BIRS Workshop
Moment maps in various geometries
21 – 26 May 2005

MEALS

Breakfast (Continental): 7:00 - 9:00 am, 2nd floor lounge, Corbett Hall, Sunday - Thursday
*Lunch (Buffet): 11:30 am - 1:30 pm, Donald Cameron Hall, Sunday - Thursday
*Dinner (Buffet): 5:30 - 7:30 pm, Donald Cameron Hall, Saturday - Wednesday
Coffee Breaks: As per daily schedule, 2nd floor lounge, Corbett Hall
***Please remember to scan your meal card at the host/hostess station in the dining room for each lunch and dinner.**

MEETING ROOMS

All lectures are held in the main lecture hall, Max Bell 159. *Please note that the meeting space designated for BIRS is the lower level of Max Bell, Rooms 155-159. Please respect that all other space has been contracted to other Banff Centre guests, including any Food and Beverage in those areas.*

SCHEDULE

	Saturday	Sunday	Monday	Tuesday	Wednesday	Thursday
7:00-9:00		Continental Breakfast, 2nd floor lounge, Corbett Hall				
9:00-10:00		Landweber	Apostolov	Weitsman	Bursztyn	Problem
10:00-10:30		Coffee Break				Session
10:30-11:30		Sjamaar	Problem session	Alekseev	Fernandes	Coffee
11:45-12:00				Group Photo[1]		
11:30-13:30		Buffet Lunch, Donald Cameron Hall				
13:30-14:30		Guided Tour[2]	Konno	free afternoon	Pflaum	
14:30-15:30		Mare	Proudfoot	free afternoon	Abreu	
15:30-16:00	Check-in begins	Coffee Break		free afternoon	Coffee Break	
16:00-17:00		Guillemin	Hausel	free afternoon	Wilkin	
17:30-19:30	Buffet Dinner, Donald Cameron Hall					
20:00	Informal gathering[3]					

[1] A group photo will be taken on Tuesday at 11:45 am, directly after the last lecture of the morning. Please meet on the front steps of Corbett Hall.

[2] A free guided tour of The Banff Centre is offered to all participants and their guests on Sunday starting at 1:00 pm. The tour takes approximately 1 hour. Please meet in the 2nd floor lounge in Corbett Hall.

[3] The informal gathering will occur in the 2nd floor lounge in Corbett Hall. Beverages and snacks will be available on a cash honor-system basis.

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ABSTRACTS
(in alphabetic order by speaker surname)

Speaker: **Miguel Abreu** (Instituto Superior Tecnico)
Title: *Moment Maps, Symplectomorphism Groups and Compatible Complex Structures*
Abstract: In the first part of this talk I will describe Donaldson's general moment map geometry for the action of a symplectomorphism group on the corresponding space of compatible almost complex structures.

In the second part I will discuss the particularly interesting example of rational ruled surfaces, where the moment map geometry suggests a new approach to study the topology of their symplectomorphism groups and spaces of compatible complex structures (joint work with Gustavo Granja and Nitu Kitchloo).

Speaker: **Anton Alekseev** (University of Geneva)
Title: *Ginzburg-Weinstein via Gelfand-Zeitlin*
Abstract: Let $U(n)$ be the unitary group, and $u(n)^*$ the dual of its Lie algebra, equipped with the Kirillov Poisson structure. In their 1983 paper, Guillemin-Sternberg introduced a densely defined Hamiltonian action of a torus of dimension $(n-1)n/2$ on $u(n)^*$, with moment map given by the Gelfand-Zeitlin coordinates. A few years later, Flaschka-Ratiu described a similar, 'multiplicative' Gelfand-Zeitlin system for the Poisson Lie group $U(n)^*$.

By the Ginzburg-Weinstein theorem, $U(n)^*$ is isomorphic to $u(n)^*$ as a Poisson manifold. Flaschka-Ratiu conjectured that one can choose the Ginzburg-Weinstein diffeomorphism in such a way that it intertwines the linear and nonlinear Gelfand-Zeitlin systems. Our main result gives a proof of this conjecture, and produces a *canonical* Ginzburg-Weinstein diffeomorphism.

