yang.

Speaker: **Emmanuel Humbert** (Nancy, France)
Title: *Surgery and harmonic spinors.*
Abstract: Let M be a compact spin manifold with Riemannian metric g. Suppose that N is obtained from M by a surgery of codimension at least 2. We prove that N carries a Riemannian metric h such that the dimension of the kernel of the Dirac operator on (N, h) is not larger than the dimension of the kernel of

the Dirac operator on (M, g). As an application, we show that for generic metrics on a spin manifold, the dimension of the kernel of the Dirac operator attains the lower bound given by the index theorem.

Speaker: **Seongtag Kim** (Inha, Korea)
Title: *Bach-flat manifolds with small L_2 curvature.*
Abstract: In this talk, I will sketch the rigidity of noncompact complete Bach-flat 4-manifolds with small L_2 curvature norm. Let (M, g) be a Riemannian four-manifold with Weyl curvature W and Ricci curvature R_{ij}. The bach tensor B_{ij} is defined by $B_{ij} \equiv \nabla^k \nabla^l W_{kijl} + \frac{1}{2} R^{kl} W_{kijl}$. The Bach tensor is conformal invariant. (M, g) is called Bach-flat if $B_{ij} = 0$. Important examples of Bach-flat metrics are Einstein metrics, conformally Einstein metrics and self-dual Einstein metrics. We describe geometric conditions that noncompact complete Bach-flat manifolds with zero scalar curvature and small L_2 curvature norm are flat. For this, we use elliptic estimations on the equation comes from $B_{ij} = 0$.

Speaker: **Andrea Malchiodi** (SISSA, Italy)
Title: *Existence of conformal metrics with constant Q-curvature.*
Abstract: We discuss the problem of finding conformal metrics on a compact four manifold which have constant Q-curvature. This consists in solving a fourth order elliptic PDE with exponential nonlinearity and with variational structure. When the total Q-curvature is large, the Euler functional is unbounded from below, and critical points have to be found via min-max methods. Using a new variational scheme based on concentration of conformal volume, we solve the problem for a large class of manifolds.

Speaker: **Rafe Mazzeo** (Stanford, USA)
Title: *Geometric foliations near infinity in asymptotically hyperbolic manifolds.*
Abstract: I will describe recent work with Frank Pacard concerning the existence and uniqueness of foliations near infinity in asymptotically hyperbolic spaces such that each leaf has constant mean (or σ_k) curvature, and the relationship of these foliations to the conformal infinity of these spaces.

Speaker: **Robert McCann** (Toronto, Canada)
Title: *Regularity and counterexamples in optimal transportation.*
Abstract: I shall give a rapid survey of new and old results in optimal transportation, including the regularity theory of Ma, Trudinger, Wang and Loeper, and counterexamples to regularity due to Gregoire Loeper.

Speaker: **Frank Pacard** (Paris 12, France)
Title: *Elliptic aspects of extremal Kähler metrics.*
Abstract: I will report some recent results with C. Arezzo and M. Singer about the existence of extremal Kähler metrics on the blow up at finitely many points of manifolds which carry an extremal Kähler metric.

Speaker: **Jie Qing** (Santa Cruz, USA)
Title: *Metrics degeneration and its applications in conformal geometry*
Abstract: In this talk we develop a bubble tree structure for a degenerating class of Riemannian metrics satisfying some global conformal bounds on compact manifolds of dimension 4. Applying the bubble tree structure, we establish a gap theorem, a finiteness theorem for diffeomorphism type for this class, and raise a question concerning a diameter bound for a family of conformal metrics satisfying a suitable curvature equation.

Speaker: **Jalal Shatah** (New York, USA)
Title: *Free boundary problems for the Euler equation.*

Speaker: **Michael Struwe** (ETH Zürich, Switzerland)
Title: *Partial regularity for harmonic maps, revisited.*
Abstract: Via gauge theory, we give a new proof of partial regularity for harmonic maps in dimensions

$m \geq 3$ into arbitrary targets. This proof avoids the use of adapted frames and permits to consider targets of "minimal" C^2 regularity. The proof we present moreover extends to a large class of elliptic systems of quadratic growth.

Speaker: **Robert Van der Vorst** (Amsterdam, Holland)
Title: *Closed characteristics on non-compact hypersurfaces.*
Abstract: C.Viterbo proved that any $(2n-1)$-dimensional compact hypersurface $M \subset (\mathbb{R}^{2n}, \omega)$ of contact type has at least one closed characteristic. This result proved the Weinstein conjecture for the standard symplectic space $(\mathbb{R}^{2n}, \omega)$. Various extensions of this theorem have been proved since, all for compact hypersurfaces. In this paper we consider *non-compact* hypersurfaces $M \subset (\mathbb{R}^{2n}, \omega)$ coming from mechanical Hamiltonians, and prove an analogue of Viterbo's result. The main result provides a strong connection between the homology groups $H_k(M)$, $k = n, \cdots, 2n-1$, and the existence of closed characteristics in the non-compact case (including the compact case).

Speaker: **Paul Yang** (Princeton, USA)
Title: *Minimal surfaces in pseudo-hermitian geometry.*
Abstract: I will discuss the notion of minimal surfaces in a pseudo-hermitian manifold, the existence and uniqueness questions as well as that of the regularity. It turns out that in dimension three, there is serious regularity question. These are joint works with J. Cheng, J. Huang and Andrea Malchiodi. I will also discuss the Sobolev inequality a pseudo-hermitian manifold, and report on some joint work with S. Chanillo.