Operator methods in fractal analysis, wavelets and dynamical systems
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Ola Bratteli (University of Oslo, Norway), bratteli@math.uio.no
Palle Jorgensen (The University of Iowa, USA), jorgen@math.uiowa.edu
David Kribs (University of Guelph, Canada), dkribs@uoguelph.ca
Gestur Olafsson (Louisiana State University, USA), olafsson@lsu.edu
Sergei Silvestrov (Lund University, Sweden), sergei.silvestrov@math.lth.se

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1 Introduction about the meeting

Recently our understanding of some of the most exciting new scientific discoveries has proved to rely on fractal features. They are understood by the coming together of mathematics, communication theory, computer graphics, signal/image processing, medical imaging, and quantum theory.

Leading researchers on fractals, wavelets, dynamics and their analysis and uses as well as talented younger researchers have converged on the Banff Centre in the week of December 2–7, 2006 for a hosted by Banff International Research Station international workshop on new developments in Operator methods in fractal analysis, wavelets and dynamical systems.

More then 40 participants from around the World: Europe, Scandinavian countries, Asia, Australia, the US and Canada took part.

The event consisted of three elements: (1) more then 25 scheduled research presentations covering the themes of the workshop; (2) discussions helping to bridge the separate topics; and (3) joint research interaction, including work on joint research papers and projects.

The conference activities aimed at creation of new international cooperation patterns and research advances as well as research training of young promising researchers.

This event was co-organized by Professors Ola Bratteli (University of Oslo, Norway), Palle Jorgensen (The University of Iowa, USA), David Kribs (University of Guelph), Gestur Olafsson (Louisiana State University, USA) and Sergei Silvestrov (Lund University, Sweden). The names Bratteli and Jorgensen are synonymous with significant advances in these fields. Professor David Kribs is conducting pioneering work in quantum computation, and Professors Gestur Olafsson and Sergei Silvestrov are leading mathematicians in areas covered by the workshop and their applications to neighboring fields such as engineering, physics, and medicine. The expert group that has been assembled focused on the hottest new results in fractal theory and related topics. The workshop brought together experts in areas of pure and applied mathematics who have made independent advances and it has provided a unique opportunity for advancing the field through teamwork and collaboration. Among participants there were a substantial number of distinguished senior researchers as well as junior researchers both with theoretical and applied backgrounds. This mix ensured a long-term impact of the program on the area, and on the mathematics and its applications as a whole. The research agenda was full of interesting problems of common interest to most of the participants.

2 Overview of the Field

Fractals are everywhere in nature and in technology. When you look at them in a microscope or in a telescope, you see hidden patterns as similar repeated structures and features, repeated at different scales. On occasion they are well hidden, for example in huge data sets from the internet. Fractal analysis and data mining are the tools that reveal these features, repeated at varying scales of resolution; and making up fundamental constituents in a yet new and relatively uncharted domain of science.

This workshop aimed at developing new approaches and mathematical foundations for wavelet analysis, dynamical and iterated function systems, spectral and tiling duality, fractal iteration processes and non-commutative dynamical systems. The basic methods involved in this work derive from a combination of operator theory, harmonic analysis and representation theory.

The program is in mathematics. However, the issues represented in the program originate both from within mathematics and from observed natural phenomena and the engineering practice. The interplay and unified approaches to these significantly interrelated areas of mathematics is of great significance both for mathematics itself and its connections to other subjects and applications.

Wavelet theory stands on the interface between signal processing, operator theory, and harmonic analysis. It is concerned with the mathematical tools involved in digitizing continuous data with a view to storage and compression, and with the synthesis process, recreating the desired picture (or time signal) from the stored data. The algorithms involved go under the name of filter banks, and their spectacular efficiency derives in part from the use of hidden self-similarity in the data or images that are analyzed. Self-similarity is built into wavelets too as they are intrinsically defined using dynamical and iterated function systems. This makes wavelets also closely related to fractals and fractal processes. Investigation of this relation has huge theoretical and practical potential and thus it is becoming a subject of growing interest both in and outside mathematics. It has been recently shown (by Palle Jorgensen, Ola Bratteli, David Larson, X. Dai and others) that a unifying approach to wavelets, dynamical systems, iterated function systems, self-similarity and fractals may be based on the systematic use of operator analysis and representation theory.

3 Recent Developments and Open Problems

Connections of operator methods to the applications are manifold.

