BIRS Workshop

Nonlinear Dynamics of Thin Films and Fluid Interfaces

November 29th – December 4th, 2003

MEETING ROOMS

All lectures are held in the main lecture hall, Max Bell 159. *Please note that the meeting space designated for BIRS is the lower level of Max Bell, Rooms 155-159. Please respect that all other space has been contracted to other Banff Centre guests, including any Food and Beverage in those areas.*

SCHEDULE

	Sat. 29th	**Sun. 30th**	**Mon. 1st**	**Tues. 2nd**	**Weds. 3rd**	**Thurs. 4th**
7.00-9.00	—	Continental Breakfast, 2nd floor lounge, Corbett Hall				
9.00-9.45	—	Wilson	Schwartz	Cummings	Ben Amar	Howard
9.45-10.30	—	Schatz	Kondic	Bernoff	Giacomelli	Münch
10:30-11:00	—	Coffee Break, 2nd floor lounge, Corbett Hall				
11.00-11.45	—	Tilley	Diez	Wieland	Grün	Wagner
11.45-12.30	—	Golovin	Grigoriev	Edmonstone	Novick-Cohen	—
12.30-12:45	—	Group Photo[1]	—	—	—	—
12:30-13:30	—	Buffet Lunch, Donald Cameron Hall				
14.00-14.45	—	Smolka	Bowen	Guided Tour[2]	Behringer	—
14.45-15.30	—	Fontelos	King	—	Shearer	
15:30-16:00	—	Coffee Break, 2nd floor lounge, Corbett Hall (except Tues.)				—
16:00-16.45	—	Bush	Pugh	—	Miksis	—
16:45-17.30	—	—	Slepcev	—	Wetton	—
17:30-19:30	Buffet Dinner, Donald Cameron Hall					—
19:30-20.15	—	Lathrop	—	—	—	—

MEALS

Breakfast (Continental): 7:00 - 9:00 am, 2nd floor lounge, Corbett Hall, Sunday - Thursday
*Lunch (Buffet): 11:30 am - 1:30 pm, Donald Cameron Hall, Sunday - Thursday
*Dinner (Buffet): 5:30 - 7:30 pm, Donald Cameron Hall, Saturday - Wednesday
Coffee Breaks: As per daily schedule, 2nd floor lounge, Corbett Hall
***Please remember to scan your meal card at the host/hostess station in the dining room for each lunch and dinner.**

[1] A group photo will be taken on Sunday at 12:30 pm, directly after the last lecture of the morning. Please meet on the front steps of Corbett Hall.

[2] A free guided tour of The Banff Centre is offered to all participants and their guests on Tuesday starting at 2:00 pm. The tour takes approximately 1 hour. Please meet in the 2nd floor lounge in Corbett Hall.

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1 Sunday

- **Stephen K. Wilson**, University of Strathclyde (Department of Mathematics)

  Rivulet Flow

  Rivulet flows occur in a wide range of physical contexts ranging from industrial coating processes to geophysical flows. In this talk we review recent developments in the analysis of flow of thin rivulets incorporating a variety of physically important effects including temperature-dependent surface tension and viscosity, non-planar substrates, perfectly wetting fluids, and non-Newtonian rheology. In each case we shall attempt to show how a judicious combination of asymptotic and numerical methods can bring new insight into the situations investigated.

- **Michael Schatz**, Georgia Tech. (Department of Physics)

  Transient Amplification and Contact Line Instability

  The role played by transient disturbances in flow instabilities (including the transition to turbulence) is poorly understood for many important problems in hydrodynamics. We present experimental and theoretical results on the temperature-induced surface-tension-driven spreading of a thin liquid film on a horizontal solid substrate. Perturbations with well-defined spatial and temporal characteristics are applied by optically heating the film prior to instability onset, and the subsequent evolution of rivulets arising from contact line instability are measured using image time series. Transient amplification rates are extracted from the experimental data and compared quantitatively to the predictions of generalized stability theory that accounts for the non-normal character of the operator that governs the evolution of small disturbances.

