BIRS Workshop 06w5065
Advances in Computational Scattering
Feb. 18 2006 - Feb. 23 2006

MEALS

Breakfast (Continental): 7:00 - 9:00 am, 2nd floor lounge, Corbett Hall, Sunday - Thursday
*Lunch (Buffet): 11:30 am - 1:30 pm, Donald Cameron Hall, Sunday - Thursday
*Dinner (Buffet): 5:30 - 7:30 pm, Donald Cameron Hall, Saturday - Wednesday
Coffee Breaks: As per daily schedule, 2nd floor lounge, Corbett Hall. Posters by graduate students and recent PhDs will be on display here.
***Please remember to scan your meal card at the host/hostess station in the dining room for each lunch and dinner.**

MEETING ROOMS

All lectures are held in the main lecture hall, Max Bell 159. *Please note that the meeting space designated for BIRS is the lower level of Max Bell, Rooms 155-159. Please respect that all other space has been contracted to other Banff Centre guests, including any Food and Beverage in those areas.*

SCHEDULE

Please note: the last talk of the workshop will end on Feb. 23 at 10:30 am.
Adjustments to the schedule, if necessary, will be made to the Monday, Tuesday morning and Wednesday schedule.

	Saturday	**Sunday**	**Monday**	**Tuesday**	**Wednesday**	**Thursday**
7:00-9:00	X	Continental Breakfast, 2nd floor lounge, Corbett Hall				
8:45-9:00	X	Opening remarks	X			
9:00-9:30	X	Antoine	Nicholls	Sadov	Maischak	Monk
9:30-10:00	X	Hagstrom	Luneville	Discussion	Hsiao	Demkowicz
10:00-10:30	X	Coffee Break and Posters, 2nd floor lounge, Corbett Hall				Schoetzau
10:30-11:00	X	Warburton	Bruno	Discussion	Cakoni	X
11:00-11:30	X	Coyle	Benamou	Discussion	Bonnet-Ben Dhia	X
11:45-12:00	X	X	Group Photo[1]	X	X	X
11:30-13:30	X	Buffet Lunch, Donald Cameron Hall				
13:00-14:00	X	X	Guided Tour[2]	free afternoon	X	X
14:00-14:00	X	Steinbach	Chandler-Wilde	free afternoon	Colton	X
14:30-15:00	X	Buffa	Nedelec	free afternoon	Nigam	X
15:00-15:30	X	Coffee Break and Posters,, 2nd floor lounge, Corbett Hall, (except Tues.)				
15:30-16:00	X	Beauwens	Martin	free afternoon	Tsynkov	X
16:00-17:00	X	X	X	free afternoon	X	X
17:30-19:30	Buffet Dinner, Donald Cameron Hall					X

BIRS Workshop
Name of Workshop
Date of Workshop

ABSTRACTS
(in alphabetic order by speaker surname)

Speaker: **Antoine, Xavier** (Institut National Polytechnique de Lorraine)
Title: *The technique of On-Surface Radiation Conditions and its applications*
Abstract: The technique of On-Surface Radiation Conditions has been introduced in the middle of the eighties to simplify the numerical solution of scattering problems for a wide frequency range. The aim of this talk is to review some recent developments of this method with regards to the challenging problem of prospecting high-frequency acoustic and electromagnetic scattering problems. Moreover, two applications of the OSRCs will be discussed: the development of accurate and local artificial boundary conditions for smooth geometries and the construction of well-posed and well-conditioned integral equations for the iterative solution of high-frequency scattering problems.

Speaker: **Beauwens, Robert, Universite Libre de Bruxelles**
Title: *Linear iterative methods*
Abstract: We present here an overview of the principles supporting the complex iter- ative schemes used today to solve large sparse linear systems of equations. Such methods are needed everywhere in mathematical physics and engineer- ing when numerically solving the equations arising from the discretization of multidimensional partial differential equations.
They rest on the combined use of two kinds of methods that can to some ex- tent be presented separately, the preconditioning methods and the convergence acceleration methods. Preconditioning methods aim at building an approxi- mate system close to the one to be solved but inexpensive to solve both in terms of computing time and memory requirements. They are used to gener- ate successive approximations that will move an arbitrary initial approxima- tion hopefully closer to the exact solution. Convergence acceleration methods are used to transform slowly converging sequences or even diverging sequences into fast converging sequences and are applied here to the sequences of suc- cessive approximations generated by the method of successive corrections just mentioned.
We first consider preconditioning starting with the basic classical precon- ditioners and the basic devices used to build evolved preconditioners through additive and/or multiplicative composition of simpler ones. We stress here the use of basic blocks such as approximate factorization methods for instance in these assembly processes.
We next indicate the general principles of the acceleration methods mostly used today, i.e. the polynomial acceleration or Krylov subspace methods among which we wish to cite the generalised minimal residual

[1] A group photo will be taken on Tuesday at 11:45 am, directly after the last lecture of the morning. Please meet on the front steps of Corbett Hall.