Operator analysis. Observations or time signals are functions, and classes of functions make up vector spaces. The most useful spaces are Hilbert spaces of square integrable functions on domains in n-dimensional Euclidean space, but several applications require more general spaces, like Sobolev spaces or even spaces of distributions and also functional spaces on fractal sets and sets with fractal boundaries. The basic idea in wavelet theory is to study the operators of dilations and translation, and subspaces invariant under those operators. The computational aspect is represented in sampling and approximation. Given measured data one finds the elements in the subspace closest to those data. The interplay between those two aspects was the focus of the workshop.

Representation theory and operator algebras. Representation theory associates to abstract algebraic objects operators on Hilbert spaces or more general locally convex topological vector spaces satisfying the algebraic relations. We are interested in representations defined by filters, wavelets and similar objects. The continuous wavelet transform gives rise to square integrable representations of topological groups; the scaling identity gives rise to representations of the Cuntz algebra defined by the high pass and low pass filters. The high pass filters alone define transfer operators. A main point is the study of intertwining operators between, on one side, the "discrete world" of high-pass/low-pass filters of signal processing, and on the other side, the "continuous world" of wavelets. There are significant operator-algebraic and representation-theoretic issues on both sides of the 'divide', and the intertwining operators throw light on central issues for wavelets in higher dimensions.

Dynamical systems, operator algebras and operator analysis. Operators of multiplication and dilations and more general weighted composition operators with more general may be non-linear and non-invertible transformations of variables are central for wavelet analysis and analysis on fractals. The properties of dynamical and iterated function systems defined by these transformations govern the spectral properties and corresponding subspace decompositions for these operators. Operator algebras generated by such operators are concrete representations of generalized crossed product type algebras and C^*-algebras defined via various kinds and generalizations of covariance commutation relations. Invariant sets and measures for the dynamical and for the iterated function systems such as for example orbits or attractors and invariant or quasi-invariant measures on them are directly linked to such operator algebras. The interplay between dynamical and iterated function systems and actions of groups and semigroups on one side and operator algebras on the other side bring fruitful results and new methods for both areas. This yields new powerful results and methods for wavelets and fractal analysis and geometry.

There are many applications of the above mentioned methods and analysis to engineering and physics problems, as well as reach possibilities to gain insight into numerical analysis of corresponding applications.

Tools from diverse areas of analysis, as well as from dynamical systems and operator theory, merge into the research on wavelet analysis. The diversity of techniques is also a charm of the subject, which continues to generate new graduate students and postdoctoral activity.

4 Scientific Progress Made

The operator methods enter in that wavelets, signals and information may be realized as vectors in a real or complex Hilbert space, or in symbolic graph or path spaces. For the purpose of transmission, these vectors are encoded in for example a set of linear coefficients. In the case of images, including fractal images, this was worked out using wavelet and filter functions, for example corresponding to ordinary Cantor fractal subsets of $\mathbb{R}$, as well as for fractal measure spaces corresponding to Sierpinski Gasket fractals.

Several fractals, like a finitely summable infinite tree, and the Sierpinski gasket fit naturally within this framework. In these cases, we show that our spectral triples do describe the geodesic distance and the Minkowski dimension as well as. Furthermore, in the case of the Sierpinski gasket, the associated Dixmier-type trace coincides with the normalized Hausdorff measure.

More generally, fractals are related to more traditional wavelets, those of $L_2(\mathbb{R}^d)$. Two computational features were addressed: (a) Approximation of the father or mother functions by subdivision schemes, and (b) matrix formulas for the wavelet coefficients. A variety of data were considered; typically for fractals, L_2-convergence is more restrictive than is the case for $L_2(\mathbb{R}^d)$-wavelets.

A variety of wavelet applications were considered, involving a construction of certain groups of measure preserving transformations, and groups and algebras of operators, with special algebraic properties. Other results include applications of a theory of projection decompositions of positive operators, and a theory of operator-valued frames.

Operator algebra constructions of covariant representations are used in the analysis of orthogonality in wavelet theory, in the construction of super-wavelets, and orthogonal Fourier bases for affine fractal measures.

