Prospects for the String Axiverse

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Abstract

It has been more than 15 years since the "string axiverse" first came to prominence as a framework to explore the physics of the axion dark sector and its connection to string theory. In the intervening years, huge advances have been made in astrophysical and experimental searches for axions. In tandem, advances in computational geometry now permit explicit realisations of Calabi-Yau compatifications with hundreds of axions, and computation of the masses and interaction strengths. This report summarises the findings of a workshop devoted to the axiverse, held at the Banff International Research Station in January 2025. The report reviews key aspects of theory and phenomenology of axions, and identifies the key questions to pursue in the next decade of the axiverse.

Contents

1 Executive Summary and Introduction

The Standard Model of particle physics is incomplete: it does not explain the existence of dark matter or dark energy, and cannot accommodate a quantum theory of gravity. It is therefore certain that new physics beyond the current Standard Model must exist: finding a theory of and evidence for this new physics is one of the most important frontiers for modern physics.

A leading candidate for new physics, and in particular for a theory of dark matter, is the existence of one or more so-called axions or axion-like particles. The axion was originally proposed as a solution to the strong CP problem, which is the observation that the neutron electric dipole moment is many orders of magnitude smaller than what naive calculations in quantum field theory predict. The QCD axion solves this discrepancy by introducing a new field that can dynamically relax this dipole moment to zero.

The QCD axion, along with other axion-like particles (pseudoscalar fields) are also well-motivated as dark matter candidates. Axions are naturally weakly coupled to Standard Model fields, and are equipped with a simple mechanism for populating the observed dark matter relic abundance through coherent oscillations in the early universe.

The landscape of ongoing and planned experiments searching for axions is vast: there are many observations that probe axion couplings to photons and other Standard Model fields, as well as very sensitive experiments that search for axions as ambient dark matter particles. The parameter space that is actively being explored is wide enough that the prospect of a positive signal in our lifetimes is realistic. Axions are one of the most promising candidates for a new physics discovery.

At the same time, the theoretical motivation for axions is extremely strong. The Standard Model cannot accommodate gravity as a quantum theory, and our leading candidate for a theory that encompasses both modern particle physics and gravity is string theory. A general key feature of solutions of string theory is the presence of a number of axions, often hundreds, ranging over many decades in mass [1–3]. This aspect of string theory solutions is known as the string axiverse [4]. The existence of the string axiverse provides strong motivation for axions, both as dark matter candidates and otherwise. Detection of key features of the axiverse would provide strong evidence for string theory as the theory that describes our universe. At the same time, understanding aspects and patterns of the string axiverse can provide insight into where to focus our experimental efforts.

Since the original proposal of the axiverse in 2009 our understanding of it has grown massively. The original proposal noted key observational signatures of birefringence, large scale structure clustering, black hole superrradiance, and decaying heavy axions. Every one of these proposed signatures has been explored in significant depth in hundreds of publications. New features have been identified, in particular in large scale structure, thanks to pioneering breakthroughs in our ability to simulate axions in the non-linear regime [5]. From the phenomenology side, the most significant advances have been made in axion direct detection. In 2009, the field was led by ADMX, which in this year made its first science run with sensitivity to the QCD axion [6]. Now, there are dozens of axion experiments around the world probing a vast range of different particles masses. In tandem with this, the understanding of axions in string theory has also grown significantly. The original axiverse proposal used simple scaling arguments to predict a logarithmic distribution of masses, and estimated a decay constant in the range 10^{16} GeV, based on understanding of string compactifications and moduli stabilisation in models with small numbers of axions and moduli. Since 2018, it has been possible to study the axiverse far more broadly, in particular with large ensembles of explicit constructions containing hundreds of axions and moduli [7]. It is now possible to explicitly compute the distribution of masses and decay constants. A key finding is that the decay constants can be much smaller, with the value being tied to control of the effective field theory and topological complexity. Thanks to all of these advances, it is now timely to bring this knowledge together, and plot a way forward for the next decade of the axiverse.

The workshop "Prospects for the String Axiverse," held at the Banff International Research Station in January 2025, was aimed at precisely strengthening the symbiotic relationship between efforts to detect axions experimentally and the program of analysing axion physics as an output of string theory. The event convened experts in axion phenomenology and detection, as well as in string theory and stringy axion physics. Talks were aimed across these divides, structured around being accessible for the wide range of backgrounds in the audience. A number of interactive sessions were held in which the prospects for axion detection and our ability to make string theoretic predictions for axion physics were discussed.

