REPORT ON THE CONFERENCE TOPOLOGY AT BIRS, AUGUST 27 - SEPTEMBER 1, 2005 ORGANIZERS: IAN HAMBLETON, MATTHIAS KRECK AND RON STERN

1. Introduction

The idea of the conference was to bring together distinguished senior and some of the best junior mathematicians representing a broad variety of subjects in topology. Topology has become - like many other areas of mathematics - a field which is a collection of many areas all having the size to justify a conference of its own. Most conferences nowadays are of the latter type. If one looks back, important progress often was based on a combination of methods and ideas from these subareas and also from some neighbouring areas. To mention a few examples:

- The Donaldson and Seiberg-Witten theory addresses problems in topology of 4-manifolds and uses methods ranging from partial differential equations, differential geometry, index theory, algebraic geometry to algebraic topology.
- Novikov conjecture and related conjectures where high-dimensional manifold theory, in particular surgery, index theory, algebraic K-theory and geometric group theory are centrally involved.
- The attempts to create elliptic cohomology use methods ranging from stable homotopy theory, algebraic geometry, index theory to theoretical physics (conformal field theory).

Almost no mathematician is able to be familiar with all these subjects and even to follow the main results and fundamental ideas is very hard. A conference like this provides an unusual opportunity to hear some of the most important and fundamental developements and - even more important - to discuss ideas with experts from other areas.

The conference was attended by forty participants. When the organizers selected the participants, they had the difficulty that to cover all these areas with leading experts left rather limited room for young people. And so they had to give up some very prominent names. The result seemed to us a good mixture of leaders and excellent young people some of which are already leaders themselves. One indication of success: we heard during the conference that *four* of our main speakers had been asked to give talks at the next ICM 2006 in Spain.

To give enough time to discussions between individuals and in groups, we limited the talks to five per day and 45 minutes each. All three of the outstanding developements mentioned above were represented in the talks. We had asked the speakers to address a broad audience and most of them succeeded very well. Our impression was that our goal was fully achieved. We have asked the participants to send us comments and we quote from them after the problem list.

We were uncertain about a problem session and finally decided against one. But on Wednesday evening a group of about twenty people met in the lounge and spontaneously a problem discussion came up. More precisely, we asked everybody to formulate her/his favourite problem. Since the list looks very nice, we gave those who did not participate in this round the chance to add their favourite problems afterwards. The problems are attached after the summaries of talks.

Many participants asked for another conference of this type. We like the idea, and are planning to apply again for 2007.

2. Program

Sunday, August 28, 2005

8:45-9:00 Introduction and Welcome to BIRS by BIRS Station Manager, Max Bell 159
9:00-9:45 Bruce Kleiner, Univ. of Michigan: Geometrization and uniformization of metric spaces
10:00-10:45 Arthur Bartels, Univ. Münster: The Farrell-Jones Conjecture for groups acting on trees
11:15-12:00 Denis Auroux, MIT: Fiber sums of Lefschetz fibrations
16:00-16:45 Jacob Lurie, Harvard: Elliptic Cohomology and Derived Algebraic Geometry
17:00-17:45 Jongil Park, Seoul National University: Rational blow-downs and smooth 4-manifolds with one basic class

Monday, August 29, 2005

9:00-9:45 Weimen Chen, University of Massachusetts: Pseudo holomorphic curves and finite group actions in dimension 4
10:00-10:45 Shmuel Weinberger, University of Chicago: A Sullivan conjecture for equivariant structure sets
11:15-12:00 Martin Bridson, Imperial College London: Limit Groups: non-positive curvature, logic, and group theory
16:00-16:45 Thomas Mark, Southeastern Louisiana University: Ozsvath-Szabo invariants of fiber sums
17:00-17:45 William Dwyer, Notre Dame: Duality in algebra and topology

Tuesday, August 30, 2005

9:00-9:45 Oleg Viro, Uppsala University: Virtual links, their relatives and Khovanov homology
10:00-10:45 Jesper Grodal, University of Chicago: p-compact groups and their classification
11:15-12:00 Peter Ozsvath, Columbia University: Heegard Floer homology of links
16:00-16:45 Jacob Rasmussen, Princeton University: Differentials on Khovanov-Rozansky homology
17:00-17:45 Yongbin Ruan, University of Wisconsin: Twisted K-theory on orbifolds and its stringy product

Wednesday, August 31, 2005

7:00-9:00 Wolfgang Lück, Münster: L^2-invariants and their applications

10:00-10:45 Stefano Vidussi, Univ. of California, Riverside: Taubes conjecture and twisted Alexander invariants
11:15-12:00 Karen Vogtmann, Cornell University: Tethers and homology stability
16:00-16:45 Andras Stipsicz, Alfrd Rnyi Institute of Mathematics: Contact Ozsvath–Szabo invariants and tight structures on 3-manifolds
17:00-17:45 Walter Neumann, Columbia University: Graph manifolds and singularities

3. Summaries of Talks

Bruce Kleiner: Uniformization and Geometrization of metric spaces

I discussed the problem of parameterizing metric spaces by nice model spaces. More precisely, the goal was to find conditions on a metric space Z which guarantee that there is quasisymmetric homeomorphism $f : X \to Z$, where the model space X is either optimal in some way, or at least canonical. This recoginition problem is motivated by a long development in Geometric Mapping Theory and by rigidity questions in Geometric Group Theory.

Arthur Bartels: The Farrell-Jones Conjecture for groups acting on trees

In my talk the Farrell-Jones Conjecture in algebraic K-theory was discussed. This conjecture proposes a computation of the algebraic K-theory of group rings RG as equivariant homology groups. If the conjecture holds for a group G, then $K_*(RG)$ is in some sense computable in terms of $K_*(RV)$, where V runs over the family of virtually cyclic subgroups of G. For torsion free groups the conjecture implies the vanishing of the Whitehead group.

The result presented in this talk is joint work with Wolfgang Lueck and Holger Reich and assertes that the conjecture holds for groups G that act properly, cocompactly and simplicially on a tree. The proof uses controlled algebra and the (negatively curved) geometry of the tree.