[2] A free guided tour of The Banff Centre is offered to all participants and their guests on Sunday starting at 1:00 pm. The tour takes approximately 1 hour. Please meet in the 2nd floor lounge in Corbett Hall.

(GMRES) and the quasi- minimal residual (QMR) methods as typical examples of general applicability as well as the Chebyshev and conjugate gradients (CG) methods as optimal ones (in some sense) when the system to be solved is symmetric positive definite(SPD). We also mention the possibility to view some polynomial methods as preconditioners and thus use them like other basic blocks in the building of elaborate preconditioners. To conclude, we briefly introduce recent developments concerning multi- level and recursive ordering methods on the one hand and the parallelization of preconditioned Krylov subspace methods on the other hand.

Speaker: **Benamou, Jean-David** (INRIA)
Title: *Numerical microlocal analysis of harmonic wavefields*
Abstract: Asymptotic high frequency representation of wavefields (Geometrical Optics in its simple form) is a computationally attractive approach for large high (and medium) frequency scattering problems because the discretization is, to a large extent, independent of the frequency. Unfortunately this technique has both theoretical and practical limits and a lot of efforts have been put (with some success) in combining or coupling usual “frequency aware” (or “full”) wavefield and “asymptotic techniques”.
While it is easy to compute a “full” wavefield representation from its (constructive) asymptotic representation, the opposite extraction (or “analysis”) from a given wavefield of its frequency independent assymptotic representation is far from obvious.
We will try to sketch an overview of available techniques and present a numerical method we have devel-loped which, given an analytical or numerical solution of the Helmholtz equation in a neighborhood of a fixed observation point and assuming that the geometrical optics approximation is relevant, determines at this point the number crossing rays and computes their directions and associated complex amplitudes.
This is joint work with O. Runborg (KTH) and F. Collino (CERFACS).

Speaker: **Bonnet-Ben Dhia,Anne-Sophie** (INRIA-ENSTA)
Title: *Acoustic scattering in presence of a mean flow*
Abstract: Nowadays, a better understanding of the effects induced by a mean flow on acous- tic scattering is required, in order to develop efficient techniques to reduce the noise produced by the planes, in the neighborhood of the airports. However, there exist no satisfactory way to solve the Linearized Euler Equations in harmonic regime and in unbounded domains. We develop an original approach, which consists in solving a linearized equation, set on the perturbation of displacement, the so-called Galbrun’s equation. First we derive an augmented formulation of the problem, which allows to work in the H1 framework. This formulation includes a term which is non-local in space, linked to the convection of vortices along the stream lines. Then we show that it is possible to combine this idea with Perfectly Matched Layers, leading to a Fredholm formulation of the problem, which can be solved by finite elements. Some numerical illustrations will be presented on the case of acoustic radiation in a 2D waveguide in the presence of a parallel shear flow.

Speaker: **Bruno, Oscar** (California Institute of Technology)
Title: *Efficient solution of wave scattering problems*
Abstract: We present a new set of algorithms and methodologies for the numerical solution of problems of scattering by complex bodies in three-dimensional space. These methods, which are based on integral equations, high-order integration, fast Fourier transforms and highly accurate high-frequency methods, can be used in the solution of problems of electromagnetic and acoustic scattering by surfaces and penetrable scatterers — even in cases in which the scatterers contain geometric singularities such as corners and edges. In all cases the solvers exhibit high-order convergence, they run on low memories and reduced operation counts, and they result in solutions with a high degree of accuracy. In particular, our algorithms can evaluate accurately in a personal computer scattering from hundred-wavelength-long objects by direct solution of integral equations — a goal, otherwise achievable today only by supercomputing. A new class of high-order surface representation methods will be discussed, which allows for accurate high-order description of surfaces from a given CAD representation. A class of high-order high-frequency methods

which we developed recently, finally, are efficient where our direct methods become costly, thus leading to a general and accurate computational methodology which is applicable and accurate for the whole range of frequencies in the electromagnetic spectrum.

Speaker: **Buffa, Annalisa** (IMATI-CNR, Pavia)
Title: An optimal variational preconditioner for the electric field integral equation.
Abstract: The electric field integral equation (EFIE) arises in the scattering theory for harmonic electromagnetic waves. We describe an optimal preconditioning technique for the conforming Galerkin approximation of the EFIE via Raviart-Thomas finite elements. At the continuous level, i.e., before discretization, Calderon formulas provide an explicit representation of the inverse operator of electric field integral operator up to compact pertubations. In this talk, we present a stable discretization of the Calderon formula and we obtain then an optimal preconditioner for the linear system which arises from the Galerkin discretization of the EFIE.
This is joint work with S. H. Christiansen (University of Oslo, Norway)