One of the biggest takeaways of the workshop was a crystallisation of the questions that string theorists and phenomenologists must work towards answering in the coming years in order to truly make contact with detection in a meaningful way. These questions include the following:

- What are the observational differences between "closed string/p-form axions" and "field theoretic/open-string/higher axions"?
- Is it possible to engineer a light QCD axion as such a field theoretic axion in string theory?
- Previous studies on the string axiverse have focused on axion physics at "generic" points in moduli space. How do explicit moduli stabilisation schemes affect axion physics? In other words, how does the distribution of axion observables compare between points in moduli space where scalar fields are all explicitly given a mass, versus randomly selected points?
- Given a collection of axion observables (such as axion-photon or axion-fermion couplings), how can one solve the "inverse problem" of estimating stringy UV parameters such as geometrical and topological data?
- What set of axion observations would serve as evidence for 10-dimensional string theory in the supergravity regime? What constitutes an observation of the axiverse?

- How can we efficiently explore predictions for axion physics across the landscape? What stringy features can robust machine learning algorithms probe, versus which structures can only be uncovered analytically? How do we efficiently encode string compactification data to predict axion properties in an ML usable way?

- What are the most promising experimental and observational probes in the next 10-20 years? Which observations directly shed light on our place in the string theory landscape? Which experimental endeavors are most favored by string theory expectations?

In terms of specific observational probes, we believe the following are key future priorities:

- Astrophysical and direct detection probes of miniclusters. Can these be used to distinguish if the QCD axion is closed or open string (in typer IIB) and what does such a distinction tell us specifically?

- Searches for QCD axion dark matter. How does the QCD axion mass specifically locate us in the landscape? Are there more precise statements to be made from string theory on the theoretical expectation for QCD axion mass and couplings?

- Cosmological probes of axions. Signatures of ultralight axions seem confined to low Hodge numbers, birefringence seems correlated to the UV behaviour of electromagnetism. Can these links be made more precise, and how can other observable signatures of axions in cosmology be brought to similar level of understanding?

- How does the cosmology of axions in string theory relate to models of inflation and reheating, given various pressures for these phenomena to happen at lower scales than naive expectations?

The remainder of this report is structured as follows. In §2, we review several talks and discussions related to the broader topic of axions in string theory. We also summarise the remainder of the workshop contributions related to this theme. In §3, we provide analogous summaries for sessions related to the broader topic of axion phenomenology and detection prospects. In §4, we briefly conclude.

2 Axions in String Theory

2.1 Jakob Moritz: Axions in String Theory Review

Axions are ubiquitous in string theory, but their microscopic origins and features are diverse. A broad but useful classification of axions that arise in string theory is the following:

1. Open-string axions

2. Closed-string axions.

Open-string axions are open strings ending on spacetime-filling branes in the compactification manifold. These axions arise as phases of matter fields:

$$\Phi = |\Phi| e^{i\psi} \tag{2.1}$$

where ψ is the axion. This is completely analogous to the standard "field-theory" axions as originally envisioned in [].

Closed string axions, on the other hand, are closed strings propagating in the bulk of spacetime. These axions arise from the dimensional reduction of p-form field strengths in the ten dimensional theory. Schematically,

$$\theta = \int_{\Sigma^{(p)}} A_p \tag{2.2}$$

where A_p is a gauge field in the higher-dimensional theory, and $\Sigma^{(p)}$ is a $p - cycle$ in the compactification manifold. The number of approximately massless axions is given by the dimension of the p-th de Rham cohomology group of the compactification manifold, which can easily lie in the hundreds, e.g. for typical Calabi-Yau manifolds drawn from the Kreuzer-Skarke ensemble [8]. Moreover, the Chern-Simons action of D-branes [9, 10] leads precisely to the dimension five interaction with gluons that are required to solve the strong CP problem[1], and the analogous interactions with photons through which axions are intensely searched for experimentally.