As a corollary of this result and of work on Nilgroups of virtually cyclic groups by Kuku and Tang, Grunewald one obtains rational vanishing results for Waldhausen's Nilgroups appearing in his work on amalgamated free products and HNN-extensions.

Denis Auroux: Fiber sums of Lefschetz fibrations

It is a key problem in 4-manifold topology to understand which smooth compact oriented 4-manifolds carry a symplectic structure (i.e., a non-degenerate closed 2-form). Symplectic 4-manifolds are much more general than complex projective surfaces, but are still a very special class of 4-manifolds. One way to approach symplectic 4-manifolds is via Lefschetz fibrations.

A Lefschetz fibration is a map $f : M^4 \to S^2$ with isolated non-degenerate critical points, near which f behaves like a complex Morse function. Hence, the generic fiber is a smooth closed oriented surface, and the singular fibers present ordinary double point singularities only, obtained by pinching a simple closed loop (the "vanishing cycle") on the regular fiber. A theorem of Gompf states that (almost) every Lefschetz fibration carries a symplectic structure; conversely, Donaldson has shown that, after blowing up a finite set of "base points", every compact symplectic 4-manifold can be presented as a Lefschetz fibration (with a distinguished set of -1-sections).

The topology of a Lefschetz fibration is encoded by its monodromy, which is a morphism from a free group (the fundamental group of the complement of a finite set in S^2) to the mapping class group of the fiber, mapping the standard generators to Dehn twists. Choosing a set of generating loops, we can express the monodromy by a "factorization" of the identity element as a product of positive Dehn twists in the mapping class group. Moreover, the various factorizations corresponding to a same Lefschetz fibration are equivalent up to two operations: global conjugation, and Hurwitz moves. There is therefore a one to one correspondence between isomorphism classes of Lefschetz fibrations, and Hurwitz and conjugation equivalence classes of factorizations in the mapping class group.

The classification of Lefschetz fibrations is well-understood in genus 0 and 1 (classical results of Moishezon and Livne), and in genus 2 in the absence of reducible singular fibers (Siebert and Tian). However, many "exotic" examples have been constructed in higher genus, and the classification there is not understood at all.

A simpler question is that of classification up to stabilization by fiber sums. The main result that one can get is the following. For any genus, there exists a Lefschetz fibration f_g^0 such that, given any two genus g Lefschetz fibrations $f_1 : M_1 \to S^2$ and $f_2 : M_2 \to S^2$ such that (1) M_1 and M_2 have same Euler characteristic and signature, (2) f_1 and f_2 have the same numbers of singular fibers of each type, (3) f_1 and f_2 each admit a section with the same self-intersection, for all large enough n, after fiber summing with n copies of f_g^0 the Lefschetz fibrations f_1 and f_2 become isomorphic.

Using Donaldson's theorem, a corollary is the following "symplectic Wall's theorem": given two compact symplectic 4-manifolds with $[\omega]$ integral and the same values of $(c_1^2, c_2, c_1.\omega, \omega^2)$, they become symplectomorphic after performing on each of them a certain number of blow-ups and fiber sums with some f_g^0.

The proof is almost purely group-theoretic, and involves a study of factorizations in the mapping class group of a surface with one boundary component.

Jacob Lurie: Elliptic Cohomology and Derived Algebraic Geometry

Let E be an elliptic curve over a commutative ring R. If certain mild hypotheses are satisfied by E, then Landweber's exact functor theorem ensures the existence of an essentially unique (elliptic) cohomology theory A such that $A(*) \simeq R$ and $A(\mathbf{CP}^\infty)$ is the ring of functions on the formal completion of the elliptic curve E. In particular, these conditions are satisfied whenever E is classified by an étale map

$$\operatorname{Spec} R \xrightarrow{\phi} \mathcal{M},$$

where $\mathcal{M}$ denotes the moduli stack of elliptic curves; let A_ϕ be the associated cohomology theory.

The assignment

$$\phi \mapsto A_\phi$$

may be viewed as a presheaf of cohomology theories on the moduli stack of elliptic curves. The work of Goerss, Hopkins, and Miller implies that this presheaf of cohomology theories can be refined (in an essentially unique way) to a presheaf of E_∞-ring spectra $\mathcal{O}$ on the moduli stack of elliptic curves. It then makes sense to take the (right-derived functor of) global sections, giving an E_∞-ring spectrum

$$\mathrm{tmf}[\Delta^{-1}] = R\Gamma(\mathcal{M}, \mathcal{O}).$$

A more refined approach (which includes the "point at ∞" on $\mathcal{M}$) yields a spectrum tmf, the spectrum of *topological modular forms*, so named for the existence of a ring homomorphism from $\pi_* \mathrm{tmf}$ to the ring of integral modular forms, which is an isomorphism after inverting 6. The spectrum tmf may be regarded as a universal elliptic cohomology theory, and is a suitable target for "elliptic" invariants such as the Witten genus.

It is natural to think of the presheaf $\mathcal{O}$ as a kind of structure sheaf on the moduli stack $\mathcal{M}$ of elliptic curves. This can be made precise using the language of *derived algebraic geometry*: a generalization of algebraic geometry in which E_∞-ring spectra are allowed to play the role of commutative rings. The pair $(\mathcal{M}, \mathcal{O})$ may naturally be viewed as a Deligne-Mumford stack in the world of derived algebraic geometry, which is a kind of "derived version" of the classical moduli stack of elliptic curves. One may then ask if $(\mathcal{M}, \mathcal{O})$ has some moduli-theoretic significance in derived algebraic geometry; our main result is an affirmative answer to this question.