Speaker: **Cakoni, Fioralba** (University of Delaware)
Title: *On Mathematical and Computational Aspects of Inverse Electromagnetic Scattering Problems.*
Abstract: Since the invention of radar, scientists and engineers have striven not only to detect but also to identify unknown objects through the use of electromagnetic waves. A significant step forward in the resolution of this problem was the use of synthetic aperture radar (SAR). However, (SAR) suffers from limitations arising from the incorrect model assumptions which ignore both multiple scattering and polarization effects. In recent years, in an effort to overcome the limitations of such an incorrect model, considerable effort has been put into the development of nonlinear optimization techniques which avoids incorrect modeling assumptions. The success of such an approach is based on strong a priori knowledge of the scattering object and hence is inappropriate for many, if not most, practical applications. In view of the problems inherent in the weak scattering and nonlinear optimization approaches to target identification, a new class of methods has been developed in the past few years loosely called qualitative methods in inverse scattering theory. In particular, the main theme of this talk is the use of one such qualitative method, the linear sampling method, to solve inverse electromagnetic scattering problems.

We first introduce the main mathematical ideas of the linear sampling method for the simple case of electromagnetic scattering by a perfect conductor. Then we show how to use the linear sampling method to determine both the shape and the surface impedance of a partially coated perfect conductor without knowing a priori whether the obstacle is coated and if so what the extent of the coating is. In the case of an inhomogeneous background, we present a new method which avoids the need to compute the Green's function of the background media. In all of the above problems we present numerical examples showing the practicality of our inversion approach and suggest some open questions that need mathematical attention.

Speaker: **Chandler-Wilde, Simon N.** (University of Reading)
Title: *Boundary Element Methods for High Frequency Scattering Problems*
Abstract: In this talk I will give an overview of recent work on boundary element methods for high frequency scattering problems. I will address two areas. First I will discuss what is known about the dependence of the conditioning of boundary integral equations on frequency and on the choice of coupling parameters in combined layer-potential formulations. Second I will discuss attempts to reduce the number of degrees of freedom by incorporating some of the oscillatory behaviour of the solution in the basis functions used in the boundary element method. The talk will be punctuated by many open problems and will include mention of some of my own recent work with co-authors including Langdon (Reading), Heinemeyer (Goettingen), Potthast (Goettingen/Reading), Monk (Delaware).

Speaker: **Colton, David** (University of Delaware)
Title: *Some open research problems in computational inverse scattering*
Abstract: Some open problems and research directions will be described in this talk.

Speaker: **Coyle, Joseph** (Monmouth)
Title: *Hierarchic finite element bases on unstructured tetrahedral meshes*
Abstract: Construction and implementation issues for hierarchic bases employed in finite element discretizations of the spaces $\mathcal{H}^1$ and $\mathcal{H}(\text{curl})$ are discussed. The setting is on unstructured tetrahedral meshes where a non-uniform order of approximation may be utilized. Specifically, enforcing the appropriate conformity properties of the approximation across element interfaces is typically a difficult task in this case. The talk will first illustrate this difficulty by relating it to the intrinsic orientation of the edges and faces as well as the global numbering of the basis functions. Observing that an appropriate reordering of the local numbering of the vertices allows any global tetrahedron to be reduced to one of two possible reference tetrahedra then leads the way for the construction of the hierarchic bases where ease of implementation is not sacrificed. The material presented in this talk is the result of joint work with Mark Ainsworth at Strathclyde University sponsored by EPSRC contract/grant number GR/M59426.

Speaker: **Demkowicz, Leszek** (University of Texas at Austin)
Title: Fully Automatics hp-adaptive simulations for elliptic and Maxwell problems: a progress report.
Abstract: I will begin with a short review of the current status of the theory of hp-discretizations for Maxwell problems summarizing the main points of the projection-based interpolation theory [1,2], convergence results for Maxwell eigenvalues [3] and recent results on the existence of polynomial preserving extension operators in H(curl) and H(div) spaces.

I will focus then on the subject of goal-oriented hp-adaptivity, presenting an extension of the original, energy-based hp-algorithm and its applications to borehole logging EM simulations [4,5].

Finally, I will illustrate the impact of the automatic hp-adaptivity in simulations involving the use of PML. The automatic reproduction of "boundary layers" by the hp-adaptivity reduces significantly a tedious design and tuning of PML's.

[1] Demkowicz, L. and Buffa, A., "H1, H(curl) and H(div) -Conforming Projection-Based Interpolation in Three Dimensions. Quasi-Optimal p-Interpolation Estimates", Comput. Methods Appl. Mech. Engr, 194, 267-296, 2005.

[2] Cao, W. and Demkowicz, L., "Optimal Error Estimate for the Projection Based Interpolation in Three Dimensions", Comput. Math. Appl., 50, 359-366, 2005.

[3] Boffi, D. and Dauge, M. and Costabel, M. and Demkowicz, L., "Discrete Compactness for the hp Version of Rectangular Edge Finite Elements", SIAM J. on Numer. Anal., in print.

