Banff International Research Station
for Mathematical Innovation and Discovery

Measurable Dynamics: Theory and Applications
August 5th-10th

MEALS

Breakfast (Continental): 7:00–9:00 am, 2nd floor lounge, Corbett Hall, Sunday–Thursday
*Lunch (Buffet): 11:30 am–1:30 pm, Donald Cameron Hall, Sunday–Thursday
*Dinner (Buffet): 5:30–7:30 pm, Donald Cameron Hall, Saturday–Wednesday
Coffee Breaks: As per daily schedule, 2nd floor lounge, Corbett Hall
***Please remember to scan your meal card at the host/hostess station in the dining room for each lunch and dinner.**

MEETING ROOMS

All lectures will be held in Max Bell 159 (Max Bell Building accessible by bridge on 2nd floor of Corbett Hall). Hours: 6 am–12 midnight. LCD projector, overhead projectors and blackboards are available for presentations. *Please note that the meeting space designated for BIRS is the lower level of Max Bell, Rooms 155–159. Please respect that all other space has been contracted to other Banff Centre guests, including any Food and Beverage in those areas.*

SCHEDULE

Saturday

16:00	Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
17:30–19:30	Buffet Dinner, Donald Cameron Hall
20:00	Informal gathering in 2nd floor lounge, Corbett Hall Beverages and small assortment of snacks available on a cash honour-system.

Sunday

7:00–8:45	Breakfast
8:45–9:00	Introduction and Welcome to BIRS by BIRS Station Manager, Max Bell 159
9:00–9:45	Jim Yorke (Maryland), Prevalence: a concept of "almost every" in Banach spaces
9:50–10:35	Oliver Jenkinson (Queen Mary, London), A partial order on $\times 2$-invariant measures
10:40–11:10	Coffee Break, 2nd floor lounge, Corbett Hall
11:15–12:00	Erik Bollt (Clarkson), Mostly Conjugate: Relating Dynamical Systems — Beyond Homeomorphism
12:00–13:30	Lunch
13:50–14:35	Gerhard Keller (Erlangen), How chaotic are strange nonchaotic attractors?
14:40–15:25	Judy Kennedy (Delaware), The Application of Inverse Limits to Models in Economics
15:30–16:00	Coffee Break, 2nd floor lounge, Corbett Hall
16:05–16:50	Geon Ho Choe (KAIST, Korea), Rotation numbers of piecewise linear circle homeomorphisms
17:30–19:30	Dinner

Monday

7:00–9:00	Breakfast
9:00–9:45	Bryna Kra (Northwestern), Parallelepipeds, Gowers norms, and ergodic theory
9:50–10:35	Wael Bahsoun (Victoria/Manchester), Rigorous Numerical approximation of escape rates
10:40–11:10	Coffee Break, 2nd floor lounge, Corbett Hall
11:15–12:00	Gary Froyland (UNSW, Australia), Transfer Operators, Eigenfunctions, and Ocean Dynamics
12:00–13:00	Lunch
13:00–14:00	Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
14:10–14:55	Sinan Gunturk (Courant), Piecewise Maps of Sigma-Delta Quantization
15:00–15:10	Group Photo; meet on the front steps of Corbett Hall
15:10–15:40	Coffee Break, 2nd floor lounge, Corbett Hall
15:40–16:25	William Ott (Courant), Does weak product recurrence imply distality
17:30–19:30	Dinner
20:00	Problem Session

Tuesday

7:00–9:00	Breakfast
9:00–9:45	Jim Meiss (Colorado), Volume-Preserving Mappings
9:50–10:35	Peter Ashwin (Exeter, UK), Piecewise isometric dynamics; from one to two dimensions
10:40–11:10	Coffee Break, 2nd floor lounge, Corbett Hall
11:15–12:00	Matt Nicol (Houston), Some large deviation results for nonuniformly hyperbolic systems
12:00–13:30	Lunch
	Free Afternoon
17:30–19:30	Dinner

Wednesday

7:00–9:00	Breakfast
9:00–9:45	Vitaly Bergelson (Ohio State), IP convergence and measurable dynamics
9:50–10:35	Ryszard Rudnicki (Silesian University, Poland), Markov semigroups and their applications
10:40–11:10	Coffee Break, 2nd floor lounge, Corbett Hall
11:15–12:00	Rua Murray (Waikato, New Zealand), Invariant measures, approximately invariant measures and convex optimization
12:00–13:30	Lunch
13:50–14:35	Evelyn Sander (George Mason), Unstable dimension variability and scaling laws for a global bifurcation in three dimensions
14:40–15:25	Ian Melbourne (Surrey, UK), The planar periodic Lorentz gas and approximation by Brownian motion
15:30–16:00	Coffee Break, 2nd floor lounge, Corbett Hall
16:05–16:50	Arno Berger (Canterbury, New Zealand), Digits of nonautonomous orbits via measurable shadowing
17:30–19:30	Dinner

Thursday

7:00–9:00	Breakfast
9:00–10:00	Informal Discussions
10:00-10:30	Coffee Break, 2nd floor lounge, Corbett Hall
11:30–13:30	Lunch

Checkout by 12 noon.