Given an E_∞-ring spectrum R, there is a natural notion of an *elliptic curve* over R in derived algebraic geometry (which specializes to the usual notion of elliptic curve when R is an ordinary commutative ring). Any elliptic curve E has a formal completion $\hat{E}$; we define an *orientation* of E to be an equivalence $\mathrm{Spf}\, R^{\mathbf{CP}^\infty} \simeq \hat{E}$ of formal groups over R. The main result then asserts that there is a natural homotopy equivalence

$$\{\text{ Oriented Elliptic Curves } E \to \mathrm{Spec}\, R\} \Leftrightarrow \mathrm{Map}(\mathrm{Spec}\, R, (\mathcal{M}, \mathcal{O}));$$

in other words, $(\mathcal{M}, \mathcal{O})$ classifies *oriented* elliptic curves in derived algebraic geometry.

This result, and the accompanying ideas, can be used to shed light on virtually all aspects of the theory of elliptic cohomology.

References

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Jongil Park: Rational blow-downs and smooth 4-manifold

It has been known that most simply connected smooth 4-manifolds with b_2^+ odd and large enough admit infinitely many distinct smooth structures due to the gauge theory, in particular, Seiberg-Witten theory. But we still do not know which smooth 4-manifolds with b_2^+ small have more than one smooth structure. Though it is not known yet whether the most fundamental 4-manifolds such as $S^4, \mathbf{C}P^2$ and $S^2 \times S^2$ admit more than one smooth structure, it has been some progress in last couple of decades.

In the case when $b_2^+ = 1$, S. Donaldson first proved that a Dolgachev surface is not diffeomorphic to $\mathbf{C}P^2 \sharp 9\overline{\mathbf{C}P}^2$ ([D]) and D. Kotschick proved in the late 1980's that the Barlow surface is not diffeomorphic to $\mathbf{C}P^2 \sharp 8\overline{\mathbf{C}P}^2$ ([K]). Recently, I constructed a new simply connected symplectic 4-manifold with $b_2^+ = 1$ and $b_2^- = 7$ ([P1]), and then R. Fintushel, R. Stern, A. Stipsicz and Z. Szabó found many new exotic smooth 4-manifolds with $b_2^+ = 1$ using rational blow-downs and knot surgeries in double node neighborhoods ([FS2], [PSS], [SS1]). So it has been proved up to now that rational surfaces $\mathbf{C}P^2 \sharp n\overline{\mathbf{C}P}^2$ with $n \geq 5$ admit infinitely many distinct smooth structures. Second, in the case when $b_2^+ = 3$, it was also known in the mid 1990's that the $K3$ surface $E(2)$ and the topological 4-manifold $3\mathbf{C}P^2 \sharp n\overline{\mathbf{C}P}^2$ with $n \geq 14$ admit infinitely many distinct smooth structures. And later, the same statement with $n \geq 10$ was also proved. Recently, Stipsicz and Szabó constructed infinitely many distinct smooth structures on $3\mathbf{C}P^2 \sharp 9\overline{\mathbf{C}P}^2$ ([SS2]), and then I proved that the topological 4-manifold $3\mathbf{C}P^2 \sharp 8\overline{\mathbf{C}P}^2$ also admit infinitely many distinct smooth structures ([P2]).

In this talk I would like to survey these recent developments.

References

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Weimen Chen: Pseudo holomorphic curves and finite group actions in dimension 4

As for the abstract of my talk, the main point is to propose to study a class of smooth finite group actions on 4-manifolds, the so-called symplectic symmetries. (These are smooth finite group actions which preserve some symplectic structure of the 4-manifold.) The hope is that the symplectic symmetries will form an interesting and large enough class of smooth finite group actions to study, which on one hand are more tractable than the general smooth actions while on the other hand are more flexible than the holomorphic actions.

From the technical point of view, the equivariant Seiberg-Witten-Taubes theory allows one in principle to detect the fixed-point set structure of a symplectic symmetry by looking at the induced action in a neighborhood of a 2-dimensional, pseudoholomorphic subset. Such information is crucial in studying finite group actions. A key issue is how the regularity of the pseudoholomorphic subset is related to the fixed-point data of the symplectic symmetry. More generally, one can consider the orbifold version of the Seiberg-Witten-Taubes-Gromov theory, which may find applications beyond finite group actions on 4-manifolds.

Shmuel Weinberger: A Sullivan Conjecture for Equivariant Structure Sets

This talk discussed the problem of classifying topologically tame G-manifolds up to equivariant homeomorphism within an equivariant homotopy type. After a quick review of classical surgery and the obstacles one faces in finding an equivariant variant of it, I discussed two key ideas: stratfied surgery, which suffices formally to solve the problem in the isovariant setting, ixed set and then the second idea is purely homotopical and uses categorical ideas (such as the homotopy fixed set and the Goodwillie calculus) to relate isovariance versus equivariance to spaces of Poincare embeddings, and ultimately to ordinary embeddings. Unfortunately, there was not enough time to discuss examples. This talk was based on a combination of results that were joint with Cappell, Klein, and Yan in various combinations.

Martin Bridson: Limit groups: non-positive curvature, logic and group theory

I am interested in exploring the universe of finitely presented groups. In this lecture, I want to focus on the region immediately adjacent and ask what natural class of groups best approximate free groups? Having identified the right class (and we shall see that there really is "a right class"), I want to set about the task of proving that groups in this class enjoy many of the non-trivial properties of free groups. The property that I am particularly interested in is the one that first got me thinking about this area: some years ago, Howie, Miller, Short and I proved that a subgroup of a direct product of n free groups is of type F_n if and only if has a subgroup of finite index that is itself a direct product of (at most n) free groups. Interest in extending this result became more interesting when work of Delzant and Gromov showed that understanding the subdirect products of surface groups is important in addressign the question of which finitely presented groups are fundamental groups of compact Kahler manifolds. The theorem of BHMS extends from free groups to surface groups, but the proof is rather mysterious and one would like a more coherent explanation of why this type of splitting theorem works.

