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Abstracts

BIRS workshop on *Rigidity, Group Actions, and Dynamics*
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Uri Bader, *Boundary representations for negatively curved groups — irreducibility and rigidity.*

Abstract. I will report about a recent joint work with Roman Muchnik. Let M be a compact negatively curved manifold, G be its fundamental group and X its universal cover. Denote the boundary of X by B. B is endowed with the Paterson-Sullivan measure. The associated unitary representation $L^2(B)$ is called a boundary unitary representation. Fixing G, but changing the metric on M, we get a different boundary (given by a different measure on the same topological boundary), and a different boundary representation. We will explain the setting and indicate the proof of

Theorem 1: The boundary representations are irreducible.

Theorem 2: Two boundary representations are equivalent if and only if the associated marked length spectrums are proportional.

The marked length spectrum is the assignment associate to a free homotopy class of closed loops in M the length of a shortest geodesic in it. The proof of the theorem is based on the mixing property of the geodesic flow on M.

Emmanuel Breuillard, *A uniform Tits alternative.*

Abstract. I will describe a joint work with Tsachik Gelander.

Using the theory of arithmetic groups (e.g. Borel–Harish-Chandra and Kazhdan–Margulis theorems) we proved the following generalizations of Tits and Eskin-Mozes-Oh theorems:

Theorem 1: If Γ is a finitely generated non-virtually solvable linear group, then there is an integer $m = m(\Gamma)$ such that for every symmetric generating set S of Γ, the m'th ball S^m contains 2 generators of a non-abelian free group.

Theorem 2: For any d there is $m = m(d)$ such that if k is a local field and $S \subset GL_d(k)$ generates a discrete subgroup, then either $[x_1^m, [x_2^m, [..., x_m^m, x_{m+1}^m]...] = 1$ for any $x_1, ..., x_{m+1} \in S$ or S^m contains 2 generators of a free semigroup.

These results have many applications, e.g.: 1) Non-amenable linear groups are uniformly non-amenable. 2) The growth of discrete groups is uniform for a given dimension (i.e. independent not just of the generating set but also of the group).

Serge Cantat, *Zimmer's conjecture for holomorphic actions on Kähler manifolds.*

Abstract. Let G be a connected simple Lie group and Γ a lattice in G. Let M be a compact Kähler manifold. I will describe the proof and the main questions related to the following result: if the dimension of M is strictly less than the rank of G, any morphism from Γ to the group of diffeomorphisms of M has finite image.

S.G. Dani, *Asymptotic behaviour under iterated random linear transformations.*

Abstract. Let V be a finite dimensional real vector space, ν be finite measure on V, and μ be a probability measure on $\mathrm{GL}(V)$. We discuss conditions under which the sequence of measures $\mu^i * \nu$ (where μ^i denotes the i th convolution power of μ) to converge, in the vague topology, to the zero measure on V; the latter condition signifies dissipation of the sequence of measures, from V. The related issue of asymptotic behaviour of convolution powers of probability measures on $\mathrm{GL}(V)$ will also be discussed.

Alex Eskin, *Quasi-isometries of solvable Lie groups and their lattices.*

Abstract. I will describe a new method for studying quasi-isometries of solvable Lie groups. This method yields a proof of a long-standing conjecture concerning the group of quasi-isometries of the three dimensional, simply connected, unimodular Lie group Sol. This conjecture is known to imply that any finitely generated group quasi-isometric to Sol is (up to standard issues of finite index) a lattice in Sol.

Benson Farb, *Isometries, rigidity, and universal covers.*

Abstract. In this talk I'll describe joint work with Shmuel Weinberger. We have found all closed, aspherical Riemannian manifolds M whose universal cover has indiscrete isometry group. One sample application is the theorem that any such M with word-hyperbolic fundamental group must be isometric to a negatively curved, locally symmetric manifold. Another application is the classification of all contractible Riemannian manifolds covering both compact and (noncompact, complete) finite volume manifolds. There are also applications to the Hopf Conjecture, the Zimmer program, a new proof of Kazhdan's Conjecture (Frankel's Theorem) in complex geometry, etc. Ideas in the proof come from Lie theory, transformation groups, harmonic maps, and large-scale geometry. I may also discuss our recent extension of parts of this to the non-aspherical case, and some pathological examples we've discovered.