- **Burt S. Tilley**, Franklin W. Olin College of Engineering

  On undercompressive shocks in countercurrent two-layer flows

  We consider the flow of two incompressible immiscible viscous fluids in an inclined channel. From a lubrication approximation based on the ratio of the channel height to the downstream disturbance wavelength, we derive a nonlinear system of evolution equations that govern the interfacial shape separating the two fluids and the leading-order pressure. This system, which is based on fluids with disparate density and dynamic viscosity ratios, includes the effects of viscosity stratification, inertia, shear, and capillarity. Experimentally, the boundary conditions for this effective system is unclear, but we consider initially two ways to drive the flow: either by fixing the volumetric flow rate of the gas phase or by fixing the total pressure drop over a downstream length of the channel. The latter forcing results in a single evolution equation whose dynamics depends nonlocally on the interfacial shape. From both of theses driven systems, admissible criteria for Lax shocks, undercompressive shocks and rarefaction waves are determined. Interestingly, these criteria, through a numerical verification, do not depend significantly on the inertial effects within the more dense layer. This classification is useful in understanding the dynamics in a more realistically driven system, which allows for the far-field interfacial height to vary in time. Connections to these results and the phenomena found in these systems in applications are discussed. This is joint work with Tetyana M. Segin, and Lou Kondic.

- **Alexander A. Golovin**, Northwestern University (Engineering Science and Applied Mathematics)

The role of wetting interactions in the formation of quantum dots in thin solid films

The nonlinear dynamics of self-organization of quantum dots in thin solid films in the presence of wetting interactions between the film and the substrate is studied. Two main mechanisms for the formation of quantum dots are considered: epitaxial stress due to lattice misfit and thermodynamic instability due to large anisotropy of the film surface-tension. It is shown that in both cases the presence of wetting interactions between the film and the substrate qualitatively changes the film instability spectrum, can stop the coarsening and lead to the possibility of the formation of spatially-regular patterns (arrays of quantum dots) with different symmetries.

- **Linda B. Smolka**, Duke University (Department of Mathematics)

Exact solution for the extensional flow of a viscoelastic fluid

We consider the free boundary problem of an axisymmetric, cylindrical liquid filament stretching in an extensional flow through a quiescent fluid of negligible viscosity. Our approach provides a systematic framework in which the known Newtonian solution can be generalized to various viscoelastic constitutive models using a condition for the existence of cylindrical solutions. By assuming a power series expansion for the stress, we obtain an analytic solution that describes the filament motion for a viscoelastic filament. We examine this solution in the weakly and strongly viscoelastic limits, as well as in the transient and long time limits. Comparisons of this exact solution with experimental measurements using a viscoelastic polymer solution show strong quantitative agreement. As $t \to \infty$, both the solution and the observations scale in the Newtonian limit. This transition from viscoelastic to Newtonian scaling provides insight as to how the molecular dynamics of the polymer couple to the filament's motion. This is joint work with Thomas Witelski (Duke), and Andrew Belmonte and Diane Henderson (Penn State).

- **Marco A. Fontelos**, Universidad Rey Juan Carlos

Self-similarity and drop dynamics in the beads-on-string configuration for polymeric liquids

Due to Rayleigh's instability, a jet of Newtonian fluid breaks-up into drops. It is a well known fact that the addition of a minute concentration of polymers to a Newtonian fluid is able to delay the break-up process in several orders of magnitude. Before break-up, the jet evolves into a so-called beads-on-string structure where drops of the polymer solution are connected by long and thin threads whose diameter decays exponentially. There exist some self-similar features of the evolution process as well as a rich dynamics of the drops along the beads-on-string configuration including drop merging, migration, coalescence and oscillation.

These issues have been investigated using asymptotic and numerical analysis on a mathematical model deduced under the slenderness assumption. Namely, assuming that the characteristic diameter of the fluid tube is much smaller than the characteristic length of the perturbation in the axial direction. The rheological constitutive law we use is Oldroyd-B. We will show how the model is able to explain much of the experimental evidence and discuss on its limitations and the way to solve them through the use of nonlinear constitutive laws of Giesekus and FENE type.

- **John Bush**, MIT (Department of Mathematics)

  New structures in free surface flows

  We report striking new structures revealed through exploratory experimental investigations of free surface flows, and outline accompanying theoretical developments. Particular attention is given to fluid chains and fishbones generated by the collision of two laminar jets, and to polygonal hydraulic jumps.