For this and other reasons I want to approximate free groups. In this talk, we looked at Gromov-Hausdorff limits of free groups, limits coming from representations of finitely generated groups into free groups (which in turn comes from looking at algebraic geometry over groups), we also looked at "fully residually free groups" (groups whose balls of arbitrary radius can be injected into a free groups), and we looked at groups whose first order logic is that of a free group (existential and/or universal theory). Remarkably, all approaches lead to the class of "limit groups" with the subclass of "elementarily free groups", these being the groups that have the same universal theory as a non-abelian free group. The hardest parts of this classification are due to Zlil Sela.

I described the beautifully simple structure theory of the groups in this class, the simple classifying spaces, with their metrics of negative and non-positive curvature and graph-of-groups decompositions. I finished by quickly mentioning some of the results that one can prove about this class. The basic message is that the programme of extending from free groups to limit groups non-trivial theorems is working. The most striking example is the splitting theorem for subdirect products of limits groups (proved by Howie and I). Other examples include recent work with my students Wilton and Tweedale in which we prove that elementarily free groups are measure equivalent to free groups. Further examples, proved by Hoiwe and I, include the fact that a non-trivial, finitely generated normal subgroup of a limit group must be of finite index, and having finitely generated H_1 is equivalent to being finitely generated.

Thomas Mark: Ozsváth-Szabó invariants of fiber sums (joint work with Stanislav Jabuka)

Ozsváth-Szabó 4-manifold invariants associate to a closed $Spin_c$ 4-manifold (X, σ) having $b^+(X) \geq 2$ a function $\Phi_{X,\sigma} : \mathbf{A}(X) \to \mathbb{Z}$, where $\mathbf{A}(X)$ is the graded algebra $\Lambda^*(H_1(X)/tors) \otimes \mathbb{Z}[U]$. Here $\mathbf{A}(X)$ is graded such that elements of $H_1(X)$ carry degree 1, while U is of degree 2. The function $\Phi_{X,\sigma}$ is nonzero only on homogeneous elements of degree $d(\sigma) = \frac{1}{4}(c_1^2(\sigma) - 2e(X) - 3\sigma(X))$, where $e(X)$ is the Euler characteristic and $\sigma(X)$ is the signature. Furthermore, there are at most finitely many $Spin_c$ structures σ for which $\Phi_{X,\sigma}$ is nontrivial.

Our goal here is to understand the behavior of these invariants under fiber sum of 4-manifolds. Recall that if $f_i : \Sigma \hookrightarrow X_i$ $(i = 1, 2)$ are embeddings of a closed oriented surface Σ in 4-manifolds X_i such that each embedding has trivial normal bundle, the *fiber sum* $Z = X_1 \#_\Sigma X_2$ of X_1 and X_2 along Σ is defined by removing a neighborhood of $f_i(\Sigma)$ from each of X_1 and X_2 and gluing the resulting manifolds (which have boundary diffeomorphic to $\Sigma \times S^1$) along their boundaries using f_i to identify the Σ factors and conjugation in the S^1 factor. (In general, the resulting manifold depends on the embeddings f_i.) We assume throughout that $[\Sigma] \in H_2(X_i; \mathbb{Z})$ is a primitive nontorsion element.

To simplify the statment of the results, we make the assumption that X_1 and X_2 have (strong) *simple type*, which is to say that the only $Spin_c$ structures σ_i for which $\Phi_{X_i,\sigma_i} \neq 0$ have $d(\sigma_i) = 0$. Furthermore, we will consider only the sum of invariants corresponding to $Spin_c$ structures differing by elements of $H^2(Z; \mathbb{Z})$ dual to *rim tori*: these are tori of the form $\gamma \times S^1$ in $\Sigma \times S^1 \subset X_1 \#_\Sigma X_2$, where γ is a circle on Σ. Specifically, if $\mathbf{R} \subset H^2(Z; \mathbb{Z})$ is the subspace spanned by the Poincaré duals of rim tori, we set $\Phi_{Z,\sigma}^{rim} = \sum_{r \in \mathbf{R}} \Phi_{Z,\sigma+r}$. Implicit in the results below is the fact that the fiber sum of two manifolds of simple type is again of simple type.

Theorem 3.1. Assume that the genus of Σ is $g \geq 2$, and suppose σ_i are $Spin_c$ structures on X_i such that $\langle c_1(\sigma_i), [\Sigma] \rangle = 2k$, with $|k| = g - 1$. Let $\sigma \in Spin^c(Z)$ satisfy $\sigma|_{X_i \setminus \Sigma \times D^2} = \sigma_i|_{X_i \setminus \Sigma \times D^2}$ for $i = 1, 2$. Then

$$\sum_{n \in \mathbb{Z}} \Phi_{Z,\sigma+nPD[\Sigma]}^{rim} T^n = \left(\sum_{n_1 \in \mathbb{Z}} \Phi_{X_1,\sigma_1+n_1PD[\Sigma]} T^{n_1} \right) \left(\sum_{n_2 \in \mathbb{Z}} \Phi_{X_1,\sigma_1+n_2PD[\Sigma]} T^{n_2} \right)$$

as polynomials in the formal variable T. If $0 < |k| < g - 1$, we have

$$\sum_{n \in \mathbb{Z}} \Phi_{Z,\sigma+nPD[\Sigma]}^{rim} T^n = 0.$$

Theorem 3.2. Suppose the genus of Σ is 1, and σ_i are $Spin_c$ structures as above with $\langle c_1(\sigma_i), [\Sigma]\rangle = 0$. Then for any glued $Spin_c$ structure $\sigma \in Spin^c(Z)$ as above, we have

$$\sum_{n\in\mathbb{Z}} \Phi^{rim}_{Z,\sigma+nPD[\Sigma]} T^n = (T^{1/2} - T^{-1/2})^2 \left(\sum_{n_1\in\mathbb{Z}} \Phi_{X_1,\sigma_1+n_1PD[\Sigma]} T^{n_1} \right) \left(\sum_{n_2\in\mathbb{Z}} \Phi_{X_1,\sigma_1+n_2PD[\Sigma]} T^{n_2} \right).$$

These formulae can be used, for example, to compute the Ozsváth-Szabó invariants of elliptic surfaces: the result is in accord with the conjecture that the Ozsváth-Szabó and Seiberg-Witten invariants are identical. We should note, however, that Theorem 1 admits a generalization for manifolds that are not of simple type, for which an analogue in Seiberg-Witten theory is not known.

