Banff International Research Station
for Mathematical Innovation and Discovery

Algebraic groups, quadratic forms and related topics
September 2-7, 2006

MEALS

Breakfast (Continental): 7:00–9:00 am, 2nd floor lounge, Corbett Hall, Sunday–Thursday
*Lunch (Buffet): 11:30 am–1:30 pm, Donald Cameron Hall, Sunday–Thursday
*Dinner (Buffet): 5:30–7:30 pm, Donald Cameron Hall, Saturday–Wednesday
Coffee Breaks: As per daily schedule, 2nd floor lounge, Corbett Hall
*Please remember to scan your meal card at the host/hostess station in the dining room for each lunch and dinner.

MEETING ROOMS

All lectures will be held in Max Bell 159 (Max Bell Building accessible by bridge on 2nd floor of Corbett Hall). Hours: 6 am–12 midnight. LCD projector, overhead projectors and blackboards are available for presentations. *Please note that the meeting space designated for BIRS is the lower level of Max Bell, Rooms 155–159. Please respect that all other space has been contracted to other Banff Centre guests, including any Food and Beverage in those areas.*

SCHEDULE

Saturday
16:00	Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
17:30–19:30	Buffet Dinner, Donald Cameron Hall
20:00	Informal gathering in 2nd floor lounge, Corbett Hall
	Beverages and small assortment of snacks available on a cash honour-system.

Sunday
7:00–8:45	Breakfast
8:45–9:00	Introduction and Welcome to BIRS by BIRS Station Manager, Max Bell 159
9:00–10:00	**Florian Pop** (University of Pennsylvania)
	Fields of cohomological dimension 1
10:00–10:30	Coffee Break, 2nd floor lounge, Corbett Hall
10:30–11:30	**John Swallow** (Davidson College)
	Galois module structure of Galois cohomology
11:30–13:30	Lunch
13:45–14:45	**Prakash Belkale** (University of North Carolina Chapel Hill)
	A new product in the cohomology of flag varieties G/P
	and applications to geometry and representation theory
14:45–15:15	Coffee Break, 2nd floor lounge, Corbett Hall
15:15–16:15	**Alexander Nenashev** (Glendon College/York University)
	A geometric approach to push-forwards in Balmer-Witt theory
16:30–17:30	**Marco Schlichting** (Louisiana State University)
	Stabilized Witt-groups
17:30–19:30	Dinner

Monday

7:00–9:00	Breakfast
9:00–10:00	**Patrick Brosnan** (University of British Columbia)
	On the essential dimension of an algebraic stack
10:00–10:30	Coffee Break, 2nd floor lounge, Corbett Hall
10:30–11:30	**Angelo Vistoli** (University of Bologna)
	Stacks of curves of low gonality and group actions
11:30–13:30	Lunch
13:00–14:00	Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
14:15	Group Photo; meet on the front steps of Corbett Hall
14:30–15:30	**Alexandr Vishik** (Institute for Information Transmission Problems, R.A.S.)
	On the u-invariant
15:30–16:00	Coffee Break, 2nd floor lounge, Corbett Hall
16:00–17:00	**Kirill Zainoulline** (University of Bielefeld)
	Chow motives of generically split projective homogeneous varieties
17:30–19:30	Dinner

Tuesday

7:00–9:00	Breakfast
9:00–10:00	**Philippe Gille** (Universit Paris XI)
	Algebraic actions of arithmetic groups
10:00–10:30	Coffee Break, 2nd floor lounge, Corbett Hall
10:30–11:30	**Uzi Vishne** (Bar Ilan University)
	Traces in arithmetic lattices, congruence conditions and the length of geodesics
11:30–13:30	Lunch
	Free Afternoon
17:30–19:30	Dinner