- **Daniel P. Lathrop**, University of Maryland (Department of Physics Institute for Research in Electronics and Applied Physics Institute for Physical Sciences and Technology University of Maryland)

  Highly nonlinear fluid flows in water, glycerin, or sodium

  This talk will be a broadside of strongly nonlinear fluid flows: jets, cones, drips, shears, and vortices. How can advances in our understanding of the mathematics of Partial Differential Equations help us to parse these violent phenomena? This connection is relatively easy to make for isolated structures - but connecting it to self-affine fields observed in turbulence is a challenge. Some suggestions to how applied mathematics and experiments can jointly advance this will be discussed, while giving a cross-section of experimental results on divers flows.

2 Monday

- **Leonard W. Schwartz**, University of Delaware (Department of Mechanical Engineering)

  Modeling of Thin-Layer Fluid Mechanics: "I'd Rather Watch the Paint Dry"

  We have developed theoretical and numerical techniques for predicting the slow free-surface flow of liquids on solid substrates with a variety of driving mechanisms. Such flows have many industrial applications. An obvious application is the flow behavior of paint and other liquids.

  Using the lubrication approximation, the three-dimensional unsteady motion of both Newtonian and Generalized Newtonian liquid layers and isolated droplets is simulated. We introduce a so-called "disjoining pressure term" into the flow equations. This allows interfacial energetics to influence the flow via the specification of equilibrium contact angles. This is a particularly attractive technique for simulating wetting and spreading behavior since apparent contact lines appear naturally in a time-dependent simulation without the need for explicit front tracking.

  Topics include the approximate treatment of non-planar substrates, the development of "fingers," evaporation/drying modeling with developed Marangoni flow effects, and the role of surfactants. Some unpublished and highly-speculative results relevant to biological cell division will also be included.

- **Lou Kondic**, NJIT (Department of Mathematics)

  Instabilities in the flow of thin films on heterogeneous surfaces

  I will present computational and experimental results about instability development in the gravity driven flow of thin fluid films on heterogeneous surfaces. In particular, we concentrate on the dynamics of the fluid fronts, i.e., on the contact line. We show that heterogeneity of the solid surface can have significant effect on the flow dynamics. Since the effect of heterogeneity often competes with the basic instability mechanism that would occur even on homogeneous surfaces, the result is an elaborate interplay of various instability mechanisms. The computational results presented here outline both the flow on surfaces perturbed by regular patterns, and on surfaces perturbed by irregular, noise-like perturbations. We relate these computational results to the pattern formation process in the experiments of gravity driven flow down an incline. This is joint work with Javier Diez.

- **Javier Diez**, Instituto de Física Arroyo Seco, Universidad Nacional del Centro de la Provincia de Buenos Aires

  Spreading of a thin two-dimensional strip of fluid on a vertical plane: Experiments and modeling

  We report experimental results on the spreading of a constant volume of silicon oil (PDMS) down a vertical glass plate. The initial condition has the shape of a long strip and is generated from a horizontal fluid filament with typical cross sections between 3×10^{-4} cm^2 and 15×10^{-4} cm^2. To characterize the early stages of the spreading we employ an optical technique based on the use of an anamorphic lens. By comparing these measurements with one dimensional numerical simulations, we are able to estimate the thickness of the precursor film needed in the calculations. For later stages, we use another technique based on the Schlieren method that captures the bi-dimensional pattern of the transversal film instability. By means of these techniques we determine the film thickness profiles, and the evolution of the moving contact line, including its shape and Fourier spectra. We also develop a linear model for the instability based on an adiabatic approximation, which extends the results of the stability analysis of the constant flux problem to the constant volume case. This is joint work with A. G. González, J. Gomba, R. Gratton, and L. Kondic.

- **Roman Grigoriev**, Georgia Tech. (Department of Physics)

Using feedback control to study the dynamics of driven contact lines

The ability to dynamically apply well controlled perturbations gives us unprecedented capabilities to study the dynamics of spreading thin liquid films. In this talk I will describe a novel experimental technique allowing optical manipulation of a broad class of capillary-dominated flows simultaneously at multiple spatial locations. I will also discuss modeling the contact line dynamics in the presence of feedback and present some new results concerning transient dynamics and asymptotic stability of spreading films driven by temperature gradients.