William Dwyer: Duality in Algebra and Topology

The talk, which represents joint work with John Greenlees and Srikanth Iyengar, discusses the idea of interpreting properties of ordinary commutative rings so that they can be extended to the more general rings that come up in homotopy theory. Among the rings that arise are Eilenberg-MacLane ring spectra, the cochains on a space with coefficients in a commutative ring spectrum, or the chains on a loop space with similar coefficients. It is something of a surprise that differential graded algebras or ring spectra can appear naturally even in purely algebraic settings. One line of reasoning leads to a new homological formula for the injective hull of the residue class field of a local ring; essentially the same formula in another setting gives, for any prime p, the p-summand of the Brown-Comenetz dual of the sphere spectrum. A homotopical interpretation of the notion of Gorenstein ring gives a common way of understanding Gorenstein rings, Poincare duality spaces, and the formal component of Gross-Hopkins duality. The main theme here is that it is interesting to take ring spectra seriously and to try to manipulate them as if they were ordinary rings.

Oleg Viro: Virtual links, their relatives and Khovanov homology

We extend Khovanov homology to links in the projective space. Unexpectedly, full fledged Khovanov homology with integer coefficients are defined only for non-zero homologous links. For zero-homologous links any construction over $\mathbb{Z}$ fails, provided it is based on $1 + 1$ TQFT.

More generally, integer Khovanov homology extends to the case of link in an oriented 3-manifold fibered over a surface with fiber $\mathbb{R}$, if the projection of the link realizes w_1(the surface).

The construction requires a study of several new kinds of virtual links: twisted virtual links (generalizing the usual ones), blunted Gauss diagrams, checkerboard virtual links, etc. Most of them admit not only combinatorial 1-dimensional, but also 3-dimensional interpretation.

Jesper Grodal: p-compact groups and their classification

In this talk I'll announce and explain a proof of the classification of 2-compact groups, joint with K. Andersen, hence completing the classification of p-compact groups at all primes p. A p-compact group, as introduced by Dwyer-Wilkerson, is a homotopy theoretic version of a compact Lie group, but with all its structure concentrated at a single prime p. Our classification states that there is a 1-1-correspondence between connected 2-compact groups and root data over the 2-adic integers (which will be defined in the talk). As a consequence we get the conjecture that every connected 2-compact group is isomorphic to a product of the 2-completion of a compact Lie group and copies of the exotic 2-compact group $DI(4)$, constructed by Dwyer-Wilkerson. The major new input in the proof over the proof at odd primes (due to Andersen-Grodal-Mωller-Viruel) is a thorough analysis of the concept of a root datum for 2-compact groups and its relationship with the maximal torus normalizer. With these tools in place we are able to produce a proof which to a large extent avoids case-by-case considerations.

Peter Oszvath: Heegaard Floer homology for links

I will describe recent joint work with Zoltan Szabo, in which we define an invariant for links, generalizing an earlier construction for knots. The filtered Euler characteristic of this theory is closely related to the multi-variable Alexander polynomial.

Jacob Rasmussen: Differentials on Khovanov-Rozansky homology

I discussed a conjecture (joint with Nathan Dunfield and Sergei Gukov) which describes how the knot Floer homology should be related to the $sl(N)$ knot homologies defined by Khovanov and Rozansky. For each $N > 0$, their construction assigns to a knot K a sequence of bigraded homology groups $H_N(K)$ whose graded Euler characteristic is the $sl(N)$ knot polynomial of K. Work of Gornik suggests that these homology groups should be equipped with a family of differentials $d_n (0 < n < N)$. For each such n, $H_N(K)$ is itself the underlying group of a chain complex with differential d_n. The homology of this chain complex is expected to be $H_n(K)$. This suggests that we should be able to take a limit of the H_N's to obtain a triply graded homology theory with graded Euler characteristic the HOMFLY polynomial of K. The conjecture suggests that this homology should be equipped with anticommuting differentials d_n, not only for $n > 0$ (which would be provided by Gornik's construction) but also for $n \leq 0$ as well. In particular, the homology with respect to d_0 is expected to give the knot Floer homology. In the actual talk, I sketched the construction of Gornik's differentials, formulated the conjecture, and finally, ended by describing a simple class of "thin" knots for which at least part of the conjecture can be seen to hold. (For such knots, the $sl(N)$ homology is determined by the HOMFLY polynomial and signature.) It can be shown that two-bridge knots are thin.

Yongbin Ruan: Twisted K-theory on orbifolds and its stringy product

Wolfgang Lueck: L^2-invariants and their applications

The purpose of this talk is to present recent developments about L2-invariants and their applications to problems in other areas such as topology, group theory, K-theory, geometry and global analysis. It addresses non-experts. We begin with a list of theorems which a priori have nothing to do with L^2-invariants but whose proofs uses L^2-methods. We develope the basic definitions of L^2-Betti numbers and basic tools. Then we mention some important theorems about L^2-Betti numbers and explain in some cases how the theorems in the first list are proved using L^2-methods. Finally we discuss open problems about L^2-invariants.