Renato Feres, *The Dynamical Liouville Problem.*

Abstract. Starting from a countable group Γ, a probability measure μ on the group, and a compact Γ-space X, one defines a discrete Laplacian acting on continuous functions of X. The question we propose to study is this: for what kinds of actions does the Liouville property for harmonic functions hold? In other words, is a harmonic function of X constant on orbits of Γ. We describe a number of results answering the question in special situations both positively and negatively. We also indicate some connections of the problem with the general rigidity program.

David Fisher, *First cohomology and local rigidity.*

Abstract. In 1964, Weil gave a criterion for local rigidity of a homomorphism from a finitely generated group Γ to a finite dimensional Lie group G in terms of cohomology of Γ with coefficients in the Lie algebra of G. Recently I have generalized Weil's result to a class of homomorphisms into certain infinite dimensional Lie groups, namely groups of diffeomorphism compact manifolds. This gives a criterion for local rigidity of group actions which implies local rigidity of: (1) all isometric actions of groups with property (T), (2) all isometric actions of irreducible lattices in products of simple Lie groups and certain more general locally compact groups and (3) a certain class of isometric actions of a certain class of cocompact lattices in $\mathrm{SU}(1, n)$.

John Franks, *Distortion elements in group actions on surfaces.*

Abstract. If G is a finitely generated group with generators $\{g_1, \ldots, g_j\}$, then an infinite order element f in G is called a distortion element of G provided $|f^n|/n$ grows sublinearly, where $|f^n|$ is the word length of f^n in the generators. Let S be a closed orientable surface and let $\mathrm{Diff}(S)_0$ denote the identity component of the group of C^1 diffeomorphisms of S. Our main result shows that if S has genus at least two and if f is a distortion element in some finitely generated subgroup of $\mathrm{Diff}(S)_0$, then every f-invariant Borel probability measure has support in the fixed point set of f. Related results are proved for $S = T^2$ or S^2.

For μ a Borel probability measure on S, denote the group of C^1 diffeomorphisms that preserve μ by $\mathrm{Diff}_\mu(S)_0$. We give several applications of our main result showing that certain groups, including a large class of higher rank lattices, admit no homomorphisms to $\mathrm{Diff}_{mu}(S)$ with infinite image.

The results described in this talk represent joint work with Michael Handel.

Alex Furman, *Cocycle superrigidity for general products of groups with semisimple targets.*

Abstract. This is a joint work (in progress) with Nicolas Monod. In the framework of Zimmer's program, we study actions of general products of groups, and lattices in products, by volume preserving diffeos on a compact manifold. We obtaining strong restrictions on such actions under rather mild assumptions, such as strong ergodicity of each of the factors.

The main tool is a new superrigidity result for measurable cocycles of the form $c : G \times X \to H$ where G is a product of at least two lc groups, or a (cocompact) lattice in such a product, and H is a semi-simple group, where $log\|c(g,-)\|$ is integrable.

Hillel Furstenberg, *Some questions on boundaries of groups.*

Abstract. We will discuss a number of questions that have never been satisfactorily answered regarding various notions of group boundaries. one of these is identification of the Poisson bounday in a number of natural situations. Other questions relate to the connection between boundaries and irreducible affine representations.

Alex Gorodnik, *Ergodic theory of semisimple lattices.*

Abstract. We extend the ergodic theorems for semisimple Lie groups (due to Margulis, Nevo, Stein) to lattices. Namely, consider a measure-preserving action of a lattice (in semisimple Lie group) on a probability measure space. For such actions, we prove strong maximal inequality, mean and pointwise ergodic theorems. For lattices satisfying property (T), we get ergodic theorems with exponential rate of convergence. Our methods can be also applied to some infinite volume homogeneous spaces. This is joint work with Amos Nevo.

Michael Handel, *Fixed points for abelian actions on $\mathbb{R}^2$ and S^2.*

Abstract. We prove that if F is a finitely generated free abelian group of orientation preserving C^1 diffeomorphisms of $\mathbb{R}^2$ and if F has a compact invariant set then there is a global fixed point for F.