** 5-day workshops are welcome to use the BIRS facilities (2nd Floor Lounge, Max Bell Meeting Rooms, Reading Room) until 3 pm on Thursday, although participants are still required to checkout of the guest rooms by 12 noon. **

Measurable Dynamics: Theory and Applications
August 5th –10th, 2006

ABSTRACTS
(in alphabetic order by speaker surname)

Speaker: **Peter Ashwin** (University of Exeter)
Title: *Piecewise isometric dynamics; from one to two dimensions*
Abstract: Piecewise isometric dynamical systems are of interest from the point of view of a number of applications, and also as examples of nonhyperbolic dynamics that is nonetheless highly nontrivial. In recent years a good understanding of one dimensional interval exchange transformations has been gained, but the picture for higher dimensions is still quite mysterious.

This talk will discuss some recent and some ongoing work with Arek Goetz on how the dynamics of two-dimensional piecewise isometries (formed of cone exchanges) can be related to the better understood one-dimensional interval exchange transformations. This will include some examples where a nontrivial renormalization structure can be found as well as general properties that build on properties of the interval exchange dynamics. Finally we outline of some of the challenges that remain.

Speaker: **Wael Bahsoun** (University of Victoria/University of Manchester)
Title: *Rigorous Numerical approximation of escape rates*
Abstract: An interval map with holes is a mathematical model which is used in the study of nonequilibrium statistical mechanics. We use Ulam's method to approximate the escape rate for an interval map with holes and find a bound on the approximation error.

Speaker: **Vitaly Bergelson** (The Ohio State University)
Title: *IP convergence and measurable dynamics*
Abstract: While the traditional ergodic theory concerns itself with the study of the limiting behavior of various Cesaro averages, the IP ergodic theory utilizes the notion of IP-convergence which is based on Hindman's finite sums theorem. This usually allows one to refine and enhance the results obtained via the Cesaro averages. We will review some of the applications of the IP ergodic theory to multiple recurrence and "generic chaos" and discuss some natural open problems and conjectures.

Speaker: **Arno Berger** (University of Canterbury, New Zealand)
Title: *Digits of nonautonomous orbits via measurable shadowing*
Abstract: The versatile technique of shadowing provides an important tool (both theoretical and computational) for the analysis of dynamical systems, and it lies at the heart of hyperbolic dynamics. In this talk we briefly discuss a version of the shadowing lemma particularly suited for an analysis of nonautonomous iterations of maps on the line which are more or less power-like. With this result, and by utilising a reference system whose behaviour is well understood, fairly general statements can be made about the distribution of (digits of) orbits in such a nonautonomous setting.

Speaker: **Erik Bollt** (Clarkson University)
Title: *Mostly Conjugate: Relating Dynamical Systems — Beyond Homeomorphism*

Abstract: A centerpiece of Dynamical Systems is comparison by an equivalence relationship called topological conjugacy. Generally, there is no easy way to determine if two systems are conjugate or to explicitly find the conjugacy between systems that are known to be equivalent. We present a method to produce conjugacy functions based on a functional fixed point iteration scheme. In specific cases, we prove that although the conjugacy function is strictly increasing, it is a.e. differentiable, with derivative 0 everywhere that it exists — a Lebesgue singular function. When applied to non-conjugate dynamical systems, we show that the fixed point iteration scheme still has a limit point, which is a function we now call a "commuter" — a non-homeomorphic change of coordinates translating between dissimilar systems. This translation is natural to the concepts of dynamical systems in that it matches the systems within the language of their orbit structures. We introduce methods to compare nonequivalent systems by quantifying how much the commuter functions fails to be a homeomorphism, an approach that gives more respect to the dynamics than the traditional comparisons based on normed linear spaces, such as L^2.

Our discussion includes a principled approach toward understanding the degree to which a "toy model" might be representative of a more complicated system, an important concept to clarify since it is often used loosely throughout science.

(joint with Joseph D. Skufca, Department of Mathematics, Clarkson University)

Speaker: **Geon Ho Choe** (KAIST, Korea)
Title: *Rotation numbers of piecewise linear circle homeomorphisms*
Abstract: Exact values of rotation numbers are obtained for some circle homeomorphisms given by piecewise linear mappings by finding the invariant measures based on the numerical investigations.

