



# Diophantine approximation and analytic number theory November 20 – 25, 2004



## MEALS

Breakfast (Continental): 7:00 – 9:00 am, 2<sup>nd</sup> floor lounge, Corbett Hall, Sunday – Thursday

\*Lunch (Buffet): 11:30 am – 1:30 pm, Donald Cameron Hall, Sunday – Thursday

\*Dinner (Buffet): 5:30 – 7:30 pm, Donald Cameron Hall, Saturday – Wednesday

Coffee Breaks: As per daily schedule, 2<sup>nd</sup> floor lounge, Corbett Hall

\*Please remember to scan your meal card at the host/hostess station in the dining room for each lunch and dinner.

## MEETING ROOMS

All lectures will be held in **Max Bell 159** (Max Bell Building accessible by bridge on 2<sup>nd</sup> floor of Corbett Hall). LCD projector, overhead projectors and blackboards are available for presentations. Hours: 6 am – 12 midnight. Please note that the meeting space designated for BIRS is the lower level of Max Bell, Rooms 155-159. Please respect that all other space has been contracted to other Banff Centre guests, including any Food and Beverage in those areas.

## SCHEDULE

### Sunday, November 21

7:00-9:00 Breakfast

9:15-9:30 Welcome and Introduction to BIRS, Max Bell 159

9:30-10:20 Paula Cohen "Some Hyperbolic Equidistribution Problems" :

Abstract : We generalize to Hilbert modular varieties of arbitrary dimension the work of W. Duke on the equidistribution of Heegner points. Similar results have been obtained by Zhang, Clozel, Ullmo using other methods. We use techniques going back to Siegel and Maass, in the spirit of Duke's work. We also prove by these methods vanishing results for limits of cuspidal Weyl sums associated with analogous problems for the Siegel upper half plane of degree 2.

10:30-10:50 Coffee, 2<sup>nd</sup> floor lounge, Corbett Hall

10:50-11:40 Michael Filaseta "Applications of Pade approximations of  $(1-z)^k$  to Number Theory"

Abstract : We survey old and new applications of function theoretic approximations to a variety of number theoretic problems.

11:40-2:00 Lunch break

2:00-2:50 Akshay Venkatesh "Descent bounds for rational points on varieties"

Abstract : (joint with J. Ellenberg) We will present some new bounds for the number of rational points on curves and some higher-dimensional varieties.

2:50-3:10 Coffee, 2<sup>nd</sup> floor lounge, Corbett Hall

3:10-4:00 Kevin Ford " Fractional parts of imaginary parts of zeta zeros"

Abstract : We describe some irregularities in the distribution of the fractional parts of the imaginary parts of the zeros of the Riemann zeta function and other L-functions.

## Monday, November 22

7:00-9:00 Breakfast

9:30-10:20 Pietro Corvaja "Integral points on surfaces and higher dimensional varieties"

Abstract : It is well known that an affine algebraic curve with at least three points at infinity has only finitely many integral points. The purpose is to generalize this result to higher dimensional varieties. In case of affine surfaces, we have nearly optimal results stating that if the divisor at infinity satisfies certain geometric conditions then the integral points are not Zariski dense. These conditions involve intersection products of the component at infinity, and are satisfied if the divisor at infinity is sufficiently reducible. We also obtain some degeneracy results for integral points on varieties of arbitrary dimension, still under suitable conditions on the divisor at infinity. These results are obtained in joint work with U. Zannier.

10:30-10:50 Coffee.

10:50-11:40 Jordan Ellenberg " $X^2+Y^4=Z^n$ "

Abstract : We bound the degree of rational isogenies of  $Q$ -curves over quadratic number fields, by combining Mazur's formal immersion method with an analytic argument showing that Jacobians of certain twisted modular curves admit quotients with Mordell-Weil rank 0. As a consequence, we show that the generalized Fermat equation  $A^4 + B^2 = C^p$  has no nontrivial primitive solutions for  $p$  sufficiently large. In order to make "sufficiently large" not too large, we use a refinement of a result of Duke.

11:40-1:00 Lunch break.

1:00-2:00 Tour of Banff Centre; meet in the 2<sup>nd</sup> floor lounge

2:00-2:50 Jan-Hendrik Evertse "Linear equations with unknowns from a multiplicative group in a function field" (joint work with Umberto Zannier)

Abstract: Let  $k$  be an algebraically closed field of characteristic 0 and  $K/k$  an extension of transcendence degree at least 1. Let  $G$  be a subgroup of  $(K^*)^n$  such that  $(k^*)^n$  is contained in  $G$  and  $G/(k^*)^n$  has rank  $r$ . We proved the following result: The equation  $a_1x_1 + \dots + a_nx_n = 1$  has (up to multiplication with  $(k^*)^n$ ) at most  $R$  non-degenerate solutions in  $(x_1, \dots, x_n)$  from  $G$ , where  $R = \binom{2}{3}^r + \binom{2}{4}^r + \dots + \binom{2}{n+1}^r$  and  $a \binom{2}{b} = a(a-1)\dots(a-b+1)/b!$