We also prove that if F is a finitely generated free abelian group of orientation preserving C^1 diffeomorphisms of S^2 then there is a global fixed point for a subgroup of index at most two.

TJ Hitchman, *Super-rigidity for cocycles via Bochner methods.*

Abstract. I will discuss a theorem concerning existence and rigidity of harmonic maps from a locally symmetric space X, associated to a semisimple Lie groups G with no compact factors and property (T), into certain infinite dimensional spaces. This result is implies a version of Zimmer's cocycle superrigidity theorem for "L^2" cocycles and is new when G has any simple factors isomorphic to $SP(1,n)$ or F_4^{-20}. (Partial results in these cases have been obtained by Corlette-Zimmer and Korevaar-Schoen.) This result has myriad applications to rigidity of group actions.

A key point in the proof is to prevent harmonic maps from escaping to infinity. We show that escape to infinity is only possible if there are non-trivial classes in $H^1(\Gamma, V)$ where $\Gamma < G$ is a lattice and V is an infinite dimensional non-unitary Γ module. We prove a new Bochner-type estimate which implies that $H^1(\Gamma, V) = 0$.

Dmitry Kleinbock, *Why unipotent flows are even more non-divergent than they appeared to be, and why it is good for number theory.*

Elon Lindenstrauss, *Compact orbits of the diagonal group and the Linnik principle.*

Abstract. I will talk about distribution properties of compact orbits of the diagonal group on the space of lattices, and in particular will show how the classification of invariant measures with positive entropy can be used to study these zero entropy measures. This is a joint work with M. Einsiedler, P. Michel, and A. Venkatesh.

Hee Oh, *Equidistribution of rational matrices in their conjugacy classes.*

Abstract. Let G be a connected real noncompact simple algebraic group defined over $\mathbb{Q}$ and Γ a cocompact congruence subgroup. For any homogeneous manifold $x_0H \subset \Gamma\backslash G$ of finite volume, and $a \in G(\mathbb{Q})$, we show that the Hecke orbit $T_a(x_0H)$ is equidistributed on $\Gamma\backslash G$ as $\deg(a) \to \infty$, provided H is a non-compact commutative reductive subgroup of G. As a corollary, we generalize the equidistribution result of Hecke points to homogeneous spaces G/H. As a concrete application, we describe the equidistribution result in the rational matrices with a given characteristic polynomial. This is a joint work with Yves Benoist.

Leonid Polterovich, *Quasi-morphisms in symplectic topology.*

Abstract. A quasi-morphism of a group is "a homomorphism up to a bounded error". This notion was introduced in the 80-ies in connection with bounded cohomology theory (Gromov, Brooks). It proved to be a useful tool in geometry, topology and dynamics. In the present talk I focus on quasi-morphisms on groups of symplectic diffeomorphisms. They come from Floer theory – the cornerstone of modern symplectic topology. I discuss a link to the theory of quasi-states - a recently emerged branch of functional analysis originated in quantum mechanics. The talk is based on joint works math.SG/0205247, math.SG/0410338 with M.Entov and math.SG/0307011 with P.Biran and M.Entov.

Sorin Popa, *Strong rigidity for* II_1 *factors and orbit equivalence relations.*

Abstract. I will present some recent rigidity results for free, ergodic, measure preserving actions of groups on the probabilty space, as well as for the orbit equivalence (OE) relations and group measure space II_1 factors they entail. A key role is played by the notion of *relative property* (T) for subalgebras and OE relations, defined by using *deformations* by completely positive maps, in analogy with the relative property (T) for a group-subgroup (Kazhdan-Margulis). One such result, of "strong rigidity" type, shows that any isomorphism betwwen two group measure space factors, one coming from an action with "good deformation" properties (e.g. a Bernoulli action) of an infinite conjugacy class (ICC) group, the other from an arbitrary action of an ICC group having an infinite normal subgroups with the relative property (T), is implemented by an isomorphism of the probability spaces that conjugates the two actions, modulo perturbation by inner automorphisms and automorphisms coming from characters of the groups. The following "OE superrigidity" result is a consequence: If a Bernoulli action of a property (T) ICC group is orbit equivalent to an arbitrary action of an ICC group, then the two actions are conjugate. The proofs use "deformation/rigidity" and "intertwining" techniques, for which the operator algebra framework is crucial. Through these methods one can also obtain Bass-Serre type OE results and explicit calculations of fundamental groups and outer automorphism groups for many group measure space factors and OE relations.