- **Mark Bowen**, University of Nottingham (School of Mathematical Sciences)

Large time asymptotics for the one-dimensional thin film equation

We investigate the large time behaviour of solutions to the thin film equation

$$h_t = -(h^n h_{xxx})_x$$

by application of asymptotic analysis and self-similar solutions. Of particular interest is whether singularities of the form $h \to 0$ can appear in the solution (corresponding to rupture of the film). The analysis is supported by numerical simulations. This is joint work with John King.

- **John R. King**, University of Nottingham (School of Mathematical Sciences)

A sixth-order thin-film equation

We seek to describe the properties of compactly-supported solutions to a prototype sixth-order degenerate parabolic equation describing the motion of a thin viscous fluid film overlain by an elastic plate. While for sufficiently small n its properties are very similar to those of the corresponding (capillary-driven) fourth-order equation, for larger n new phenomena manifest themselves.

- **Mary Pugh**, University of Toronto (Department of Mathematics)

Blowing-up exact solutions of long-wave unstable thin film equations

Long-wave unstable thin film equations

$$h_t = -(h^n h_{xxx})_x - (h^m h_x)_x$$

are a fourth-order analogue of the the semilinear heat equation. A "reaction" term destabilizes a "diffusion" term, allowing for a competition between effects. Bertozzi and Pugh proved that if $n = 1$ then the initial value problem can yield solutions that blow up in finite time in the critical ($m = 3$) and super-critical ($m > 3$) cases. Witelski, Bertozzi, and Bernoff have done extensive computations and asymptotics on the $n = 1$ case suggesting this blow-up is self-similar. We consider the critical ($m = n + 2$) case and present exact solutions with compact support and zero contact angles that blow up in a self-similar manner. These solutions exist if $0 < n < 3/2$ and cannot exist if $n \geq 3/2$. This is joint work with Dejan Slepcev (University of Toronto).

- **Dejan Slepcev**, University of Toronto (Department of Mathematics)

Linear stability of self-similar solutions of a critical case unstable thin-film equation

A critical case long-wave unstable thin-film equation

$$h_t = -(hh_{xxx})_x - (h^3 h_x)_x$$

displays a rich dynamical behaviour. Its solutions can spread indefinitely, converge to a steady state or blow up in finite time. Witelski, Bernoff and Bertozzi have shown numerically that the equation possesses a family of linearly stable source-type (spreading) self-similar solutions and families of both linearly stable ("single-bump") and linearly unstable ("multi-bump") blowup self-similar solutions. The existence of these solutions was proven by Beretta (source-type) and Pugh and I (blowup). I will show how to carry out linear stability analysis of self-similar solutions using the gradient flow structure of the equation.

3 Tuesday

- **Linda Cummings**, University of Nottingham (School of Mathematical Sciences)

  Thin film models for nematic liquid crystals

  Nematic Liquid Crystals (NLCs) are made up of rod-like molecules, which have a local preferred direction of orientation, described by a (unit) vector field, $\mathbf{n}$, imparting a strongly anisotropic character to the fluid.

  We use lubrication theory scalings on the flow equations for NLCs to derive a simplified model describing the evolution of a thin film of NLC, of height $z = h(x, y, t)$, flowing under gravity on a rigid substrate $z = 0$.

  Such a thin film model for NLCs was first considered by Ben Amar and Cummings, for the case in which the director field is specified at both $z = 0$ and $z = h$, boundary conditions known as 'strong anchoring'. Here, that work is extended to the case in of 'weak anchoring' at one or both surfaces; that is, the NLC molecules have a preferred alignment (given by a specified vector $\mathbf{p}$) at each surface, which is modelled by an anisotropic surface energy, which takes a minimum value when $\mathbf{n}$ is aligned with $\mathbf{p}$. Lubrication-type equations governing the evolution of the film height h are derived, and stability properties of the various models are considered.