Speaker: **Gary Froyland** (University of New South Wales, Australia)
Title: *Transfer Operators, Eigenfunctions, and Ocean Dynamics*
Abstract: The isolated spectrum of the Perron-Frobenius operator controls the rate of decay of correlations of chaotic systems. The eigenfunctions of large isolated spectral values carry important geometrical information and can be used to define almost-invariant sets; regions of state space that dissipate very slowly. We give an example of a 2D hyperbolic map with an isolated eigenvalue and demonstrate via a new procedure how to uncover the dynamical geometry carried by the corresponding eigenfunction. We present preliminary work on applying transfer operator methods to mapping Southern Ocean circulation based on a state-of-the-art 1/4 degree resolution global ocean model.

Speaker: **Sinan Gunturk** (Courant Institute)
Title: *Piecewise Maps of Sigma-Delta Quantization*
Abstract: Sigma-delta quantization is a method of approximating bounded real-valued functions by judiciously chosen binary sequences whose low frequency spectra approximate those of the target functions. Applications include digital halftoning algorithms used in printers and the analog-digital interface in many digital audio devices. Underlying this method is a class of piecewise-affine maps on the Euclidean space that are designed to yield the above approximation property. In this talk, we will present some of the recent progress in this subject from a dynamical systems point of view.

Speaker: **Oliver Jenkinson** (Queen Mary, University of London)
Title: *A partial order on $\times 2$-invariant measures*
Abstract: I will describe a partial order on $\times 2$-invariant measures which describes relative dispersion, and identify the minimal elements for this order (the "least spread out" measures). This is related to ergodic optimization (i.e. study of the largest and smallest possible ergodic averages of a given function).

Speaker: **Gerhard Keller** (Universität Erlangen–Nürnberg, Germany)
Title: *How chaotic are strange nonchaotic attractors?*
Abstract: We show that the classic examples of quasi-periodically forced maps with strange nonchaotic attractors described by Grebogi/Ott/Pelikan/Yorke and Herman in the mid-1980s have some chaotic properties. More precisely, we show that these systems exhibit sensitive dependence on initial conditions, both

on the whole phase space and restricted to the attractor. The results also remain valid in more general classes of quasiperiodically forced systems. Further, we clarify the structure of the attractor in an example with two-dimensional fibers also introduced by Grebogi et al. The proofs use a dichotomy between equicontinuity and sensitive dependence due to Glasner and Weiss and some more recent findings related to the approximation properties of the fiber rotation number in quasiperiodically forced circle homeomorphisms. The results presented are joint work with Paul Glendinning (Manchester) and Tobias Jaeger (Erlangen).

Speaker: **Judy Kennedy** (University of Delaware)
Title: *The Application of Inverse Limits to Models in Economics*
Abstract: We discuss the application of the theory of inverse limits to solve problems coming from models in economics that have the property of not being well-defined forward in time. Here we focus on the cash-in-advance model. The work involves putting an appropriate measure on the resulting inverse limit space, proving that integration then makes sense, and computing relevant integrals. When the model is chaotic, we have been able to compute the integral of the associated utility function, thus giving us an expected utility and allowing us to rank inverse limit spaces according to expected utility. Thus, for this simple idealized model, we can say something about monetary policy.

(Joint work with Brian Raines and David Stockman)

Speaker: **Bryna Kra** (Northwestern University)
Title: *Parallelepipeds, Gowers norms, and ergodic theory*
Abstract: In his proof of Szemeredi's Theorem, Gowers introduced certain norms that play an important role in additive combinatorics. Host and I introduced seminorms that are closely related to the Gowers norms. I will discuss the relations between these norms anad seminorms, and the general setting in which they can be defined.

Speaker: **James Meiss** (University of Colorado–Boulder)
Title: *Volume-Preserving Mappings*
Abstract: Volume preserving dynamics is, in some sense, intermediate between the well-studied case of area-preserving dynamics and the more difficult case of symplectic dynamics. Moreover, it has applications to the mixing of passive scalars in fluids and the configurations of magnetic field lines. We discuss several simple model maps and some aspects of their dynamics including the onset of chaotic motion, the behavior of their stable and unstable manifolds, the appearance and destruction of invariant tori, and normal forms for their simplest bifurcations.

Speaker: **Ian Melbourne** (University of Surrey, UK)
Title: *The planar periodic Lorentz gas and approximation by Brownian motion*
Abstract: The planar periodic Lorentz gas with finite horizon was introduced by Sinai and can be viewed as a simple deterministic model for Brownian motion.