[4] Pardo, D. and Demkowicz, L. and Torres-Verdin, C. and Tabarovsky, L., " A Goal-Oriented *hp*-Adaptive Finite Element Method with Electromagnetic Applications. Part 1: Electrostatics, Int. J. Num. Meth. Eng., in print.

[5] Paszynski, M. and Demkowicz, L. and Pardo, D., "Verification of Goal- Oriented hp-Adaptivity", Comput. Math. Appl.,50,1395-1404, 2005.

Speaker: **Hagstrom, Thomas** (University of New Mexico)
Title: *Radiation Boundary Conditions for Time-Domain Simulations*
Abstract: As the radiation of energy to the far field is a fundamental feature of most wave propagation problems, accurate and efficient methods for near-field domain truncation are necessary for practical computations. In this talk we will review the state-of-the-art in the construction, analysis, and application of arbitrarily-accurate radiation boundary conditions for time-domain simulations. Specific topics include:
(i.) Experiments with nonlocal boundary conditions employing efficient compressions of the time-domain kernels;
(ii.) Reformulated local boundary condition sequences and their use in polygonal domains and stratified and anisotropic media;
(iii.) Speculations on potential improvements of the local boundary condition sequences and extensions to inhomogeneous media and nonlinear problems.

Speaker: **Hsiao, George** (University of Delaware)

Title: *On an Inverse Scattering Problem for Periodic Structures*

Abstract: The scattering theory in periodic structures has many applications in micro-optics, where periodic structures are often called diffraction gratings. The treatment of inverse problem, recovering the periodic structure or the shape of the grating profile from the scattered field, is useful in quality control and design of diffractive elements with prescribed far field patterns. In this lecture we consider an inverse diffraction grating problem to recover a two-dimensional periodic structure from scattered waves measured from above and below the structure. From measured Rayleigh coefficients for several incidence directions, we wish to reconstruct the grating.

The problem is reformulated as an optimization problem including regularization terms. The solution is obtained as the minimizer of the optimization problem, where the objective function consists of three terms: the first is the residual of the Helmholtz equation, the second the deviation of the computed Rayleigh coefficients from the measured data, and the third is a regularization term to cope with the ill-posedness of the inverse problem. The solvability and the dependence on the parameter of regularization is analyzed. Some numerical experiments are included based on the finite element discretization for the Helmholtz equation as well as for the corresponding optimization problem in order to demonstrate the practicability of our inversion algorithm.

This lecture is based on a joint work with J. Elschner and A. Rathsfeld of WISE (Weierstrass-Institut für Angewandt Analysis und Stochastik, Berlin, Germany).

Speaker: **Luneville, Eric** (ENSTA)

Title: *Exact Transparent Condition*
for Time-Harmonic Maxwell Equations
in a Waveguide

Abstract: In an acoustic waveguide, assumed to be semi-infinite along one propagation axis, one can easily construct from spectral theory of a simple transverse operator an "exact" transparent condition. More precisely, such condition is based on an explicit diagonalisation of the Dirichlet to Neuman operator (see e.g [1]). The situation for Maxwell equation is more intricate. Indeed, the application which associates the electrical field to its derivative (equivalent to a Dirichlet to Neuman operator) is not adapted because transverse and longitudinal Maxwell operator remains coupled and no explicit diagonalisation may be performed. We propose in this paper a new transparent condition in a two dimensional case for the regularized Maxwell equation, based on the diagonalization of an operator which involves mixte unknowns, say the couple of the electric tangential component and the divergence of the electric field and the couple of electric normal component and the rotational of the electric field. Unfortunately, this transparent condition implies to deal with mixte variational formulation where, for example, the divergence on the transparent boundary appears as a new unknown of the problem. However, this formulation is well-posed and its approximation by Lagrange finite element is convergent. This approach is an alternative way to other methods such that integral equation or Perfectly Matched Layer techniques. It is of interest to point out that it appears as a theoritical tool in proof of convergence of PML techniques too (see [2],[3]). Such transverse decomposition are also related to modal approximation (see [4] or [5]). This approach may also be used for elastodynamic problem. In that case, the spectral theory of the transverse operator is not obvious.