- **Andrew J. Bernoff**, Harvey Mudd College (Department of Mathematics)

  Domain Relaxation in Polymer Monolayers

  I will report on both experimental and theoretical studies of a polymer monolayer (with a typical width of a 100 microns) on the surface of a quiescent subfluid. When stretched (by a transient applied stagnation point flow), these domains take the form of a "bola" or dumb-bell, consisting of two roughly circular reservoirs connected by a thin tether. This shape will then relax very slowly to the minimum energy configuration of a circular domain. The tether is never observed to rupture, even when the it is more than a hundred times as long as it is thin. We model these experiments by taking previous descriptions of the full hydrodynamics (by McConnell and Stone, Lubensky and Goldstein, etc.), identifying the dominant effects via dimensional analysis, and reducing the system to a more tractable form. We derive relaxation rates for perturbations of a uniform strip and a circular patch. Lubrication theory for the evolution of the tether yields the thin film equation $h_t = -(h^n h_{xxx})_x$ with $n = 2$. We show that this evolution equation appears not to manifest rupture, in agreement with the experiments. Finally, we speculate on which physical properties of the system (such as line tension) can be deduced by comparison of the theory to the experiment. This is joint work with James Alexander, J. Adin Mann III (Case Western Reserve University) and Elizabeth K. Mann (Kent State University).

- **Sandra Wieland**, University of Bonn (Department of Mathematics)

  Modeling and mathematical analysis of contaminated thin films

  We consider Navier-Stokes equations for an incompressible fluid coupled with a convection-diffusion-equation for surfactant molecules on the free surface. The lubrication approximation leads to a coupled system of parabolic equations, consisting of a degenerate fourth order equation for the film height and a second order equation for the surfactant concentration. A technical proof based on energy estimates shows the existence of global weak solutions which in addition fulfill an integral inequality (entropy inequality) to ensure positivity of the film height on sets of positive measure.

- **Barry D. Edmonstone**, Imperial College London (Department of Chemical Engineering and Chemical Technology)

Flow of Surfactant Laden Films down an Incline

A theoretical model is formulated to describe the dynamics of surfactant laden thin film flow down an inclined plane. Use of lubrication theory yields a coupled pair of partial differential equations for the film height and surfactant monolayer concentration. Base flow models for constant flux and fixed volume fluid configurations are examined over various inclination angles, with propagating wave fronts achieved for both configurations. Application of a transverse perturbation analysis highlights significant instabilities at the leading edge of the film and within the surfactant monolayer concentration. Expanding the model to a third dimension results in fingering formation consistent with these predicted instabilities. Several key features of the flow, including fingering formation, are shown to change due to inclusion of the surfactant. This is joint work with O. K. Matar.

4 Wednesday

- **Martine Ben Amar**, Ecole Normale Superieure (De'partement de Physique)

  Transition of a moving contact line from smooth to angular

  Abstract: We consider the motion of a small droplet sliding under gravity down an inclined plane. Experimentally it has been observed that such a droplet will develop an angular point in the contact line at its rear if its velocity is sufficiently high (if the plane is inclined sufficiently steeply to the horizontal). The angular point first appears at some critical value U_c and the discontinuity increases monotonically from zero at $U = U_c$. We use a simple mathematical model based on lubrication theory which allows to find an exact solution in the subcritical regime and predict the formation of an angle in the contact line above the threshold. This analysis suggests a further bifurcation also observed in the experiment (cf. L. Limat et al.) This is joint work with L. Cummings (University of Nottingham) and Y. Pomeau.

- **Lorenzo Giacomelli**, University of Rome (Dipartimento di Metodi e Modelli Matematici per le Scienze Applicate)

  Rigorous lubrication approximation — a model case

  Liquid films are characterized by a separation of length scales: the region occupied by the liquid is thin and gently sloping, and the evolution is slow. The lubrication approximation capitalizes on this separation of scales to substantially reduce the complexity of the appropriate bulk fluid model, thus allowing for a more efficient study of the free surface. In the talk I will present a rigorous justification of lubrication approximation, obtained together with Felix Otto, for a liquid thin film which spreads on a solid, driven by surface tension. We consider a two–dimensional Darcy liquid in the "complete wetting" regime as simple model case. Of particular interest to us is the codimension–two free boundary, i. e. the triple junctions where solid, liquid and vapor meet. In the considered regime of complete wetting, the contact angle vanishes throughout the evolution. We show in particular that this contact angle condition is preserved in the lubrication approximation.