Wednesday

7:00–9:00	Breakfast
9:00–10:00	**Daniel Krashen** (Yale University)
	Index reduction for genus 1 curves
10:00–10:30	Coffee Break, 2nd floor lounge, Corbett Hall
10:30–11:30	**Eva Bayer-Fluckiger** (Ecole Polytechnique Federale de Lausanne),
	Multiples of G-quadratic forms
11:30–13:30	Lunch
13:45–14:45	**Detlev Hoffmann** (University of Nottingham)
	The graded Witt ring of bilinear forms in characteristic 2
14:45–15:15	Coffee Break, 2nd floor lounge, Corbett Hall
15:15–16:15	**Ricardo Baeza** (University of Talca)
	Relations of bilinear and quadratic Pfister forms in characteristic two
16:30–17:30	**Ahmed Laghribi** (Universit d'Artois)
	On the norm theorem for singular quadratic forms
17:30–19:30	Dinner

Thursday

7:00–9:00	Breakfast
9:00–10:00	TBA
10:00–10:30	Coffee Break, 2nd floor lounge, Corbett Hall
10:30–11:30	TBA
11:30–13:30	Lunch

Checkout by 12 noon.

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ABSTRACTS

Speaker: **Ricardo Baeza** (University of Talca)
Title: *Relations of bilinear and quadratic Pfister forms in characteristic two*
Abstract: (joint work with J.Arason) Let K be a field of characteristic two, $W(K)$ the Witt ring of symmetric bilinear forms and $Wq(K)$ the Witt group of quadratic forms over K. Let I be the maximal ideal of $W(K)$. We determine a complete set of relations of the n-fold bilinear Pfister forms in I^n and of the n-fold quadratic Pfister forms in $I^n Wq(K)$ for all n.

Speaker: **Eva Bayer-Fluckiger** (Ecole Polytechnique Federale de Lausanne)
Title: *Multiples of G-quadratic forms*
Abstract: Let k be a field of characteristic $\neq 2$ and let G be a finite group. A *G-quadratic form* is by definition a symmetric bilinear form $q : V \times V \to k$ over a finite dimensional k-vector space V invariant by the action of G. Let us denote by $W(k)$ the Witt ring of k, and by $I = I(k)$ its fundamental ideal. Let $d = \mathrm{cd}_2(k)$. The aim of this talk is to present the following

Theorem. *Let q and q' be two G-quadratic forms defined over the same $k[G]$-module, and let ϕ be a quadratic form over k. If $\phi \in I^d$, or if G has no quotient of order 2 and $\phi \in I^{d-1}$, then $\phi \otimes q \simeq \phi \otimes q'$.*

Using additional invariants (signatures, G-discriminant) one obtains similar criteria for fields of virtual cohomological dimension d and for arbitrary groups.

Such statements were first conjectured in the framework of trace forms of G-Galois algebras, and proved in several special cases by Chabloz, Monsurro, Morales, Parimala and Schoof. The proofs use results of Berhuy as well of Parimala, Sridharan and Suresh.

Speaker: **Prakash Belkale** (University of North Carolina Chapel Hill)
Title: *A new product in the cohomology of flag varieties G/P and applications to geometry and representation theory*
Abstract: (joint work with S. Kumar) Let G be a simple complex algebraic group (e.g $SO(n)$ or $SP(n)$, $SL(n)$) and P a maximal parabolic. The (topological) cohomology $H^*(G/P)$ appears in questions in representation theory : decomposing tensor products, possible eigenvalues of product of elements in the compact form of G etc. An inductive characterisation of non-vanishing structure coefficients (in the schubert basis) is sought for. A deformation of the cohomology will be introduced (which is the "correct" ring for these problems). The deformation is algebro-geometric, and measures the "amount of moving" required to obtain a transverse intersection. If time permits, I will report on ongoing work on these problems for Symplectic and Odd orthogonal groups.

Speaker: **Patrick Brosnan** (University of British Columbia)
Title: *On the essential dimension of an algebraic stack*
Abstract: The essential dimension $ed(G)$ of an algebraic group G is a numerical measure of the complexity

of the torsors for that group. I will discuss joint work with Reichstein on an extension of the notion of essential dimension to algebraic stacks.