The talk is based on a joint work with E. Meinrenken.

Speaker: **Vestislav Apostolov** (UQAM)
Title: *Hamiltonian 2-forms and explicit Kähler–Einstein metrics on toric orbifolds*
Abstract: I will discuss the classification of compact Kähler manifolds admitting a hamiltonian 2-form. This involves two components of independent interest. The first is the notion of rigid hamiltonian torus action which leads to a fairly explicit classification: up to a blow-up, compact Kähler manifolds with a rigid hamiltonian torus action are bundles of toric Kähler manifolds. The second idea is a special case of toric geometry, which we call orthotoric. A compact orthotoric Kähler manifold is diffeomorphic to a complex projective space, but we obtain new explicit examples of Kähler–Einstein orthotoric complex surfaces. Combining these two themes provides a context for constructing new examples of extremal Kähler metrics on geometrically ruled complex manifolds.

Speaker: **Henrique Bursztyn** (University of Toronto)
Title: *Generalized moment maps*
Abstract: I will overview various generalized notions of hamiltonian actions and moment maps, recalling

the contexts in which they arise. The examples will include, on one hand, moment maps of Poisson-Lie group actions and symplectic groupoids, and on the other hand quasi-hamiltonian spaces with G-valued moment maps and quasi-Poisson spaces with D/G-valued moment maps. I will describe how these are all examples of moment maps in Dirac geometry, and hence can be naturally understood in a unified way.

Speaker: **Rui Loja Fernandes** (Instituto Superior Tecnico)
Title: *Moment maps in Poisson geometry*
Abstract: We consider a proper Poisson action of a Lie group G on a Poisson manifold M. I will explain that the orbit space M/G is a Poisson stratified space and that the action of G on M lifts to a Hamiltonian action on the "symplectization" $\Sigma(M)$ of M, for a canonical choice of a moment map. Then I will discuss in what sense symplectization commutes with reduction: $\Sigma(M/G) = \Sigma(M)//G$.

Speaker: **Victor Guillemin** (MIT)
Title: *The notion of moment map for families of symplectomorphisms*
Abstract: As Alan Weinstein observed in his paper "Symplectic geometry" (Bull.AMS 5, 1981, 1-13), the moment map associated with the Hamiltonian action of a Lie group defines a "moment Lagrangian" whose properties encode many of the main features of moment geometry. In my talk I'll describe an analogous object for families of symplectomorphisms (i.e. no groups). This is joint work with Shlomo Sternberg.

Speaker: **Tamás Hausel** (UT Austin)
Title: *Cohomology of hyperkahler spaces in gauge theory*
Abstract: I survey three techniques and their results so far on the cohomology of gauge theoretical hyperkahler manifolds. The spaces considered are moduli spaces of Yang-Mills instantons on R^4 (more generally Nakajima quiver varieties) and moduli spaces for dimensional reductions of the same equations. Such as magnetic monopoles in 3 dimensions and moduli space of Higgs bundles in 2 dimensions; and the diffeomorphic spaces of the moduli space of flat holomorphic connections on a vector bundle on a curve, and the $GL(n, C)$ representation variety of the fundamental group of the Riemann surface. The three techniques are: global analytical to determine the dimension of the space of L^2 harmonic forms on the spaces (motivated by the physics of Sen's conjecture); circle-equivariant cohomological: to calculate the equivariant cohomology (motivated by the physics of Nekrasov-Shatasvili-Moore), and finally arithmetic harmonic analysis: to calculate the zeta functions of our spaces (motivated by mirror symmetry).

Speaker: **Hiroshi Konno** (University of Tokyo)
Title: *Geometry and topology of hyperkahler quotients*
Abstract: I will talk about the geometry and topology of various examples of hyperkahler quotients, for example, toric ones, hyperkahler polygon space, the moduli space of torsion free sheaves on $\mathbb{C}^2$ (ADHM description), quiver varieties etc.