Closed-string axions are in some ways more appealing than the traditional "field-theoretic" axion constructions: they naturally come with a high quality Peccei-Quinn (PQ) symmetry[2] [2], and their decay constants often fall in the phenomenologically acceptable range [13, 14]. Furthermore, the fact that axions are a ubiquitous feature of string compactifications suggests that they may be important for understanding quantum gravity itself. Indeed, axions in string theory show striking apparently universal features such as sub-Planckian decay constants [15] which is closely related to the *weak gravity conjecture* [16]. Finally, a generic feature of string compactifications is a plethora of axion particles, the famous *string axiverse* [4], whose experimental verification would be striking evidence for string theory as *the* theory of quantum gravity.

Much progress has been made recently in understanding the detailed predictions of the string axiverse — often making direct contact between string constructions and experimental searches for axions, and cosmological and astrophysical constraints. Some highlights include 1) an assessment of the Peccei-Quinn quality problem in string compactifications [13], concluding that is absent in the generic many-axion case, 2) the explicit computation of axion-photon couplings in large ensembles of string compactifications in type IIB string theory [14] and the

[1] It has been suggested that this is a consequence of the absence of global symmetries in quantum gravity [11]

[2] In contrast, in field theoretic construction there is a tension between realising high PQ quality, and avoiding a domain wall problem [12]

heterotic string [17], 3) assessments of dark radiation from moduli decays to axions [18], 4) attempts to construct models of fuzzy dark matter in type IIB string theory [19, 20], and 5) the explorations of new corners of the string axiverse [21].

However, many fundamental questions remain unanswered. In particular, our understanding of moduli stabilisation in string compactifications in combination with realistic particle physics remains largely incomplete.[3] Yet, the intense ongoing and planned experimental efforts devoted to detecting the axion make it an urgent task for string theorists to provide realistic models of interactions between stringy axions and the visible sector!

2.2 Andreas Schachner: Towards Fully Automated Pipelines for Exploring the String Axiverse

In this section, we explore various computational aspects of the string axiverse. Specifically, we emphasise two key approaches: leveraging auto-differentiation to address optimisation challenges within the string axiverse and using stochastic optimisation to identify phenomenologically favoured compactification geometries.

2.2.1 Auto-differenation and optimisation in the string landscape

In the string axiverse, optimising parameters such as axion masses, or couplings to match physical constraints often involves navigating highly complex functions derived from the interplay of string compactifications, moduli stabilisation, and cosmological evolution equations. A key computational challenge is the intricate, multi-scale nature of these problems, where the dependence of physical observables on underlying parameters is both highly involved and computationally expensive to evaluate. For instance, the dark matter (DM) relic density of an axion with decay constant f_a and mass 10^{-28} eV $\lesssim m_a \lesssim 10^{-15}$ eV produced by vacuum realignment [25–28] with initial misalignment angle θ_a is

$$\Omega_a h^2 \approx 0.12\,\theta_a^2 \left(\frac{m_a}{4.4\cdot 10^{-19}\,\mathrm{eV}}\right)^{1/2}\left(\frac{f_a}{10^{16}\,\mathrm{GeV}}\right)^2 . \tag{2.3}$$

Provided that the parameters m_a, f_a, θ_a are suitably large, such an axion may contribute a significant fraction to DM. In string compactifications, axion properties like m_a, f_a depend on the values of moduli and thus the magnitude of $\Omega_a h^2$ varies across moduli space.

Automatic differentiation (AD) is a powerful tool in this context which, unlike numerical differentiation (error-prone due to finite differences) or symbolic differentiation (impractical for large systems), uses the chain rule at the computational graph level to provide derivatives at machine precision. By allowing precise and efficient derivative calculations for relic densities $\Omega_a h^2$ or moduli potentials, AD opens the door to systematically explore the parameter space of string compactifications. Such strategies have recently been employed in [20] to find string compactifications with large fuzzy relic abundance in models with up to seven axions.

[3]Recent progress on full moduli stabilisation include [22, 23] but do not feature realistic particle physics. In state-of-the-art standard model constructions such as [24] moduli stabilisation remains unaddressed.