K-theoretic tools have been further developed. Smale spaces are abstract topological dynamical systems characterized by canonical coordinates of contracting and expanding directions. These include basic sets from Smale's Axiom A systems as well as shifts of finite type. In general, they are chaotic and the underlying geometry is fractal. There are C^*-algebras associated with such objects and the aim is to compute their K-theory providing invariants. Other C^*-algebras were associated to shift spaces. They can be viewed as generalizing the universal Cuntz-Krieger algebra, leading to simple and purely infinite C^*-algebras, understood via their K-theory, K_0 and K_1. Further understanding of the structure of these algebras leads to new approaches to classifications and better understanding of the dynamics of arbitrary shift spaces. One version of dynamics was explained as information is stored in certain ideal structure, referring to a recent classification result for certain non-simple Matsumoto C^*-algebras [6]. New insights also have been gained into the interplay between orbit space structure for dynamical systems or more general group and semigroup actions, and structure of ideals, subalgebras and representations for crossed product algebras constructed via twisted by dynamics generalized convolution products [18].

A different class of Banach algebras enter applied analysis as follows: (a) Wiener's Lemma for twisted convolution and for the rotation algebras; (b) algebras of infinite matrices with off-diagonal decay are inverse-closed in the algebra of all bounded operators; (c) inverse-closedness plays an essential role in quantitative studies of the finite section method to solve operator equations; (d) a new construction of inverse-closed matrix algebras by approximation properties.

We further worked on complex B-splines, i.e., a generalization of ordinary B-splines to complex degrees. This results in an infinite uniform knot sequence for complex B-splines. Result is that generalized fractional divided differences can be defined via the fractional Weyl-integral with complex B-splines as densities.

In the area of quantum information, the operator theoretic view begins with a von Neumann measurement. It is replaced by a more general concept called a positive operator valued measure (POVM), which is essentially a partition of unity in terms of positive semi-definite operators. POVM's formed from equally weighted rank one operators define a tight frame, and any frame defines a POVM.

Several themes were covered in the workshop, and we have grouped them loosely under the headers, fractals, wavelets and dynamics. But we are stressing an underlying unity based on operator theoretic tools. Covariant representations represent one such. This has been a central theme in operator algebras since the 1950s, and has played a key role in numerous applications since. One of these more recent applications is to a class of wavelets called ”frequency localized” wavelets [2, 10] as well as to signal and image processing constructions. And more generally to symbolic dynamics! Another is the ”transfer operator.” This refers to a construction with origins in probabilistic path models from physics and ergodic theory. In our workshop we drew up some connections between the two areas, showed how operator algebraic ideas and representations throw new light on applications. Since several ideas are involved, participants from one area learned from the others. The workshop offered a unique opportunity for leading researchers in diverse but related areas to meet and learn from one another. One use of operator algebras (specifically, C^*-algebras) is to construction of representations, to states, and to dynamics. Initially [14], the focus was on groups and on harmonic analysis, but the notion of covariance from physics (see e.g., [4, 16]) suggested crossed products of groups with act by automorphisms on C^*-algebras ([9, 20]). Since the pioneering paper by Stinespring [17], a preferred approach (e.g., [1]) to constructing representations begins with a positive operator valued mapping, and Stinespring identified complete positivity; now widely used. At the same time, related notions of positivity are central in a variety of probabilistic path models, beginning with Doeblin [12], see also [8]. It is now a key tool in ergodic theory, [19]. As a result, over the years, Doeblin's operator has taken on many other incarnations, and it is currently known as ”the transfer” operator, the Ruelle operator, or the Perron-Frobenius-Ruelle operator [3]. The name Ruelle is from its use in statistical mechanics as pioneered by David Ruelle, see [16] and [3]. Of a more recent vintage are applications to wavelets [11], and to analysis of fractals, i.e., special and computational bases in Hilbert space constructed from a class of unitary representation of certain discrete groups of affine transformations. It was realized (e.g., [5]) that there are transfer operators T_W for wavelets, that the solution to a spectral problem for T_W yields wavelet representations; and moreover that these representations come with a useful covariance. The workshop further explored an intriguing variant of the operators T_W used in quantum error correction codes, see [7, 13]. All versions of transfer operators involve hieratical processes with branching, and probabilities assigned by a weight function; touching again on the central feature of fractals.

5 Outcome of the Meeting

Parallel efforts are under way in the United States, Canada, and Europe and merging the activities add tremendously both to generating research advances, and to advance education. Bringing together groups of participants with common interests and prominent researchers in the fields of operator algebras, representation theory, fractal analysis, dynamical and iterated function systems, wavelets and harmonic analysis have lead to substantial new advances in both the theory and application of all these fields. Finally, there have been discovered enticing connections between the focus areas of this workshop and certain aspects of quantum computing and quantum information theory. The workshop had also important educational dimension since a number of junior researchers, post-docs and PhD students, have taken part and were given opportunity to present their results and take part in discussions with senior researchers and colleagues from other groups and countries.