Stefano Vidussi: Taubes' conjecture and twisted Alexander invariants

It is well-known that the Seiberg-Witten invariants of a 4–manifold provide obstructions to the existence of a symplectic structure. When the 4–manifold is of the form $S^1 \times N$, these obstructions can be described in terms of the Alexander polynomial of N. C. Taubes formulated the conjecture that, if $S^1 \times N$ is symplectic, then N fibers over the circle. P. Kronheimer studied the case where N is obtained as 0–surgery along a knot $K \subset S^3$ and showed that the aforementioned constraints on the Alexander polynomial Δ_N give evidence to Taubes' conjecture, i.e. Δ_N must be monic and its degree must coincide with the genus of the knot. Still, these conditions are short of characterizing fibered knots. In this talk we discuss how to extend these ideas to the case of a general 3–manifold and how these conclusions can be strengthened by taking into account the twisted Alexander polynomials associated to an epimorphism of $\pi_1(N)$ into a finite group. This way we get new evidence to Taubes' conjecture and, practically, new obstructions to the existence of symplectic structures on $S^1 \times N$, even in the case of 0–surgery along a knot. As an application! o! f these results we show that if N is the 0–surgery along the pretzel knot $(5, -3, 5)$, a case that cannot be decided with the use of the Alexander polynomial, $S^1 \times N$ is not symplectic: this answers a question of Kronheimer. In a similar way, we show that Taubes' conjecture holds for knots up to 12 crossings. (*Joint work with Stefan Friedl of Rice University*)

Karen Vogtmann: Tethers and homology stabillity

I defined what it means for a sequence G_n of groups to have homology stability and pointed out some important consequences of homology stability (Quillen's finite generation of K-groups and the Madsen-Weiss computation of the stable homology of mapping class groups). I then described the method introduced by Quillen in the 1970's for proving homology stability, by looking at the equivariant homology spectral sequence of the group G_n acting on a highly-connected complex X_n, with simplex stabilizers G_{n-k-1}. I then showed how to find a suitable complex for $G_n = Aut(F_n)$, giving an action which makes the spectral sequence argument work in the simplest possibly way. This complex involves finding "enveloping spheres" for coconnected sphere systems in a 3-manifold with fundamental group F_n. The complex can alternatively be described by "tethering" the spheres to the basepoint, from both sides. This idea of tethering turns out to be useful in other contexts giving, for instance, a simplified proofs of homology stability for braid groups (first proved by Arnold in 1970), for mapping class groups of orientable surfaces (Harer 1980's), and symmetric automorphism groups of free groups.

Andras Stipsicz: Contact Ozsvath–Szabo invariants and tight structures on 3-manifolds

Recall that an oriented 2-plane field ξ on an oriented 3-manifold Y is a *contact structure* if ξ can be given as the kernel of a 1-form α satisfying $\alpha \wedge d\alpha > 0$. A contact structure is *overtwisted* if there is an embedded 2-disk D in Y such that ξ is tangent to D along ∂D; otherwise ξ is *tight*. It turns out that overtwisted structures are determined by the homotopy type of Y, while the tight structures capture important geometric information of the underlying 3-manifold.

Contact structures can be constructed by performing surgeries along *legendrian* links, that is, along links for which the tangent vectors are in ξ. The tightness of (Y, ξ) can be detected by computing its contact Ozsváth-Szabó invariant $c(Y, \xi)$, which is an element of the Heegaard-Floer homology group $\widehat{HF}(-Y)$. It is known that $c(Y, \xi)$ is zero if (Y, ξ) is overtwisted and is nonzero if (Y, ξ) is the boundary of a Stein domain.

We have studied the existence and classification problem of tight contact structure on a special class of 3-manifolds, called *small Seifert fibered* 3-manifolds. Y is small Seifert fibered if it admits a Seifert fibration over S^2 with 3 singular fibers. As an application of Donaldson's famous diagonalizability theorem for definite 4-manifolds, we find a tight contact structure which is not the boundary of any symplectic 4-manifold.

Walter Neumann: Graph manifolds and singularities

The topology of a complex singularity is determined by its 3-manifold link. The topologies are known but until recently it was rarely possible to give explicit analytic descriptions for any but the simplest topology. The "splice singularities" of Jonathan Wahl and the author do this for many rational homology spheres. The talk will describe a nice characterization of these singularities that we have (finally) proved.

4. List of problems

Problem 1. (Adem) A finite group G acts freely on a finite complex X with the homotopy type of a product of k spheres if and only if every elementary abelian subgroup in G is of rank at most k.

Problem 2. (Akbulut) Formulate and prove a Resolution Theorem for polynomial maps. This is the only missing issue to topologically characterizing real algebraic sets, i.e. to determine when a given space is a real algebraic set.

Problem 3. (Bartels) Borel conjecture. Let M and N be closed aspherical manifolds of dimension ≥ 5 that are homotopy equivalent. Then there is a homeomorphism $f : M - - > N$ that is homotop to the given homotopy equivalence.

Problem 4. (Boden) The smooth Poincaré Conjecture in dimensions three and four.

Problem 5. (Bridson) Construct counterexamples to the Andrew's Curtis Conjecture: Let $F = F_n$ be the free group of a finite rank n with a fixed set $X = \{x_1, \ldots, x_n\}$ of free generators. Is the normal closure of a set $Y = \{y_1, \ldots, y_n\}$ equals F if and only if Y is Andrews-Curtis equivalent to X, which means one can get from X to Y by a sequence of Nielsen transformations together with conjugations by elements of F?

Problem 6. (Collin) If a non-trivial Dehn surgery on a knot K in S^3 has cyclic fundamental group, must K be fibered?

Problem 7. (Edwards) The Hilbert-Smith Conjecture: If G is a compact subgroup of the homeomorphism group of a topological mannifold, then G is a Lie group.

Problem 8. (Grodal) Find a topological proof of the classification of finite simple groups.

Problem 9. (Hambleton) Formulate a local to global principal for smooth manifolds.

Problem 10. (Kirby) Is a slice knot a ribbon knot?

Problem 11. (Kreck) Is a random smooth manifold asymmetric, i.e. has no non-trivial finite group action?

Problem 12. (Lueck) The Atiyah Conjecture: Denote by $N(G)$ the group von Neumann algebra associated to G viewed as a ring (not taking the topology into account). For a $N(G)-$module M let $\dim_{N(G)}(M) \in [0.\infty]$ be its dimension. Let $\frac{1}{FIN(G)} \subset \mathbf{Q}$ be the additive abelian subgroup of $\mathbf{Q}$ generated by the inverses $|H|^{-1}$ of the orders $|H|$ of finite subgroups H of G. Notice that $\frac{1}{FIN(G)}\mathbf{Z}$ agrees with $\mathbf{Z}$ if and only if G is torsion-free. Consider a ring A with $\mathbf{Z} \subset A \subset C$. The Atiyah Conjecture for A and G says that for each finitely presented $AG-$ module M we have $\dim_{N(G)}(N(G) \otimes_{AG} M) \in \frac{1}{FIN(G)}\mathbf{Z}$.