M.S. Raghunathan, *The congruence subgroup problem.*

Abstract. This is a short survey of the progress on the congruence subgroup problem since the 1960s when the first major results on the integral unimodular groups appeared. It is aimed at the non-specialists and avoids technical details.

Omri Sarig, *Ergodic theory for horocycle flow on periodic surfaces* (joint with F. Ledrappier).

Abstract. The horocycle flow of a non-geometrically finite hyperbolic surface may have many different (infinite) ergodic invariant Radon measures.

We classify these measures for the class of periodic surfaces: regular covers of surfaces of finite volume. It turns out that in this case there are as many measures as there are positive eigenfunctions for the Laplacian of the surface.

In the special case of $\mathbb{Z}^d$-covers of compact surfaces, we can show that only one of those measures satisfies a generalized pointwise ergodic theorem. In other words: only one measure is "relevant".

Ben Schmidt, *Simplicial volumes of closed locally symmetric spaces of noncompact type.*

Abstract. In this joint work with Jean-Francois Lafont, we confirm Gromov's conjecture that all closed locally symmetric spaces of noncompact type have positive simplicial volume. Our proof is based on Thurston's method of obtaining a lower bound for simplicial volume in terms of a uniform upper bound for the volumes of "straightened" simplices. Technically, our "straightening" and estimates rely on the barycenter method of Besson, Courtois, and Gallot and its further development in the higher rank setting by Connell and Farb.

George Tomanov, *Values of decomposable forms at S-integer points and closed tori orbits on homogeneous spaces.*

Abstract. Let $\mathbb{G}$ be a reductive algebraic group defined over a number field K and let S be a finite set of non-equivalent valuations of K containing the archimedean ones. Let $G = \prod_{v\in S} \mathbb{G}(K_v)$ and Γ be an S-arithmetic subgroup of G. Let $R \subset S$ and $T_R = \prod_{v\in R} T_v$ where T_v is a sub-torus of $\mathbb{G}(K_v)$ containing a maximal K_v-split torus. We prove that if G/Γ admits a closed T_R-orbit then $R = S$ or R is a singleton. In addition, the closed T_R-orbits are always "standard"; this generalizes a joint result with Barak Weiss (Duke Math.J. , vol.119, 2003, pp.367-392). When $\#S > 1$ it turns out that for $R = S$ there are no divergent orbits and for $\#R = 1$ all closed orbits are divergent. As an application, we prove that if a collection of decomposable homogeneous forms $f_v \in K_v[x_1, \dots, x_n], v \in S$, takes discrete values at $\mathcal{O}^n$, where $\mathcal{O}$ is the ring of S-integers of K, then there exists an homogeneous form $g \in \mathcal{O}[x_1, \dots, x_n]$ such that $f_v = \alpha_v g$, $\alpha_v \in K_v^*$, for all $v \in S$.

Barak Weiss, *The Mahler question for quadratic differentials.*

Abstract. In the 1930's Mahler conjectured that almost every point on a certain curve in $\mathbb{R}^d$ is not very well approximable. This was solved by Sprindzhuk in the 1960's and spawned many related questions and results in which one studies intersections of "geometric objects" like manifolds or self-similar sets with "diophantine sets". A major development was the use of homogenous space flows by Kleinbock and Margulis to answer several unsolved questions of this type. Motivated by the many emerging parallels between homogeneous flows and flows on the moduli spaces of quadratic differentials, we formulate analogous conjectures and prove many of them. For example we prove: suppose σ is an irreducible permutation on d symbols, Δ is the $d-1$ dimensional simplex parametrizing all interval exchange transformations with permutation σ, and ℓ is an analytic curve in Δ not contained in a proper affine subspace, then a.e. interval exchange on ℓ is uniquely ergodic. Joint work with Yair Minsky.