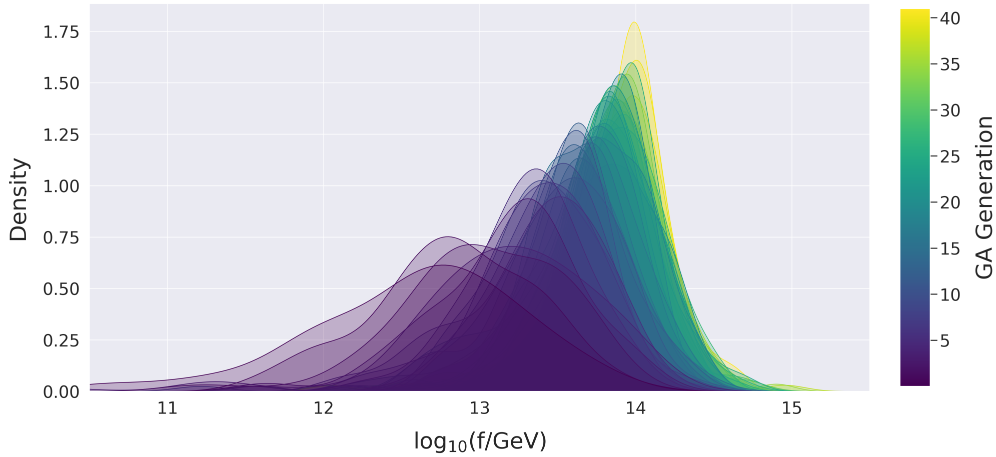

Figure 1: Evolution of the distributions of decay constants for the lightest axion in each Calabi-Yau geometry colored by the GA generation. Figure taken from [33].

Indeed, through a suitable implementation of Eq. (2.3), AD enables the use of gradient-based optimisation algorithms to identify such regions in moduli space. In this way, [20] discovered models in which the lightest C_4-axion has an untuned misalignment abundance matching the observed value $\Omega_a/\Omega_{\mathrm{DM}} = 1$. Other related applications of AD involve stabilising moduli by minimising scalar potentials with a highly intricate dependence on fluxes and other compactification parameters. The use of AD allowed the authors of [29] to develop an efficient framework facilitating optimisation techniques that seek vacua with desired properties, such as realistic hierarchies of scales, see in particular [30–32] for applications.

In the long term, such computational approaches can play an essential role in making the axiverse more predictive and for uncovering connections between string theory and observable physics. By systematically exploring compactification choices, flux configurations, and stabilisation schemes, we can begin to connect low-energy physics to high-energy theory in a more predictive framework. This includes identifying axions that align with cosmological and astrophysical observations and exploring their implications for inflation.

2.2.2 The DNA of Calabi-Yau hypersurfaces and combinatorial cosmology

Typically, when investigating cosmological or astrophysical phenomena in string theory, we adopt a top-down approach. However, it can also be beneficial to examine the following inverse problem: given a four-dimensional effective field theory (EFT) with specific values for axion observables, such as axion-photon couplings $g_{a\gamma\gamma}$, how can we determine suitable UV data within string theory?

Significant progress in this direction has recently been made in [33] through the application of stochastic search optimisation. Specifically, *Genetic Algorithms* (GAs) are utilised to explore the space of triangulations of four-dimensional reflexive polytopes Δ° [8] defining

Calabi-Yau (CY) threefold hypersurfaces [34].[4] They are inspired by natural selection processes and perfectly suited to search fitness landscapes for optimal solutions by evolving a population of individuals, here representing different CY geometries. As a proof of concept, we would like to find EFTs obtained from CY compactifications of Type IIB string theory in which the lightest C_4-axion has a decay constant $f_a \approx f_* = 10^{14}$ GeV. We show the evolution of decay constants f_a for a single GA run in Fig. 1. The distribution of f_a in each population quickly converges towards the target value $f_* = 10^{14}$ GeV within only a few generations showing that GAs indeed learn general traits of the fittest configuration. Moreover, GAs surpass conventional methods like Markov Chain Monte Carlo and Simulated Annealing in efficiency [33]. In the future, such techniques allow us to explore the string axiverse globally for more diverse and phenomenologically rich targets.