Problem 13. (Lurie) Let G be a group acting on a set X. Suppose that the action of G is simply 3-transitive on X (that is, given any two triples (x,y,z) and (x',y',z') of distinct points in X, there is a unique g in G such that (gx,gy,gz) = (x',y',z')). Suppose furthermore that every element g in G which exchanges two distinct points (so that (gx,gy) = (y,x)) has order 2. Does there exist a commutative field k such that the action of G on X can be identified with $PGL_2(k)$ acting on the projective line over k?

Problem 14. (Mark) Does every simply connected symplectic 4-manifold X satisfy $c_1^2(X) \leq 9\chi_h(X)$? Here $\chi_h(X) = \frac{1}{4}(sign(X) + e(X))$ where $sign(X)$ is the signature of the intersection form and $e(X)$ is the Euler characteristic.

Problem 15. (Mrowka-Ozsváth) Find a proof of the existence of uncountably many exotic smooth structures on $\mathbf{R}^4$ without using instantons, possibly using Seiberg-Witten or Heegaard Floer homology.

Problem 16. (Mrowka We have learned starting with the work of Furuta that subtle information can be obtained from refining the Seiberg-Witten invariants from homology classes in the suitable configuration space to a stable homotopy class of map. To what extent can a similar story be told for the Donaldson invariants and the Gromov invariants?

Problem 17. (Neumann) Lehmer Conjecture: Let $M_1(P)$ denote the Mahler measure for a univariate integer polynomial $P(x)$. Suppose that is $P(x)$ not a product of cyclotomic polynomials. Lehmer conjectured that $M_1(P) \geq M_1(1 - x + x^3 - x^4 + x^5 - x^6 + x^- x^9 + x^1 0)$. Here $M_1(P) = \exp[\int_0^1 \ln |P(e^{2\pi it})|dt]$.

Problem 18. (Park) Does there exist an exotic smooth structure on the complex projective plane $\mathbf{C}$P?

Problem 19. (Pederson) The Arf/Kervaire Invariant One Problem: Do there exist framed manifolds with Kervaire invariant one?

Problem 20. (Ranicki) Extend the algebraic surgery model for high-dimensional topological manifolds to dimensions 3 and 4. While at it, use the model to obtain combinatorial formulae for the Pontrjagin classes!

Problem 21. (Reich) Farrell-Jones conjecture. For a torsion free group Γ the so-called assembly map $A : H_n(B\Gamma; \mathcal{K}^{-\infty}(\mathbf{Z})) \to K_n(\mathbf{Z}\Gamma)$ is an isomorphism for all $n \in \mathbf{Z}$.

Problem 22. (Stern) Is every topological $n-$manifold, $n \geq 5$, a simplicial complex?

Problem 23. (Stolz) What is the geometric interpretation of elliptic cohomology and what is its relationship to conformal field theory

Problem 24. (Teichner) The $A - B$ slice problem.If $B^4 = A \cup B$ is a decomposition of the 4-ball into two smooth submanifolds, such that the intersection with S^3 is a thickening of the Hopf link, determine which side (A or B) is strong. The definition of strong must be invariant under Bing doubling (and thus the obvious homological definition does not work). If there is such a definition then the topological surgery and s-cobordism theorems are false (for free fundamental groups) in dimension 4.

Problem 25. (Vidussi) Does there exist a closed smooth 4-dimensional manifold with only finitely many exotic smooth structures?

Problem 26. (Vogtman) Using Kontsevich's identification of the homology of the Lie algebra l_∞ with the cohomology of $\mathrm{Out}(F_r)$, Morita defined a sequence of 4k-dimensional classes μ_k in the unstable rational homology of $\mathrm{Out}(F_{2k+2})$. Are theses Morita classes trivial in $H^*(\mathrm{OutF_g})$?

Problem 27. (Wahl) Get a hold on diffeomorphisms of $3-$manifolds.

Problem 28. (Weinberger) What does a random manifold mean? See problem of Kreck. The main point is that most manifolds we consider, e.g. have group actions, are not random. For example a random graph with valence less than or equal to three has no symmetries.

5. Comments by some participants

Adem:

I enjoyed the meeting at Banff, I'm glad to hear that you will reapply.

Auroux:

Thanks for putting together such a great conference! I think it was a great idea to have such a broad topology conference. It's definitely useful and can help keep the topology community united. The talks were great, and almost all speakers made a very good effort to keep things elementary.

One suggestion, though: at this meeting, some 3-/4-manifold specialists were confused during homotopy theory talks, and vice-versa. It may be useful in the future to have a series of remedial talks on the first day, planned once the main topics become clear – for this meeting, it would have been useful to have maybe a 90-minute crash-course on homotopy theory for low-dimensional topologists (introducing ring spectra, p-completions, and other monsters, giving concrete examples to make them less scary) and a 90-minute crash-course on low-dimensional topology for homotopy theorists (maybe brief overviews of SW and Ozsvath-Szabo theories ?)

Bartels:

I enjoyed the meeting very much. Most talks were very good and speakers made an (successful) effort to adress the general audiance. Given the number of talks on 4-dimensional manifolds I think it would have been a good idea to have one survey talk on 4-dimensional manifolds to set the stage for the specialliced talks. The talk of Bridson presented a class of groups that seems to be interesting to study in relation with the Farrell-Jones conjecture.

Bridson:

I think taht the idea of sustaining communication between the broad community of "topologists" is a fruitful one, and that this meeting provides an excellent example of the benefits. For the most part, speakers made a real effort to communicate to the whole audience and as a result I have a much better idea of what is happening in adjacent subfields of topology, and who I should ask which questions to. This was a meeting quite different to the highly specialised ones that happen with such great regularity these days. I think that it has played a valuable role, and I hope that it may be repeated on a regular (bi-annual?) basis.