Speaker: **Greg Landweber** (University of Oregon)
Title: *Equivariant K-theory in Symplectic Geometry*
Abstract: We give an survey of complex equivariant K-theory, the generalized cohomology theory built from equivariant vector bundles, and discuss its use in the study of Hamiltonian group actions. Many of the standard (rational or de Rham) cohomological results in equivariant symplectic geometry have K-theoretic analogues, including the Atiyah-Bott lemma (on the Euler class being not a zero-divisor) and Kirwan surjectivity. In work with Harada, we have found that K-theory behaves better in this regard than integral cohomology.

Speaker: **Augustin-Liviu Mare** (University of Regina)
Title: *Connectivity properties for moment maps on based loop groups*
Abstract: If G is a compact semisimple Lie group, we consider the based loop group $\Omega(G)$, which consists of all loops $\gamma : S^1 \to G$ with $\gamma(1) = e$. This is an example of an infinite dimensional Kähler manifold. The

circle S^1 acts on $\Omega(G)$ by rotating the loops (actually their argument), and the corresponding moment map is just the energy function E. I will sketch a proof of the fact that all levels of $E : \Omega(G) \to R$ are connected. This is another manifestation of the general principle which says that $\Omega(G)$ behaves in many respects like a compact Kähler manifold. The maximal torus $T \subset G$ acts on $\Omega(G)$ by pointwise conjugation. A classical result of Atiyah and Pressley says that the image of the moment map corresponding to the action of $S^1 \times T$ on $\Omega(G)$ is a convex polytope. I will sketch a proof of the fact that the regular levels of this moment map are connected. This is joint work with Megumi Harada, Tara Holm, and Lisa Jeffrey.

Speaker: **Markus Pflaum** (Goethe Universiteit)
Title: *Groupoids in symplectic geometry*
Abstract: Differentiable groupoids can be interpreted as an interpolation between the notion of a manifold and a Lie group and appear in many instances in differential geometry. In the talk I will give an introduction to the theory of Lie groupoids. Afterwards, two major applications of Lie groupoids in symplectic geometry will be explained. The first deals with the integrability of Poisson manifolds by symplectic groupoids, a concept which goes back to work of Weinstein and Karasev. As a second application we describe Moerdijk's approach to the definition of orbifolds via proper etale Lie groupoids.

Speaker: **Nick Proudfoot** (UT Austin)
Title: *Algebraic symplectic quotients*
Abstract: Let G be a reductive algebraic group acting on a smooth variety V. The cotangent bundle of V has a natural algebraic symplectic form, and the induced action of G is hamiltonian. I will discuss the symplectic quotient, emphasizing the relationship between this space and the various GIT quotients of V by G.

Speaker: **Reyer Sjamaar** (Cornell)
Title: *Varieties invariant under a Borel subgroup*
Abstract: This is joint work with Victor Guillemin. Atiyah proved that the moment map image of the closure of an orbit of a complex torus action is convex. Brion generalized this result to actions of a complex reductive group. We extend their results to actions of a maximal solvable subgroup.

We present two sets of examples of Borel invariant subvarieties of a Kähler Hamiltonian G-manifold M.

First we have generalized Schubert varieties, which were introduced by Białynicki-Birula as unstable sets of certain flows on M.

Then we have the so-called facial varieties. For each face of the moment polytope of M we find a Borel invariant subvariety whose moment polytope is equal to the face. (As pointed out by Brion, there is in general no such G-invariant subvariety.)

Speaker: **Jonathan Weitsman** (UCSC)
Title: *Measures on Banach Manifolds and Supersymmetric Quantum Field Theory*
Abstract: We show how to construct measures on Banach manifolds associated to supersymmetric quantum field theories. As examples of our construction we produce measures corresponding to spaces of maps from a Riemann surface to a semisimple Lie group (the Wess-Zumino-Novikov-Witten model) and to gauge theory in three dimensions. We show that these measures are positive, and that the Wess-Zumino-Novikov-Witten measure where the Riemann surface is P^1 has finite mass. As an application we show that formulas arising from expectations in this measure reproduce the results of Frenkel and Zhu from vertex operator algebras.