Speaker: **Philippe Gille** (Universit Paris XI)
Title: *Algebraic actions of arithmetic groups*
Abstract: This is a joint work with L. Moret-Bailly (Rennes). We give an integral generalization of the Borel-Serre's theorem. Given an algebraic action of an algebraic group $G(Z)$ over an arithmetic scheme X/Z, we show then that the orbits of $G(Z)$ on $X(Z)$ are almost determined by the local orbits.

Speaker: **Detlev Hoffmann** (University of Nottingham)
Title: *The graded Witt ring of bilinear forms in characteristic 2 under function field extensions of arbitrary quadrics*
Abstract: Let F be a field and let $I^n F$ be the nth power of the fundamental ideal IF of Witt classes of even-dimensional forms in the Witt ring WF of nondegenerate *symmetric bilinear* forms. Let $\overline{I}^n F = I^n F/I^{n+1} F$. For a field extension K/F we denote by $\overline{I}^n(K/F)$ the kernel of the restriction map $\overline{I}^n F \to \overline{I}^n K$. Let q be a quadratic form over F (where we assume the polynomial $q(X)$ to be irreducible) and let $F(q)$ be the function field of the projective quadric $q = 0$. If F is of characteristic $\neq 2$, the determination of $\overline{I}^n(F(q)/F)$ is in general quite difficult. One of the few cases where the result is known is when q is a Pfister form, and this is one of the central results in the work by Orlov, Vishik and Voevodsky on the Milnor conjecture. In characteristic 2, Aravire and Baeza computed these kernels for function fields of bilinear Pfister forms. Such function fields are in fact nothing else but function fields of certain totaly singular quadratic forms. Based on these results, we compute $\overline{I}^n(F(q)/F)$ for arbitrary quadratic form.

Speaker: **Daniel Krashen** (Yale University)
Title: *Index reduction for genus 1 curves*
Abstract: Let k be a field, and let K/k be the field of functions of a curve of genus 1 defined over k. Given a central simple algebra A/k, we ask the question of how to determine the index of the algebra $A \otimes K$. In particular, this entails describing when the algebra $A \otimes K$ is split. In this talk I will present two sets of results. The first, which is joint work with Mirela Ciperiani, gives tools for determining when the algebra $A \otimes K$ is split. The second, which is joint work with Max Lieblich, shows how we may essentially reduce the question of determining the index of $A \otimes K$ to the question of splitting. Precisely, we show that we may compute the index by having information on when the algebra $A' \otimes K'$ is split, where $A' = A \otimes E$, $K' = K \otimes E$ and E/k is a finite extension. In the case that k is a local field, this gives a totally explicit formula for the index of $A \otimes K$ in terms of the invariant of the algebra A and certain invariants of the genus 1 curve.

Speaker: **Ahmed Laghribi** (Universit d'Artois)
Title: *On the norm theorem for singular quadratic forms*
Abstract: We prove some results concerning the norm theorem for semisingular quadratic forms, i.e., quadratic forms which are neither nonsingular nor totally singular. As a consequence and by introducing the notion of quasi-hyperbolicity, we extend the Cassels-Pfister subform theorem to the case of semisingular quadratic forms.

Speaker: **Alexander Nenashev** (Glendon College/York University)
Title: *A geometric approach to push-forwards in Balmer-Witt theory*
Abstract: We construct projective push-forwards in the Witt theory of algebraic varieties by using twisted Thom isomorphisms and deformations to the normal cone.

Speaker: **Florian Pop** (University of Pennsylvania)
Title: *Fields of cohomological dimension 1*
    Abstract: The Theorem of Merkurjev-Suslin inspired Bogomolov to state his "Freeness Conjecture". In spite of the fact that there is almost no non-obvious evidence for Bogomolov's Freeness Conjecture, even

stronger conjectures were proposed by several people (Gilles, Positselski, etc.), which among other things would imply that the maximal Abelian extension of $C(x, y)$ has cohomological dimension 1. In my talk I would like to give some non-obvious evidence for the conjectures above (and even for a stronger one, which will be thoroughly explained).