2:50-3:10 Coffee

3:10-4:00 Preda Mihailescu "Kummerian results on cyclotomic norm equations"

Abstract : The general frame - equation which we consider is:  $(x^p + y^p)/(x+y) = p^e z^q$  with  $(x,y,z) = 1$  and  $e = 0$  if  $(p, x+y) = 1$  and  $e = 1$  otherwise, like usual. The talk is an extensive talk on homework kummerian investigation of these equations and covers a series of new results including:

1. Theorem: for  $p$  and  $q$  there are two cases.

A.  $(xy, p) = 1$  and  $(x \pm y, p) = 1$ . Then there are no solutions unless

$2^{p-1} = 3^{p-1} = 1 \pmod{p^2}$ , and the irregularity index  $i(p) > \sqrt{p}$  (Not so new, just adapted)

B. The remaining cases. Then, if  $y = 1$ , there are no solutions if Vandiver holds for  $p$ .

2. Proof of Catalan can be done without use of Thaine's theorem (not so spectacular)

3. Some general bounds and conditions for the case  $y = 1$  and  $p < q$ .

4. The rest of the talk covers some (surprising ?) results on  $x^p + y^p = z^q$ .

## Tuesday, November 23

7-9 am Breakfast

9:00-9:40 Jason Lucier "Polynomials and intersective sets"

Abstract : A set of integers  $H$  is called intersective if given any subset  $A$  of the positive integers with positive upper density, there exist distinct elements  $a$  and  $a'$  in  $A$  such that their difference  $a-a'$  is in  $H$ . If  $H$  is the value set of a polynomial  $h$  then  $H$  is intersective provided  $h$  has a root modulo every integer. We present a quantitative version of this result.

9:40-10:00 Coffee

10:00-10:40 Stephen Choi "Mean value theorem of Dirichlet polynomials and an application"

Abstract : Let  $F(x, \chi)$  be a certain Dirichlet polynomial. We will discuss a mean value estimate of a particular type and its application to the linear equation  $a_1 p_1 + a_2 p_2 + a_3 p_3 = b$  in prime variables  $p_i$ .

Free Afternoon

## Wednesday, November 24

7-9 am Breakfast

9:30-10:20 Yann Bugeaud "Complexity of algebraic numbers"

Abstract : Let  $x$  be an irrational real number. We give a combinatorial criterion on the expansion of  $x$  in a given integer base to ensure that  $x$  is transcendental. It follows that irrational automatic numbers are transcendental, confirming a conjecture of Loxton and van der Poorten. Furthermore, we get that the expansion of any irrational algebraic number in a given base cannot have sublinear complexity.

10:30-10:50 Coffee.

10:50-11:40 Chantal David "Vanishing of L-functions of elliptic curves over number fields"

Abstract : Let  $E$  be an elliptic curve of  $\mathbb{Q}$ , with L-function  $L_E(s)$ . For any primitive Dirichlet character  $\chi$ , let  $L_E(s, \chi)$  be the L-function of  $E$  twisted by  $\chi$ . In this talk, we use random matrix theory to study vanishing of these twisted L-functions at the central value  $s=1$ . In particular, random matrix theory predicts that there are infinitely many characters of order 3 and 5 such that  $L(1, \chi)=0$ , but that for any fixed prime  $k > 5$ , there are only finitely many characters  $\chi$  of order  $k$  such that  $L(1, \chi)=0$ . With the Birch and Swinnerton-Dyer Conjecture, those conjectures can be restated to predict the number of cyclic extensions  $K/\mathbb{Q}$  of prime degree such that  $E$  acquires new rank over  $K$ .

11:40-2:00 Lunch break.

3:00-3:50 Valentin Blomer "Subconvexity bounds for automorphic L-functions"

Abstract: We discuss different methods for obtaining subconvexity bounds for L-functions, and present results for various families of automorphic L-functions.

3:50 -- 4:10 : Coffee

4:10 -- 5:00 : Harald Helfgott "Power-free values of polynomials and integer points on irrational curves"

Abstract : Let  $f \in \mathbb{Z}[x]$  be a polynomial of degree  $r \geq 3$  without roots of multiplicity  $r$  or  $(r-1)$ . Assume that  $f(x) \not\equiv 0 \pmod{p^{r-1}}$  has a solution in  $(\mathbb{Z}/p^{r-1})^*$  for every  $p$ . Erdős conjectured that  $f(p)$  is then free of  $(r-1)$ th powers for infinitely many primes  $p$ . I prove this for every  $f$  a root of which generates its splitting field, and for some other  $f$  as well.

The proof takes advantage of a certain kind of repulsion among integer points in curves of positive genus. Probabilistic arguments also come into play.

## Thursday, November 25

7:00-9:00 Breakfast

No Talks scheduled. Informal discussions encouraged.

11:30-1:30 Lunch