- **Amy Novick-Cohen**, Technion-Israel Institute of Technology (Department of Mathematics)

  Dynamics under near wetting conditions

  We explore the dynamics of interfacial motion in the proximity of a wetting transition. We do this both in the framework of a grain boundary coupled to an exterior surface which is evolving under the influence of surface diffusion, and for antiphase boundaries coupled to interphase boundaries within the interior of a material specimen. Some of the results are rather surprising. The connection with liquid wetting is explained.

- **Günther Grün**, University of Bonn (Institute for Applied Mathematics)

  Droplet spreading under weak slippage: new interpolation inequalities and some optimal results on contact line propagation

  In this talk, we will discuss recent results for the multi-dimensional thin film equation under the assumption of a Navier-slip condition or under the assumption of even weaker slip conditions. New interpolation inequalities will be presented which serve as a multi-dimensional substitute for the well-known Bernis inequalities. These interpolation estimates are the key ingredient to control third order derivatives of appropriate powers of the solution and to prove existence for initial-boundary value problems as well as for the Cauchy problem, in both cases using a much simplified solution concept. As a further consequence, a number of qualitative results can be established, e.g. optimal propagation estimates or sufficient criteria for the occurrence of a waiting time phenomenon.

- **Robert P. Behringer**, Duke University (Department of Physics)

  Instabilities and Shocks in Marangoni-Driven Thin Films

  We describe experiments of Marangoni and gravity driven thin film flow. The basic geometry of these studies consists of a flat smooth surface (silicon in the experiments) that is tilted relative to the horizontal direction. A thin film is driven up the surface from a reservoir of liquid (silicon oil) by a temperature gradient, which in turn induces a surface tension gradient (Marangoni effect). The studies have focused on two issues: shock structures and transverse instabilities of the moving contact line. Regarding the first of these, we have experimentally documented a novel undercompressive shock that forms at the trailing edge of the film. This shock may instead take the form of a rarefaction wave depending on the pinch-off film thickness at the meniscus. The experimental studies are supported by numerical calculations using the lubrication approximation. The second issue concerns the stability of the leading edge of the film to transverse perturbations. For appropriate film thicknesses, linear theory predicts a band of wavenumbers from $q = 0$ to $q = q_c$ such that perturbations within the band will grow. We have carried out both experimental and theoretical investigations of what occurs near q_c. In order to investigate this regime experimentally, we use a novel wavelength selection process involving a swept laser beam to select a perturbation wavelength of choice. Numerical simulations indicate that the bifurcation at q_c is subcritical to a branch of marginally stable finite-amplitude fingering profiles. Experiments support this prediction. This is joint work with J.-M. Sur, A. Bertozzi and T. Witelski.

- **Michael Shearer**, North Carolina State University (Department of Mathematics)

  The connection between driven thin films and new concepts in hyperbolic conservation laws

  The lubrication approximation for the motion of a thin liquid film over a solid surface leads to a fourth order PDE, in which surface tension regularizes the underlying scalar conservation law. Numerical simulations of the PDE capture striking effects, including the appearance of undercompressive waves, represented by sharp planar fronts that are unexpectedly stable, and multiple wave structures for the same upstream and downstream conditions. In this talk, I describe joint work with Philippe LeFloch in which the theoretical devices of kinetics and nucleation are used to capture the new waves for the underlying conservation law. In particular, nucleation allows for multiple attractors in the long-time behavior of solutions, a property not encountered before in the theory of conservation laws.

- **Michael Miksis**, Northwestern University (Engineering Science and Applied Mathematics)

  Dynamics of a Bubble in an Inclined Channel

  Here we present level set calculations for the dynamics of a bubble rising in an inclined channel. Experiments show that at low Reynolds numbers, the bubble rise velocity will increase with increasing angle of inclination of the channel, but at large Reynolds number Re, the rise velocity has a maximum velocity at a finite angle of inclination. Experiments have also shown that it is possible for the bubbles to bounce along the upper wall as it rises. We will present numerical computations with capture these observations. Steady bubble profiles are obtained at small Reynolds number, but with increasing Re, the profiles can become time periodic. The numerical method allows for rupture onto the upper wall of the channel. In this case, a contact line is formed and we investigate the dynamics of this bubble as it moves up the channel.