In this talk, we describe the almost sure invariance principle (ASIP): for almost any initial condition, the position vector q(t) in the Lorentz gas approximates a two-dimensional Brownian trajectory. This answers a question of Chernov and Dolgopyat (ICM, 2006). Previously Bunimovich and Sinai (1981) proved a distributional version of this result.

More generally, we prove a vector-valued ASIP for Axiom A diffeomorphisms and flows as well as large classes of nonuniformly hyperbolic systems.

This is joint work with Matthew Nicol.

Speaker: **Rua Murray** (University of Waikato, New Zealand)
Title: *Invariant measures, approximately invariant measures and convex optimization*
Abstract: When $T : X \to X$ is non-singular with respect to an ambient measure m, we approach the problem of accessing interesting T-invariant measures via a sequence of finite and tractable numerical computations. Fix a sequence $\{h_i\}$ in $L^\infty(X, dm)$. For each n, we call a measure μ_n "approximately

invariant up to $\{h_1, \ldots, h_n\}$" (AI(n)) if $\mu_n(h_i) = \mu_n(h_i \circ T)$ for $i = 1, \ldots, n$. Let Φ be a convex functional on $L^1(X, dm)$ and select μ_n to minimize $\Phi(\frac{d\mu_n}{dm})$, subject to μ_n being AI(n). Very general conditions on T, Φ and $\{h_i\}$ guarantee convergence of $\{\mu_n\}$ to a T–invariant measure as $n \to \infty$. Interesting additional conditions are needed to reduce the solution of each optimization problem to a tractable computation. Also, some of the challenges in achieving accurate approximations will be discussed.

This is joint work with Chris Bose.

Speaker: **Matt Nicol** (University of Houston)
Title: *Some large deviation results for nonuniformly hyperbolic systems*
Abstract: We consider the asymptotic behavior of $m\{x \in X : \frac{1}{n}\sum_{j=0}^{n-1} \phi \circ f^j > \epsilon\}$ for a Hölder observable on a nonuniformly hyperbolic dynamical system (f, X, m) modeled by a Young tower. We show exponential decay if the tower has exponential tails and polynomial decay if the tower has polynomial tails. We give examples to show that the polynomial decay is optimal in the latter case.

This is joint work with Ian Melbourne

Speaker: **William Ott** (Courant Institute)
Title: *Does weak product recurrence imply distality?*
Abstract: The relationship between recurrence and rigidity in dynamical systems is subtle and interesting. A classical result in this direction states that for actions of $\mathbb{Z}$, product recurrence and distality are equivalent. More generally, Auslander and Furstenberg have studied the relationship between product recurrence and distality for general semigroup actions. We introduce the notion of weak product recurrence and we show that weak product recurrence is strictly weaker than distality, solving a problem posed by Auslander and Furstenberg in their study.

Speaker: **Ryszard Rudnicki** (Institute of Mathematics PAS and Silesian University-Poland)
Title: *Markov semigroups and their applications*
Abstract: Dynamical systems and dynamical systems with stochastic perturbations can be effectively studied using Markov semigroups. Such semigroups are generated by partial differential equations. Equations of this type appear in the theory of diffusion and jump processes, in astrophysics and in population dynamics. We give a survey of recent results concerning asymptotic properties of Markov semigroups: asymptotic stability, sweeping and Foguel alternative. We present applications of these results to biological models from population dynamics and genetics.

Speaker: **Evelyn Sander** (George Mason University)
Title: *Unstable dimension variability and scaling laws for a global bifurcation in three dimensions*
Abstract: A crisis is a global bifurcation in which a chaotic attractor has a discontinuous change in size or suddenly disappears as a scalar parameter of the system is varied. In this talk, I describe a global bifurcation in three dimensions which can result in a crisis. This bifurcation does not involve a tangency and cannot occur in maps of dimension smaller than three. The crisis produces unstable dimension variability. The crisis results in a new scaling law describing the density of the new portion of the attractor formed in the bifurcation.

Speaker: **James Yorke** (IPST, University of Maryland)
Title: *Prevalence: a concept of "almost every" in Banach spaces.*
Abstract: Many problems in mathematics and science require the use of infinite-dimensional spaces. Consequently, there is need for an analogue of the finite-dimensional notions of "Lebesgue almost every" and "Lebesgue measure zero" in the infinite-dimensional setting. The theory of prevalence addresses this need and provides a powerful framework for describing generic behavior in a probabilistic way. These ideas are particularly useful in dynamical systems for describing what we mean by typical behavior. Applications will be described.