References 1. M.Lenoir and A.Tounsi, *The localized finite element method and its application to the two-dimensional sea-keeping problem*", SIAM J. Numer. Anal., 25, 1988.
2. J.-P. Bérenger; *A perfectly matched layer for the absorption of electromagnetic waves*, J. Comput. Phys.,114, 185-200, 1994.
3.. E. Bécache and A.-S.Bonnet-Ben Dhia and G. Legendre, *Perfectly matched layers for the convected Helmholtz equation,* SIAM J. Numer. Anal.42,409,2004.
4. C. Hazard and E. Lunéville, *Multimodal approach and optimum design in non uniform waveguides*, Journées Européennes sur les Méthodes Numériques en Electromagnétisme, Toulouse, 2002.
5.V. Pagneux and A. Maurel, *Lamb wave propagation in homogeneous elastic waveguide*, To appear in Royal Society,

6. C. Hazard and M. Lenoir, *On the solution of time-harmonic scattering problems for Maxwell's equations*, SIAM J. Marth. Anal., 27, 1597, 1996

Speaker: **Maischak, Matthias.** (University of Hannover)
Title: *Preconditioning for electro-magnetic fem-bem coupling formulations*
Abstract: The symmetric coupling of finite elements and boundary elements for electro-magnetic problems [3] results in highly ill-conditioned linear systems of equations. We present a block-preconditioner for the GMRES method [4] which is based on domain decomposition methods applied to the fem-part and the bem-part separately and analyse the eigenvalue distribution of the preconditioned system [1,2]. We show that the efficiency of this method only depends on the ratio of coarse grid mesh size and the overlap. Numerical examples for the eddy-current problem underline the efficiency of this method. This is joint work with Ernst Stephan, U. Hannover.

1. T. Tran, E.P. Stephan: An overlapping additive Schwarz preconditioner for boundary element approximations to the Laplace screen and Lamé crack problems. J. Numer. Math. 12 (2004) 311–330

2. M. Maischak, T. Tran: A Block Preconditioner for an electromagnetic fem-bem coupling problem in R^3 (in preparation)

3. E.P. Stephan, M. Maischak: A posteriori error estimates for fem-bem couplings of three-dimensional electromagnetic problems. Comput. Meth. Appl. Mech. Engrg. 194 (2005) 441–452

4. P. Mund, E.P. Stephan: The preconditioned GMRES method for systems of coupled FEM-BEM equations. Numerical treatment of boundary integral equations. Adv. Comput. Math. 9 (1998) 131–144

Speaker: **Martin, Paul A.** (Colorado School of Mines)
Title: *Scattering by many spheres: a new proof of the Lloyd-Berry formula*
Abstract: We provide the first classical derivation of the Lloyd-Berry formula (published in 1967) for the effective wavenumber of an acoustic medium filled with a sparse random array of identical small scatterers. Our approach clarifies the assumptions under which the Lloyd-Berry formula is valid. More precisely, we derive an expression for the effective wavenumber which assumes the validity of Lax's quasicrystalline approximation but makes no further assumptions about scatterer size, and then we show that the Lloyd-Berry formula is obtained in the limit as the scatterer size tends to zero.

We have also obtained a similar formula in two dimensions. See C. M. Linton and P. A. Martin, Multiple scattering by random configurations of circular cylinders: Second-order corrections for the effective wavenumber, J. Acoust. Soc. Amer., 117 (2005), pp. 3413-3423.

The methods employed should extend to analogous electromagnetic and elastodynamic problems.

Speaker: **Monk, Peter** (University of Delaware)
Title: *The Ultra Weak Variational Formulation for Wave Scattering*
Abstract: When electromagnetic or acoustic waves impinge on an object, they are scattered by reflection, refraction and other mechanisms depending on the shape and properties of the scatterer. We wish to compute the scattered field resulting from the interaction of an object (the scatterer) and a known incident field.? Although such problems are usually linear and well-posed they are difficult to solve numerically because the oscillatory nature of the solution forces the use of large numbers of degrees of freedom in the numerical method, and the resulting linear system defies standard approaches such as multigrid. This is a particular problem at high frequencies when the scatterer spans many wavelengths.
In an effort to improve the efficiency of a volume based approach as the frequency increases and to allow the solution of problems at widely different frequencies on a single grid, we have investigated the use of plane waves as a basis for approximating the scattered field.? These are used in a discontinuous Galerkin

scheme based on a tetrahedral finite element mesh.? This method is termed the Ultra Weak Variational Formulation (UWVF) by its originators O. Cessenat and B. Despres.
In joint work with Tomi Huttunen (Finland) and Eric Darrigrand (France) we have developed the UWVF by addressing certain conditioning problems that may arise if the plane waves are not chosen carefully. We have also shown how to use the Perfectly Matched Layer or Fast Multipole Method to improve the artificial boundary condition needed by the method.? Interestingly the linear system from the UWVF is easier to solve than the one arising from the finite element method, and this allows a simple parallel implementation of the method. The method has been validated on a variety of problems, and extended to the acoustic-elastic fluid-structures problem.
In the talk I shall show numerical and analytic results that demonstrate both the successes and drawbacks of the method applied to both ultrasonic simulations and also to electromagnetic scattering problems.