An important outcome of the workshop is the appearance of a number of joint publications and the creation of several new collaborative research projects and initiatives involving different groups of participants. A special volume on the subject is in preparation.

Dissemination.
We have agreement with Springer Verlag about the publication of a special volume on the topic of the conference.

The aim of the volume is to broaden and deepen interplay between:
– wavelets and fractals and operator algebras, operator theory and representation theory;
– dynamics and operator algebras and operator theory;
– quantum computing and information;
– applications of the above in engineering, physics and beyond.

Contributions to the volume will address one or several of the above directions.

6 Acknowledgements

All the participants thoroughly enjoyed their stay at the BIRS facility. The venue is superb and the scenery, the hospitality and the food were fantastic. This brought together a number of important participants in the field. The scientific discussions continued long after the talks were over, in the meeting rooms, the hiking trails, and by the donation fridge. It was a meeting that will shape the future of our field for years to come. Our thanks to the staff and directorship of BIRS for this opportunity.

Participation from other countries nearby and far away from Canada have become possible thanks to crucial travel support from many national and international research foundations and networks, such as European networks (Liegrits and others), Research councils and foundations in Norway, Denmark, United states of America, Canada and other countries, STINT (The Swedish Foundation for International Cooperation in Research and Higher Education) and Crafoord foundation, private research foundations and participant's home universities. The support of all these funding institutions is gratefully acknowledged.

7 Presentation Highlights (in alphabetical order)

Florin Boca (University of Illinois)
Title: C^*-algebras and continued fractions
An AF algebra associated with the Farey tessellation and capturing the properties of the continued fraction algorithm has been considered. The Effros-Shen rotation AF algebras arise as quotients of this algebra.

Bernhard Bodmann (University of Waterloo)
Title: Optimal Redundant Packet Encoding for Loss-Insensitive Linear Transmissions
The objective of this talk was to characterize the optimal use of redundancy in transmitting a signal that is encoded in packets of linear coefficients. The signals considered here are vectors in a finite-dimensional real or complex Hilbert space. For the purpose of transmission, these vectors are encoded in a set of linear coefficients that is partitioned in packets of equal size. It was investigated how the encoding performance depends on the degree of redundancy it incorporates and on the amount of data-loss when packets are either transmitted perfectly or lost in their entirety. The encoding performance is evaluated in terms of the maximal Euclidean norm of the reconstruction error occurring for the transmission of unit vectors. The main result of this talk is the derivation of error bounds as well as the characterization of optimal encoding when up to three packets are lost.

Berndt Brenken (University of Calgary)
Title: Topological quivers as multiplicity free relations
For a discrete directed graph a certain graph C^*-algebra is invariant under a procedure that yields a multiplicity free graph. It was shown the analogue of this holds for topological quivers; a certain C^*-algebra, namely the unaugemented Cuntz-Pimsner C^*-algebra, of a topological quiver remains Morita equivalent to the C^*-algebra of an associated multiplicity free topological quiver.

Jonas D'Andrea (University of Colorado)
Fractal wavelets of Dutkay-Jorgensen Type for the Sierpinski gasket space
Several years ago, D. Dutkay and P. Jorgensen developed the concept of wavelets defined on a sigma-finite

fractal measure space, developed from an iterated affine system. They worked out in detail the wavelet and filter functions corresponding to the ordinary Cantor fractal subset of $\mathbb{R}$. In this talk the construction of Dutkay and Jorgensen was examined as applied to the fractal measure space corresponding to the Sierpinski Gasket fractal. A variety of high-pass filters was developed, and as an application, the various families of wavelets were used to analyze digital images.

Kenneth R Davidson (University of Waterloo)
Operator algebras for multivariable dynamics I
To a locally compact Hausdorff space X with n proper continuous maps of X into itself, there were associated various topological conjugacy algebras; with two emerging as the natural candidates for the universal algebra of the system, the tensor algebra and the semicrossed product. The reasons for this were discussed, including dilation theory, representations and C^*-envelopes. Generalized notions of wandering sets and recursion were used to characterize when these algebras are semisimple.

Dorin Dutkay (Rutgers University)
bf Covariant representations, scaling functions and affine fractals
(a joint work with Palle Jorgensen)
In this talk it was demonstrated how some operator algebra constructions of covariant representations can be used to analyze orthogonality in wavelet theory, to construct super-wavelets, and to obtain orthogonal Fourier bases for affine fractal measures.