Speaker: **Marco Schlichting** (Louisiana State University)
Title: *Stabilized Witt-groups*
Abstract: Let $X = U \cup V$ be a scheme covered by two open subschemes. When X is smooth, Balmer's triangular Witt groups yield a long Mayer-Vietoris exact sequence involving the Witt-groups of X, U, V and $U \cap V$. This is no longer true, in general, when X is singular. However, stabilized Witt groups (introduced by Karoubi and Ranicki) have this property, and coincide with Balmer Witt-groups in case X is smooth. The aim of the talk is to introduce stabilized Witt groups and explain some of their properties (Homotopy invariance, invariance under nilpotent thickenings, Mayer Vietoris for open covers in general, and for blow-up squares in char 0).

Speaker: **John Swallow** (Davidson College)
Title: *Galois module structure of Galois cohomology*
Abstract: Let p be a prime number, F a field containing a primitive p-th root of unity, and E/F a cyclic extension of degree p, with Galois group G. Let G_E be the absolute Galois group of E. The cohomology groups $H^i(E, \mathbb{F}_p) = H^i(G_E, \mathbb{F}_p)$ possess a natural structure as $\mathbb{F}_p G$-modules and decompose into direct sums of indecomposables. In the 1960s Borevic and Faddeev gave decompositions of E^*/E^{*p} – the case $i = 1$ – for local fields. We describe the case $i = 1$ for arbitrary fields, and then, using the Bloch-Kato Conjecture, we also determine the case $i > 1$. No small surprise arises from the fact that there exist indecomposable $\mathbb{F}_p G$-modules which never appear in these module decompositions. We give several consequences of these results, notably a generalization of the Schreier formula for G_E, connections with Demuskin groups, and new families of pro-p-groups that cannot be realized as absolute Galois groups. These results have been obtained in collaboration with D. Benson, J. Labute, N. Lemire, and J. Minc.

Speaker: **Alexandr Vishik** (Institute for Information Transmission Problems, R.A.S.)
Title: *On the u-invariant*
Abstract: The uniform construction of fields with various u-invariants will be provided.

Speaker: **Uzi Vishne** (Bar Ilan University)
Title: *Traces in arithmetic lattices, congruence conditions and the length of geodesics*
Abstract: (Joint work with M. Katz and M. Schaps) We consider the traces in congruence subgroups of arithmetic lattices in $SL_2(\mathbb{R})$. Bounds obtained from quadratic conditions are translated into a lower bound on the length of geodesics in some Riemannian surfaces, showing that in various families, the girth grows at least logarithmically with the genus.

Speaker: **Angelo Vistoli** (University of Bologna)
Title: *Stacks of curves of low gonality and group actions*
Abstract: Some interesting fibered categories can be expressed as quotient stacks for very simple group actions; this gives an approach to the calculation of algebraic invariants of the categories (for example, their Picard groups). In my talk I will describe the cases of hyperelliptic curves (joint work with A. Arsie) and trigonal curves (work in progress). I will assume no knowledge of the theory of algebraic stacks.

Speaker: **Kirill Zainoulline** (University of Bielefeld)
Title: *Chow motives of generically split projective homogeneous varieties*
Abstract: Let G be an anisotropic linear algebraic group which splits by a field extension of a prime degree. Let X be a projective homogeneous G-variety such that G splits over the function field of X. We prove that under certain conditions the Chow motive of X is isomorphic to a direct sum of twisted copies

of an indecomposable motive R. This covers all known examples of motivic decompositions of generically split projective homogeneous varieties (Severi-Brauer varieties, Pfister quadrics, maximal orthogonal Grassmannians) as well as provides new ones (exceptional varieties of types E_6 and E_8).