Speaker: **Nedelec, Jean-Claude** (Ecole Polytechnique Palaiseau)
Title: *Preconditioning the Maxwell integral equations using Calderon identities*
Abstract:

Speaker: **Nicholls, David** (University of Illinois at Chicago)
Title: *Boundary Perturbation Methods in Direct Scattering*
Abstract: Boundary perturbation methods are among the most classical techniques for approximating scattering returns from irregular obstacles. Despite a history which dates to Rayleigh's calculations in the nineteenth century, their convergence, stability, and capabilities were, for almost a century, misunderstood. The work of Bruno & Reitich (1992,1993) not only placed these methods on a secure theoretical foundation, but also provided fast, high-order computational strategies. Subsequent work by the speaker (in collaboration with Reitich, Nigam, and Shen) has further clarified the properties and limitations of these methods, and suggested new algorithms to achieve high-order approximations in a rapid and numerically stable manner. In this talk we will give an overview of these boundary perturbation methods and discuss recent enhancements.

Speaker: **Nigam, Nilima** (McGill University)
Title: *Transparent boundary conditions for elastic scattering*
Abstract: We shall present some recent work on artificial boundary conditions for the scattering of elastic waves from bounded obstacles, including extensions of the boundary perturbation approach of Bruno and Reitich, as well as investigations into an overlapping Schwarz domain decomposition method. A few open problems will also be described.

Speaker: **Sadov, Sergey** (Memorial University of Newfoundland)
Title: *High-frequency asymptotics of the symbol of the Dirichlet-to-Neumann operator in 2-dimensional exterior problem*
Abstract:Unique solvability of an exterior Dirichlet problem implies existence of an operator that maps the Dirichlet data (function on the obstacle boundary) to the normal derivative of the solution (another function on the boundary). The so defined Dirichlet-to-Neumann map DtN is a boundary pseudodifferential operator of order 1. In 2D problems, the boundary is one-dimensional, usually diffeomorfic to a circle, and DtN can be exactly (without truncation by order) described by a discrete symbol, which is a function of three parameters: boundary parameter s, Fourier series index (discrete momentum) n, and the wavenumber k. As k goes to infinity, the symbol has a nice asymptotic behaviour uniformly in s and n. This fact can be viewed as a microlocal refinement of the Kirchhoff approximation. A related numerical method will also be discussed.

(Joint research with Margo Kondratieva. Supported by NSERC)

Speaker: **Shen, Jie** (Purdue University)
Title: *Efficient and Stable Spectral Methods for the Helmholtz equation in exterior domains*

I shall present an efficient and stable spectral algorithm and their numerical analysis for the Helmholtz equation in exterior domains. The algorithm couples a boundary perturbation technique with a well-conditioned spectral-Galerkin solver based on an essentially exact Dirichlet-to-Neumann operator. Error analysis as well as numerical results will be presented to show the accuracy, stability, and versatility of this algorithm.

Speaker: **Schoetzau, Dominik** (University of British Columbia)
Title: *Title*
Abstract: Abstract Text

Speaker: **Steinbach, Olaf** (Technische Universitat Graz)
Title: *Modified Boundary Integral Equations in Acoustics*
Abstract: Starting from the well known combined boundary integral formulations due to Brakhage/Werner and Burton/Miller we will review existing modifications which are needed for the numerical analysis in the correct function spaces. While most of the proposed modifications rely on a compactness argument, we will discuss an alternative approach. Then we derive a stable approximation scheme and discuss an efficient solution of the resulting linear system.

Speaker: **Tsnykov, Symon** (North Carolina State University)
Title: *The Huygens's Principle as a Computational Tool*
Abstract: Among the well-known challenges that arise when computing the unsteady wave fields is the deterioration of numerical schemes over long time intervals (error buildup) and the unboundedness of the domain of definition. The latter is typical for many applications, e.g., for the scattering problems, when the waves are radiated toward infinity. In the literature, a standard way to deal with the first issue is to increase the order of accuracy (quite independently, paraxial approximations can be employed), whereas the second issue requires truncation of the domain and setting of the artificial boundary conditions (ABCs). According to the conventional wisdom, exact ABCs for multidimensional unsteady problems are nonlocal not only in space but also in time, and the extent of temporal nonlocality continually increases as time elapses.
It turns out, however, that in many cases both types of difficulties can be addressed using a unified approach based on exploiting the Huygens's principle. The propagation of waves is said to be diffusionless, and the corresponding governing PDE (or system) is said to satisfy the Huygens principle, if the waves due to compactly supported sources have sharp aft fronts. The areas of no disturbance behind the aft fronts are called lacunae. Diffusionless propagation of waves is rare, whereas its opposite - diffusive propagation with after-effects is common. Nonetheless, lacunae can still be observed in a number of important applications, including acoustics and electromagnetism. The key idea of using lacunae for computations is that any finite size region falls behind the propagating aft front, i.e., right into the lacuna, after a finite interval of time. In other words, any given feature of the solution will only have a finite predetermined lifespan on any fixed domain of interest. By incorporating these considerations into a numerical scheme, one can make its grid convergence uniform in time (non-deteriorating method). The same considerations facilitate design of exact unsteady ABCs with only fixed and limited (non-increasing) extent of temporal nonlocality.
We will describe recent progress made in constructing the lacunae-based numerical schemes for the d'Alembert equation, as well as for the linearized Euler equations and the Maxwell equations. We will also touch upon the technical issues that need to be addressed, in particular, when applying the methodology to systems, as opposed to scalar equations. Finally, we will discuss different physical models from the standpoint of existence of the lacunae. We will show that in some interesting cases that are technically speaking diffusive, e.g., the propagation of electromagnetic waves in dilute plasma, lacunae can still be identified in the solutions in some approximate sense.