Soren Eilers (University of Copenhagen)
Classification of C^*-algebras associated to irreducible shift spaces
A construction by Matsumoto allows an invariant association of C^*-algebras to any shift space. Somewhat exceptionally, these C^*-algebras are not always simple when the shift space is irreducible, and in previous work, mainly with Carlsen, one has endeavored to explain what dynamical information is stored in the ideal structure in those cases. In this talk this problem was reviewed and discussed in light of a recent classification result for certain non-simple Matsumoto C^*-algebras obtained in joint work with Restorff and Ruiz.

Karlheinz Groechenig (University of Vienna)
Inverse-closed Banach algebras in applied analysis
A Banach algebra A is called inverse-closed in a larger Banach algebra B, if every element in A that is invertible in B is already invertible in the smaller algebra A. For instance, the algebra of absolutely convergent Fourier series is inverse-closed in the algebra of continuous functions on the torus. This is the classical Wiener Lemma. In the talk there were presented several results about inverse-closed Banach algebras in applied analysis:
(a) Wiener's Lemma for twisted convolution and for the rotation algebras;
(b) algebras of infinite matrices with off-diagonal decay are inverse-closed in the algebra of all bounded operators;
(c) inverse-closedness playing an essential role in quantitative studies of the finite section method to solve operator equations;
(d) A new construction of inverse-closed matrix algebras by approximation properties.

Cristina Ivan (University of Hannover, Germany)
Spectral triples for fractals
The purpose of this talk was to present two possible ways of associating a spectral triple to a fractal such that it encodes geometric data of the fractal. The spectral triple is obtained in both constructions as a countable sum of unbounded Fredholm modules. In the first construction (joint work with Erik Christensen) each summand is a spectral triple for a set consisting of just two points (a "two-point" spectral triple). It was Alain Connes who first constructed in this way a spectral triple for the middle Cantor set in the unit interval. Connes showed how the metric, the Hausdordff measure and dimension are encoded by this spectral triple and its associated Dixmier trace. Connes' construction has been studied in details and extended to certain classes of fractals by Guido and Isola. Together with Erik Christensen, Cristina Ivan has investigated to which extend such a spectral triple may encode geometric data of a general compact metric space. They showed that

for any compact metric space it is possible to associate a spectral triple which is a countable sum of ”two-point” spectral triples and which reflects the Minkowski dimension of the space, and the metric induced by the spectral triple is equivalent to the given one. Explicit computations were performed for the unit interval, Cantor set and Sierpinski gasket and each time it was obtained that the spectral triple and its Dixmier trace recovers metric, dimension and volume measure of the compact metric space under discussion. In the second construction (joint work with Erik Christensen and Michel Lapidus) each summand is based on a curve in the space. Several fractals, like a finitely summable infinite tree, and the Sierpinski gasket fit naturally within this framework. In these cases, we show that our spectral triples do describe the geodesic distance and the Minkowski dimension as well. Furthermore, in the case of the Sierpinski gasket, the associated Dixmier-type trace coincides with the normalized Hausdorff measure. It is important to mention the advantage of each proposal for constructing spectral triples. The advantage of the first construction is that it is modeled for a general compact metric space. The advantage of the second construction is that, when it is possible to be done, it brings more information about the topological structure of the fractal (the spectral triple will induce a nontrivial element in the K-homology of the fractal).

Elias G. Katsoulis (East Carolina University)
Operator algebras for multivariable dynamics II
(This is a joint work with Ken Davidson.)
To a locally compact Hausdorff space X with n proper continuous maps of X into itself there were associated various topological conjugacy algebras; and two emerged as the natural candidates for the universal algebra of the system, the tensor algebra and the semicrossed product. A new concept of topological conjugacy for multidimensional systems, called piecewise conjugacy, was introduced and discussed. It was proved that the piecewise conjugacy class of the system can be recovered from the algebraic structure of either the tensor algebra or the semicrossed product. Various classification results follow as a consequence. For example, for n=2,3, the tensor algebras are (algebraically or even completely isometrically) isomorphic if and only if the systems are piecewise topologically conjugate.