- **Brian Wetton**, University of British Columbia (Department of Mathematics)

  Generalized Stefan Velocities

  We consider elliptic problems in which the domain is separated into two regions by a free boundary, on which mixed linear Dirichlet-Neumann conditions are specified. Led by the classical Stefan condition applied to change of phase models, we consider idealised numerical methods which evolve the interface by a trial method that uses the error in one of the boundary conditions as the normal velocity of the boundary. Using linear perturbation analysis of simple cases, we show exactly which interfacial conditions lead to well-posed problems and which choices of velocities lead to convergent methods. Moreover, some velocities lead to methods having superior numerical properties. Analysis of numerics representing the free boundary by a cubic spline fit is presented, followed by an example computation. This is joint work with Roger Donaldson.

5 Thursday

- **Peter Howard**, Texas A&M University (Department of Mathematics)

Nonlinear stability of viscous shock waves in equations with high order smoothing only

We consider the question of nonlinear stability for viscous shock waves arising in conservation laws with high order viscosity only. Over the last ten years or so the pointwise Green's function approach toward analyzing stability has proven effective in numerous cases, including (recently) viscous shock waves arising in multidimensional systems (Zumbrun, Hoff, Serre), traveling wave solutions arising in hyperbolic relaxation systems (Zumbrun, Mascia), and boundary layers in one-dimensional systems of conservation laws (Grenier, Rousset). Though clearly quite general, the method has certain seemingly inherent limitations based on the interplay between spatial dimension, order of the nonlinear perturbation, and the lowest order of smoothing. I will present a brief overview of the method and discuss some equations that arise as limiting cases in the study of thin films and lie at the border of the theory's limitations. In particular, I will discuss what the theory can and cannot (at this time?) assert about the nonlinear stability of viscous shock waves arising as solutions to these equations.

- **Andreas Münch**, Humboldt-Universität Berlin

Reverse undercompressive fronts in thin film flow

In this talk, we consider the evolution of a thin liquid film that spreads up an inclined silicon wafer due to a thermally induced Marangoni shear stress, with the parallel component of gravity as a counteracting force. For vertical to moderately inclined wafers, the film profile $h(x,t)$ (x is the direction up the wafer) is governed by the following evolution equation,

$$h_t + (h^2 - h^3)_x = -(h^3 \kappa_x)_x, \qquad \text{with} \quad \kappa = h_{xx}, \tag{1}$$

where the fourth order term on the right hand side represents the influence of surface tension of the liquid/air interface.

The unusual dynamical behaviour of the thin film observed by Cazabat and coworkers could be explained by the formation of a double wave structure with a leading undercompressive and a trailing Lax front. Recently, we revisited the dip coating experiments by Ludviksson-Lightfoot, where the authors observe a pinch-off instability at the meniscus. We could extend the the lubrication model into the meniscus region by replacing κ with the corresponding nonlinear expression for the surface curvature, $\kappa = h_{xx}/(1 + h_x^2)^{3/2}$. We found that the pinch-off process leads to the formation of a new type of wave structure. It consists of a *reverse* undercompressive front that propagates in the direction of a thicker state, and a trailing rarefaction wave. Moreover, we found that this reverse undercompressive front is unstable with respect to spanwise perturbations, and, in numerical simulation of the nonlinear model gives rise to fingers. Our predictions were confirmed in recent experiments focusing on the new double wave structure (Sur, Bertozzi and Behringer, *PRL* 12, 2003).

- **Barbara Wagner**, Weierstrass Institute

Destabilisation of dewetting fronts in the presence of slip

In physical experiments, dewetting fronts that form where a polymer film ruptures tend to destabilize in the spanwise direction: The ridge forms thicker and thinner regions. The thicker regions tend to stay behind, leading to bulges that stick out of the front. The appearance of this instability has been associated with the occurrence of slip at the film/substrate interface. We investigate a lubrication model (with a Van-der-Waals potential the drives the dewetting) via a linear stability analysis and show how the presence of slip affects the evolution of the base state and the evolution of the perturbation.