Speaker: **Warburton, Tim** (Rice University)
Title: *Advances in Wave Propagation with the Discontinuous Galerkin Method*

Abstract: Two important features relating to the suitability of the discontinuous Galerkin (DG) method for wave propagation will be discussed.

Recent investigations of the spectral properties of the discrete DG operators have revealed important connections with their continuous Galerkin analogs. Theoretical and numerical results will be shown which demonstrate the correct asymptotic behavior of these methods and precludes spurious solutions under mild assumptions.

Given the suitability of DG for solving Maxwell's equations and their ability to propagate waves over long distance, it is natural to seek effective boundary treatments for artificial radiation boundary conditions. A new family of far field boundary conditions will be introduced which gracefully transmit propagating and evanescent components out of the domain. These conditions are specifically formulated with DG discretizations in mind, however they are also relevant for a range of numerical methods.

This is work with Thomas Hagstrom, Jan Hesthaven, Lucas Wilcox, Nigel Nunn.

New PhDs and Graduate Students

One of the goals of this workshop is to promote interaction between young mathematicians and more senior researchers in the field of computational scattering. The discussion of state-of-the-art algorithms and open problems in talks by senior mathematicians in their talks should provoke discussions. To highlight their work, new PhDs and graduate students will be presenting posters. These posters will be posted in the lounge during all the coffee breaks, providing ample opportunity for interaction.

Binford, Tommy (Rice University)
Title:*Experiments with a Dirichlet to Neumann Map for High Order Finite Elements*
Abstract: For electromagnetic scattering problems, the number of degrees of freedom to acheive a desired accuracy can be prohibitively large depending on the domain. Artificial boundary methods are a powerful tool for treating radiation conditions while preserving the physical behavior with fewer degrees of freedom. Work by Dave Nicholls on Dirichlet to Neumann maps has provided a method of handling the radiation condition for perturbed simple geometries such as a circular boundary. In these experiments, we apply a high order finite element method in conjunction with a Dirichlet to Neuman map to solve Helmholtz' equation for a right circular cylindrical scatterer with different perturbations of a circular artificial boundary away from the scattering object.

Ecevit, Fatih (Max Planck Institute)
Title: *High-frequency asymptotics and convergence of multiple-scattering iterations in two-dimensional scattering problems*
Abstract: One of the main difficulties in high-frequency electromagnetic and acoustic scattering simulations is that any numerical scheme based on the full-wave model entails the resolution of wavelength. It is due to this challange that simulations involving even very simple geometries are beyond the reach of classical numerical schemes.

We present an analysis of a recently proposed integral equation method for the solution of high-frequency electromagnetic and acoustic scattering problems that delivers *error-controllable solutions in frequency-independent computational times*. Within single scattering configurations the method is based on the use of an appropriate ansatz for the unknown surface densities and on suitable extensions of the method of stationary phase. The extension to multiple-scattering configurations, in turn, is attained through consideration of an iterative (Neumann) series that successively accounts for multiple reflections. Here we derive a high-frequency asymptotic expansion of the successively induced currents in this latter procedure and, within this context, we derive an estimate for its convergence rate. As we show, this rate is explicitly computable and it depends solely on geometrical characteristics; in particular, it is independent of the specific incidence of radiation. Numerical results confirm the accuracy of this high-frequency estimate for the case of several interacting structures.

This is work with Fernando Reitich.

Han, Young-Ae (Caltech)

Title:*A Continuation Method for high-order parametrization of arbitrary surfaces*

To this date there had been no adequate methods to parametrize complicated surfaces in three-dimensional space by locally smooth functions. In this poster we present an approach to this problem. Our new method, whose implementation is very simple indeed, accurately renders geometric singularities such as edges and corners, and it yields super-algebraically convergent approximations to a given surface. The local parametrizations, further, can be selected so as to contain large portions of a given surface, a feature that is very useful in many applications.

Our approach is based on a very simple but previously untested idea: continuation of each smooth branch of a piecewise-smooth function into a new function which, defined on a larger domain, is both smooth and periodic. These "continuation functions" have Fourier coefficients that decay super-algebraically, and thus result in high-order approximations of the given function throughout its domain of definition—thus, in particular, producing a resolution of the Gibbs phenomenon. As we will detail, the proposed method can be utilized to advantage in a wide variety of applications. We will present a number of examples showing that our method can yield parametrizations of very high quality and accuracy for very complex surfaces, such as complete aircraft.