Chen:

Thanks for organizing such a wonderful workshop. I particularly likes this format of having a diverse range of topics.

Dwyer:

I really enjoyed the meeting, and especially the chance to hear something of what's going on across the board in topology.

Grodal:

I think the conference went great!

Kirby:

The conference went very well, thanks to the organizers and thanks to the speakers who with few exceptions did an excellent job of making their specialty accessible to everyone else. This is not easy, and is particularly hard when the audience covers all of topology. But it is vital that we have such conferences and such talks or else topology will just break up into its subareas which no longer interact.

Kleiner:

Thanks very much for organizing the conference and for the invitation to participate. I enjoyed the conference overall. The only way it could have been improved, from my own standpoint, would have been if a few of the lectures were pitched to a more general audience, closer to a colloquium style. However, I suspect that most of the other participants were better versed in homotopy theory and the fine points of surgery, so my comments simply reflect the fact that I'm more of a geometer/geometric group theorist than a topologist.

Lueck:

In my opinion this conference shall be in the format as this year, very broad and not specialized. There are enough special conferences and I like to get an impression to hear from leading representatives what happens in other fields.

In my opinion this is a very good meeting. I have no complaints about the organization or the facilities, they are excellent.

Mark:

I found the Banff workshop to be very informative and a broadening experience. Conferences such as this one, involving a range of mathematicians in various subdisciplines, are too rare. The workshop opened my eyes to problems and techniques in topology of which I was previously unaware, which is extremely valuable.

Thanks for your efforts with organization, and I hope the application for the next workshop goes well.

Mrowka:

I very much enjoyed the meeting and think that more of the same would be great for topology.

Park:

As it usually happens in any conference which puts several areas together, I hardly catch a theme of topics without introducing the contents of topics enough. So, although I am sometimes bored, what do you think that one hour talk is better than 45 minutes talk for speakers and audiences? Except this, I really like this type of conference!

Ranicki:

Thanks again to the organizers for inviting me to a most enjoyable conference. The only negative comment I have is that the organizers did not have the imagination to follow the Oberwolfach tradition (possibly initiated by Matthias himself) of distributing the abstracts of all the talks proposed, and there was no opportunity of presenting posters (e.g. in the room set aside for BIRS across the corridor from the lecture room). Also, the speakers should have been asked to provide reading lists for their talks, so that members of the audience could follow up the talks if so inclined. Thanks again, and good luck with your 2007 proposal

Rasmussen:

This is my second time at BIRS, and my impression of the place has not changed very much from the last visit. I think it is simply the best conference venue for encouraging collaborative work and interaction that I have been to. The setup (breakfast room, everyone staying in the same place, meals together) is great for encouraging interaction between people who might not otherwise get together. I had a lot of fun going to talks from other areas, but I can't say that I got ideas useful for my own reseach from them, or that I was in a position to make meaningful suggestions about them. Despite this criticism, I should say that I really had a great and productive time this week. Thanks to you and the other organizers for putting this thing together.

Stipsicz

It was a great conference, I enjoyed it a lot,

Vidussi:

Some comments on the conference. I definitely enjoyed the idea of having a meeting with people that work in different areas of topology. It is very difficult and time consuming to keep track of the developments of various areas only by reading papers. A conference's talk, instead, gives an easier access to main results and ideas, and allows interaction with a specialist. If there is an improvement that I can suggest, this would be to stress out in advance that the talks are meant for a "general" audience. (You pointed that out at the beginning of the conference but some - including possibly myself - did not fully comply with this.) Personally, my interest in some of the topics discussed at the conference grew; for example, I am currently reading a review paper of W. Lueck on L^2 invariants, and trying to understand if this may have applications in my research.

Third, the schedule and number of talks was perfect, and Banff is a great place for a conference.

Finally, I am very grateful to the organizers for inviting me and giving me the opportunity to give a talk.

Vogtman:

I thought it was great, I learned a lot about what's happening in the rest of topology. Thanks!!!

6. List of participants

Adem, Alejandro; University of British Columbia
Akbulut, Selman; Michigan State University
Auroux, Denis; Massachusetts Institute of Technology
Baldridge, Scott; Louisiana State University
Bartels, Arthur; Universitt Münster
Bauer, Kristine; University of Calgary
Boden, Hans; McMaster University
Bridson, Martin; R. Imperial College London
Chen, Weimin; University of Massachusetts at Amherst
Collin, Olivier; Universit du Qubec Montral (UQAM)
Dwyer, William; Notre Dame University
Edwards, Bob; University of California Los Angeles
Grodal, Jesper; University of Chicago
Hambleton, Ian; McMaster University
Kirby, Robion; University of California - Berkeley Kleiner, Bruce; University of Michigan
Kreck, Matthias; University of Heidelberg
Lee, Yi-Jen; Purdue University
Lück, Wolfgang Universitat Münster
Lurie, Jacob; Harvard University
Mark, Thomas; Southeast Louisiana University
Matic, Gordana; University of Georgia
Mrowka, Tom; Massachusetts Institute of Technology
Neumann, Walter; Columbia University
Ozsvath, Peter; Columbia University
Park, Jongil; Seoul National University
Pedersen, Erik; SUNY Binghamton
Ranicki, Andrew; University of Edinburgh
Rasmussen, Jacob; Princeton University
Reich, Holger; Universitt Münster
Ruan, Yongbin; University of Wisconsin
Stern, Ronald; University of California, Irvine
Stipsicz, Andras; Hungarian Academy of Sciences

Stolz, Stephan; University of Notre Dame
Teichner, Peter; University of California - Berkeley
Vidussi, Stefano; Kansas State University
Viro, Oleg; Uppsala universitet
Vogtmann, Karen; Cornell University
Wahl, Nathalie; University of Chicago
Weinberger, Shmuel; University of Chicago