Palle Jorgensen (The University of Iowa)
Wavelets on Fractals
(This is a joint work with Dorin Dutkay)
Joint work between Palle Jorgensen and Dorin Dutkay, recently led to wavelet constructions, and wavelet algorithms in Hilbert spaces built on fractals. The talk covered some highpoints, and provided a comparative study: the case of fractals was contrasted with the more traditional wavelets, those of $L^2(R^d)$. As a conclusion there were noted several instances of dichotomies; e.g., measure classes, regions of convergence, stability to mention three. Two computational features were addressed:
(a) Approximation of the father/mother functions by subdivision schemes;
(b) matrix formulas for the wavelet coefficients.
For (a) it was demonstrated that the variety of data when L^2-convergence holds is much smaller in the case of fractals than is the case for $L^2(R^d)$-wavelets.

Takeshi Katsura (Hokkaido University)
Cuntz-Krieger algebras and factor maps between topological graphs
A (one-sided) Markov chain is a topological dynamical system defined from a 0,1-matrix. Cuntz and Krieger introduced a C^*-algebra to examine a Markov chain. Although a Cuntz-Krieger algebra is defined from a 0,1-matrix, it only depends on the associated Markov chain. Later, a construction of Cuntz-Krieger algebras from Markov chains using groupoids were provided. In this talk, there were introduced topological graphs which contain 0,1-matrices and Markov chains as special cases. Also there was introduced a way to construct C^*-algebras from topological graphs, which generalizes the construction of Cuntz-Krieger algebras. Using the notion of ”factor maps” between topological graphs, it was clarified the relation of the two constructions of Cuntz-Krieger algebras from $\{0, 1\}$-matrices and from Markov chains.

David R. Larson (Texas A & M University)
Frames and Operator Theory
A few years ago David R. Larson and his collaborators developed an operator-interpolation approach to

wavelets and frames using the local commutant (i.e. commutant at a point) of a unitary system. This is really an abstract application of the theory of operator algebras to wavelet and frame theory. The concrete applications of operator-interpolation to wavelet theory include results obtained using specially constructed families of wavelet sets. The methods include the construction of certain groups of measure preserving transformations, and groups and algebras of operators, with special algebraic properties. Other results include applications of a theory of projection decompositions of positive operators, and a theory of operator-valued frames. In the talk there have been discussed unpublished and partially published results, and some brand new results, that are due to David R. Larson and his former and current students, and other collaborators.

Nadia S. Larsen (University of Oslo)
Projective multi-resolution analyses arising from direct limits of Hilbert modules
(This is a joint work with I. Raeburn.)
In joint work with I. Raeburn it has been shown how direct limits of Hilbert spaces can be used to construct multi-resolution analysis and wavelets in $L^2(R)$. This talk was devoted to enlargement of the framework of this construction and use a direct limit of Hilbert modules over a fixed C^*-algebra to produce projective multi-resolution analysis in the limit module. In certain cases, existence of standard module frames for the limit module was proved. For modules over the algebra of continuous functions on the product of n-copies of the circle, these methods shed light on work of Packer and Rieffel on projective multi-resolution analysis for specific Hilbert modules of functions on R^n. New applications arise in the context of modules over the algebra of continuous functions on the compact infinite path space of a finite directed graph.

Peter Massopust (Technische Universitat Munchen, Germany)
Dirichlet-Poisson Processes meet Complex B-Splines
(This is a joint work with Brigitte Forster.)
Complex B-splines are a generalization of ordinary B-splines to complex degrees. This results in an infinite uniform knot sequence for complex B-splines. It was shown that generalized fractional divided differences can be defined via the fractional Weyl-integral with complex B-splines as densities. This representation leads to a generalized Hermite-Genocchi formula over infinite dimensional simplices. The generalized Hermite-Genocchi formula then allows the extension of complex B-splines to non-uniform knots and their interpretation as probability densities for a class of stochastic processes, namely the Dirichlet-Poisson processes.

Sergey Neshveyev (University of Oslo)
KMS-states on Hecke algebras crossed products
(This is a joint work with M. Laca and N. S. Larsen)
It was shown that KMS-states on crossed products of abelian C^*-algebras by Hecke algebras correspond to measures scaled by Hecke operators. In this work there was considered the $GL(2)$-system of Connes and Marcolli and their analysis of the system was completed by showing that for each value of the temperature in the critical region there exists a unique KMS-state.

Johan Oinert,
Commutativity and ideals in generalized crossed products
(This is a joint work with Sergei Silvestrov)
A short review was given of G-crossed product systems and the construction of generalized algebraic crossed products following C. Nastasescu, F. Van Oystaeyen, Methods of graded rings, LNM 1836, Springer-Verlag, 2004. Thereafter, inspired by the theory of C^*-dynamical systems, some results were presented relating commutativity, ideals, group actions and zero-devisors in algebraic crossed product algebras.