This is joint work with O. Bruno and M. Pohlman.

Kurtz, Jason (U. Texas at Austin)

Title:*Fully-Automatic hp-Adaptivity for Acoustic and Electromagnetic Scattering in 3D* We present an algorithm for fully-automatic hp-adaptivity in the context of either finite element or coupled finite/infinite element discretizations for acoustic and electromagnetic scattering problems in three dimensions. The algorithm generates a sequence of optimal hp-meshes that deliver exponential convergence in the energy norm. The method is ideally suited for scatterers with geometric singularities and/or for discretizations truncated by a perfectlymatched layer (PML). We discuss three issues that have been critical in the efficient implementation of the method: namely, fast integration of element stiffness matrices, a domain-decomposition multi-frontal solver, and a "telescoping" solver for a sequence of locally nested meshes. Computational results are presented for a variety of simple scatterers based on both PML and infinite element truncation.

This is joint work with L. Demkowicz.

Sifuentes, Josef (Rice University)

Title:*GMRES performance in integral equation methods for scattering by inhomogeneous media*

Abstract: Wave Scattering through inhomogeneous media arise in a variety of engineering applications, however building numerical solvers to deal with these models remain a challenge due to the necessity of resolving high frequency waves over large scale scattering obstacles. In particular, the number of GMRES operations necessary to solve the discretized integral equation formulation increases sharply with the increase of the frequency of the incident wave. I will describe recent investigations into the dependence of the spectrum of the discretized integral operator on the wave number and the corresponding GMRES performance. Understanding these relationships are important in the development of efficient and effective preconditioning strategies.

Turc, Catalin (Caltech)

Title: *High-order solutions of 2D scattering problems from surfaces with composite roughness*

Abstract: In this poster we will present a numerical scheme capable of producing high-order solutions to scattering problems off multi-scale rough surfaces. We view the multi-scale rough surface as a perturbation of slowly varying components in terms of highly oscillatory ones, and we propose series expansions of the same type for the scattered fields. The terms in these perturbation series are radiative solutions to the Helmholtz equation that assume highly oscillatory boundary values on the slowly varying components. Each of these problems are treated using a class of recently designed high-order solvers for high-frequency scattering problems from rough surfaces. We illustrate the high-order character of the resulting numerical algorithms for a variety of incidences and profiles.

This is joint work with Fernando Reitich.

Wilcox, Lucas (Brown)
Title: *Sledge++ – A discontinuous Galerkin finite element discretization package*

The discontinuous Galerkin method was developed for hyperbolic conservation laws has been gaining attention. The method has been extended to solve elliptic equations and equations with higher order derivatives. Unlike the finite element method, there is a lack of software packages that support discontinuous Galerkin methods. Sledge++ is a nodal discontinuous Galerkin discretization framework designed for discretizing operators – these can then be used to represent time-domain, time-harmonic or other operators, e.g., Laplace. The theory for this method has been developed in a variety of papers [1-3]. Two- and three-dimensional computational domains of simplexes are supported.

Sledge++ extends the Unstructured Spectral Element Method (USEMe) [3] code in a couple of meaning full ways. First, Sledge++ is designed for the efficient application of discontinuous Galerkin operators and the efficient building of discontinuous Galerkin matrices. Adaptation of the computational mesh in h and p is also implemented through the nonconforming mesh support in Sledge++. A nonconforming mesh allows for neighboring elements with different orders (p-nonconforming) and for neighboring elements whose vertices do not coincide (h-nonconforming).

Sledge++ provides a core set of linear algebra routines to solve basic problems. This allows for rapid development and testing of new discontinuous Galerkin operators. For more advanced linear algebra and parallel support Sledge++ provides an interface to build operators supported by Trilinos [4].

An overview of the Sledge++ framework will be given through the exploration of Sledge++ applications used to solve the time-domain and time-harmonic Maxwell's equations scattering problems.

1. W. H. Reed and T. R. Hill, "Triangular mesh methods for the neutron transport equation", Los Alamos Scientific Laboratory report LA-UR-73-479, Los Alamos, NM, 1973. 2. B. Cockburn and C.-W. Shu, "The Local Discontinuous Galerkin Method for Time-Dependent Convection-Diffusion Systems", v. 35, n. 6, p. 24402463, 1998. 3. J. S. Hesthaven and T. Warburton, "High-Order Nodal Methods on Unstructured Grids. I: Time-Domain Solution of Maxwell's Equations", Journal of Computational Physics, v. 181, p. 186-221, 2002. 4. Michael A. Heroux et al., "An Overview of the Trilinos Project", ACM Transactions on Mathematical Software, v. 32, p. 397-423, 2005.