N Christopher Phillips (University of Oregon)
Crossed products of the irrational rotation algebras by the "standard" actions of $\mathbb{Z}/2\mathbb{Z}$, $\mathbb{Z}/3\mathbb{Z}$, $\mathbb{Z}/4\mathbb{Z}$, $\mathbb{Z}/6\mathbb{Z}$ are AF
Let F be a finite subgroup of $SL_2(\mathbb{Z})$ (necessarily isomorphic to one of $Z/2Z$, $\mathbb{Z}/3\mathbb{Z}$, $\mathbb{Z}/4\mathbb{Z}$, $\mathbb{Z}/6\mathbb{Z}$), and let F act on the irrational rotational algebra A_θ via the restriction of the canonical action of $SL_2(\mathbb{Z})$. Then the crossed product of A_θ by F, and the fixed point algebra for the action of F on A_θ, are AF algebras. The same is true for the crossed product and fixed point algebra of the flip action of $\mathbb{Z}/2\mathbb{Z}$ on any sim-

ple d-dimensional noncommutative torus A_θ. Along the way, there were proved a number of general results which should have useful applications in other situations. (The paper is available at arXiv:math.OA/0609784)

Ian Putnam (University of Victoria)
K-theory for Smale spaces
Smale spaces are abstract topological dynamical systems characterized by canonical coordinates of contracting and expanding directions. These include basic sets from Smale's Axiom A systems as well as shifts of finite type. In general, they are chaotic and the underlying geometry is fractal. There are C^*-algebras associated with such objects and the aim is to compute their K-theory. For shifts of finite type, this is the usual dimension group invariant. More generally, there is a spectral sequence for this, but the answer can be given in purely dynamical terms as a kind of homology theory for chaotic systems, similar in spirit to Cech homology.

Iain Raeburn (University of Newcastle, Australia)
Direct limits, the Cuntz relations and wavelets
(This is a joint work with Nadia Larsen)
A famous theorem of Mallat shows how to build a wavelet basis for the Hilbert space of square-integrable functions on R starting from a quadrature mirror filter, which is a function on the unit circle satisfying an algebraic relation. From such a filter, Bratteli and Jorgensen constructed a pair of isometries satisfying the Cuntz relations well-known to operator algebraists. This talk was devoted to an approach to Mallat's theorem which uses a direct limit construction and exploits the geometric information inherent in the Cuntz relations.

Kjetil Roysland (University of Oslo)
Transition operators on bundle maps
In a joint work Kjetil Roysland and Dorin Dutkay have studied transition operators that act on the bundle maps of a vector bundle. This talk was about the fix points of such an operator. In some situations this turn out to be a finite-dimensional non-commutative C^*-algebra.

Christian Skau (Norwegian University of Science and Technology, Trondheim) Title: AF-equivalence relations and group actions
It was shown that a group acting freely as homeomorphisms on a zero-dimensional space gives rise to an AF-equivalence relation if and only if the group is locally finite. Furthermore, it was shown that the AF-equivalence relations that occur are exactly the ones that are associated to Bratteli diagrams that have the equal path number property. It was also shown that the "super" order of the locally finite group is completely determined by the rational subdimension group of the AF-relation.

Sergei Silvestrov (Lund University, Sweden)
C^*-crossed Products and Shift Spaces
(This is a joint work with Toke Meier Carlsen, arXiv:math.OA/0512488)
In this talk Exel's C^*-crossed product of non-invertible dynamical systems was used to associate a C^*-algebra to every shift space. It was shown that this C^*-algebra is canonically isomorphic to the C^*-algebra associated to a shift space in arXiv:math.OA/0505503, has the C^*-algebra defined by Toke Meier Carlsen and Kengo Matsumoto (Math. Scand. 95, 2 (2004), 145-160) as a quotient, and possesses properties indicating that it can be thought of as the universal C^*-algebra associated to a shift space. Also there were considered its representations, relationship to other C^*-algebras associated to shift spaces, shown that it can be viewed as a generalization of the universal Cuntz-Krieger algebra, discussed uniqueness and a faithful representation, shown that it is nuclear and satisfies the Universal Coefficient Theorem, were provided conditions for it being simple and purely infinite, shown that the constructed C^*-algebras and thus their K-theory, K_0 and K_1, are conjugacy invariants of one-sided shift spaces, presented formulas for those invariants, and also presented a description of the structure of gauge invariant ideals.

Christian Svensson (Lund University, Sweden and Leiden Universty, The Netherlands)
Dynamical systems and commutants in crossed products
(This is a joint work with Marcel de Jeu and Sergei Silvestrov, arXiv:math.DS/0604581)

Given a discrete dynamical system, one may construct an associative (non-commutative) complex algebra with multiplication determined via the action defining the system - a crossed product algebra. It turns out that for large classes of systems, one obtains striking equivalences between, in particular, dynamical properties of the system and algebraic properties of the crossed product. Quite a lot has been done in this direction for C^*-crossed products. It is satisfactory to see that many of the results obtained in purely algebraic setup are analogous to those well-known for C^*-case, and at the same time further progress on interplay between structure of crossed product and dynamics can be obtained outside the C^*-context. In this work, in particular, there was described the commutant of an arbitrary subalgebra A of the algebra of functions on a set X in a crossed product of A with the integers by a composition automorphism defined via a bijection of X. The conditions on A and on the dynamics, extending topological freeness, which are necessary and sufficient for A to be maximal abelian in the crossed product are subsequently applied to situations where these conditions can be shown to be equivalent to a condition in topological dynamics. As a further step, using the Gelfand transform, for a commutative completely regular semi-simple Banach algebra, there was obtained a topological dynamical condition on its character space which is equivalent to the algebra being maximal abelian in a crossed product with the integers.

Jun Tomiyama (Tokyo Metropolitan University, Japan Women's University)
Hulls and kernels with actions of topological dynamical systems and C^*-algebras
Let $\Sigma = (X, \sigma)$ be a topological dynamical system in a compact space X with a homeomorphism σ, and let $A(\Sigma)$ be the associated C^*-crossed product. In this context there were defined hulls and kernels with the action σ, and discussed the following problems.
1. What is the meaning in C^*-theory of the kernels of the elementary sets for the dynamical system Σ?
2. What is the meaning of the Hulls of those structural ideals of the C^*-algebra $A(\Sigma)$?

8 List of Participants

Boca, Florin (University of Illinois, Urbana, USA)
Bodmann, Bernhard (University of Waterloo, Canada)
Brenken, Berndt (University of Calgary, Canada)
D'Andrea, Jonas (University of Colorado at Boulder, USA)
Davidson, Ken (University of Waterloo, Canada)
De Jeu, Marcel (Leiden University, Holland)
Dutkay, Dorin (Rutgers University, USA)
Eilers, Soren (University of Copenhagen, Denmark)
Giordano, Thierry (University of Ottawa, Canada)
Groechenig, Karl-Heinz (University of Vienna, Austria)
Ivan, Cristina (University of Hannover, Germany)
Johansen, Rune (University of Copenhagen, Denmark)
Jorgensen, Palle (The University of Iowa, USA)
Katsoulis, Elias (East Carolina University, USA)
Katsura, Takeshi (Hokkaido University, Japan)
Kribs, David (University of Guelph, Canada)
Laca, Marcelo (University of Victoria, Canada)
Lamoureux, Michael (University of Calgary, Canada)
Larsen, Nadia S. (University of Oslo, Norway)
Larson, David (Texas A & M University, USA)
Lesosky, Maia (University of Guelph, Canada)
Massopust, Peter (GSF – Institute for Biomathematics and Biometry and Technical University, Munich, Germany)
Mohari, Anilesh N. (Bose Centre for Basic Sciences, India)
Neshveyev, Sergey (University of Oslo, Norway)
Nikolaev, Igor (University of Calgary, Canada)
Niu, Zhuang (University of Calgary, Canada)

Öinert, Johan (Lund University, Sweden)
Olafsson, Gestur (Louisiana State University, USA)
Packer, Judith (University of Colorado, USA)
Phillips, N. Christopher (University of Oregon, USA)
Putnam, Ian (University of Victoria, Canada)
Raeburn, Iain (University of Newcastle, Australia)
Roysland, Kjetil (University of Oslo, Norway)
Ruskai, Mary Beth (Tufts University, USA)
Sangha, Amandip (University of Oslo, Norway)
Silvestrov, Sergei (Lund University, Sweden)
Skau, Christian (NTNU, Trondheim, Norway)
Song, Myung-Sin (Southern Illinois University Edwardsville, USA)
Strungaru, Nicolae (University of Victoria, Canada)
Svensson, Christian (Lund University, Sweden; Leiden University, The Netherlands)
Tomiyama, Jun (Tokyo Metropolitan University, Japan)
Whittaker, Michael (University of Victoria, Canada)

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