2.3 Alexander Westphal: Discussion

Axions in string theory – or in short 'string axions' – come in two varieties. In both heterotic and type I/II string theories there exist Abelian p-form gauge fields at the 10-dimensional level. In geometric Kaluza-Klein (KK) string compactifications down to four dimensions, e.g. on Calabi-Yau (CY) manifolds, these p-form gauge fields generate KK zero modes on appropriate topologically non-trivial subspaces ('cycles') of the compact extra dimensions. These p-form KK zero modes constitute 4D pseudo-scalars with non-compact shift symmetries as non-linear realisations of the underlying gauge symmetry of the p-form gauge fields. As these p-forms arise from the NSNS and/or RR-sector of closed string theories, we call their associated 4D pseudo-scalars 'closed-string axions'. In theories with open strings such as type I/II string theories, open strings ending on D-branes generate both charged matter fields and SM-like gauge fields. As these charged matter fields can trigger spontaneous symmetry breaking of some of the $U(1)$ gauge symmetries of the open string sector, the phases of some of the open string charged matter fields can acquire the properties of Peccei-Quinn like axions. Their couplings to the open string sector matter and gauge fields often show the structures typical for bottom-up Peccei-Quinn like field theory axions. Hence, these string axions are called 'open string axions' or sometimes 'field-theory-like axions'.

Open string axions have been less systematically explored in the extant literature. Hence, there is a clear need to study them more systematically, aiming for questions like:

- How field theory like are open string axions typically?
- How do their spectra of axion decay constants f_a, masses and matter, gauge field, and Chern-Simons couplings look like and distribute over the landscape of string vacua?
- Are the value ranges and structure of these couplings wide enough to accommodate all bottom-up Peccei-Quinn like EFT axion models, or do they restrict model space?

[4] As explained e.g. in [35], due to Wall's theorem [36], the homotopy type of such manifolds is determined by the induced triangulations on two-faces of Δ°. The chosen data encoding in [33] removes these redundancies building on algorithms developed in [37], allowing for faster and more efficient searches.

- A further important question is to determine how to discriminate a field-theory-like open string axion from closed string axions.

Conversely, for closed string axions some of these questions have been begun to be answered for C_4-axions arising in type IIB string theory vacua from CY orientifolds [13, 14, 19, 20, 38]. However, beyond this type IIB Calabi-Yau C_4-axiverse, there remains much to be explored. For instance, already in type IIB CY vacua we need a systematic construction and exploration of the axiverse of 2-forms B_2, C_2 on CY orientifolds with $h^{1,1}_- > 0$. Beyond type IIB, there is the B_2-axiverse of CY compactifications of the heterotic string (see [17] for an exploration of properties of the QCD axion candidate in 4D heterotic string compactifications), which is again largely unexplored. On an even wider scope, there is a clear motivation to look at the very large classes of string vacua arising on compact negatively curved spaces., which tend to produce axiverses with potentially very large numbers of 2- and 4-form axions.

Once we are given a certain type of string axiverse, we need to understand the resulting cosmological dynamics and signals much better. If an axiverse contains closed string axions, it seems to be a generic feature due to the topology of the compact extra dimensions, that all but a few of the closed string axions are 'dark states' which do not couple directly to the SM sector. We clearly need to understand both axion production and the potential signals, such as primordial gravitational wave (GW) emission or additional 'dark radiation' contributions measured by the $\Delta N_{\rm eff}$ of a 'cosmic axion background' of such a 'dark axiverse'. These depend on the production mechanisms:

- There is model-dependent production after inflation occurring via inflaton and/or modulus decay, cosmic string dynamics and non-perturbative phenomena such as preheating from parametric resonance and tachyonic preheating.
- There is also a universal production channel – all axions with non-zero mass $m < H$ during inflation will get produced by their quasi-de Sitter (dS) random walk quantum fluctuations. These generate a non-vanishing misalignment angle (the axions get displaced from their potential minimum and 'walk up the potential well'). This mechanism acts on all light pseudo/scalar degrees of freedom by virtue of the universality of gravity in GR.

For a string axiverse with high-scale $f_a \sim 10^{16^{+1}_{-2}}$ GeV this implies that every axion heavier than fuzzy dark matter [39] $m_a \gtrsim 10^{-19}$ eV will overclose the universe. Avenues for avoiding this problem include

- either anthropic tuning of the misalignment angle [40, 41] (which may get falsified in the future if DM were found to be non-axionic [42]),
- a very atypical axiverse mass spectrum with essentially an axion desert between $m_a \sim 10^{-19}$ eV and the scale of inflation [19],
- or very low scale inflation (so that most axions would have $m > H_{\rm inf.}$ avoiding their production, as well as potential CMB isocurvature constraints [19]).

The last two options present clear opportunities for future work, as neither a search for such atypical axiverses nor good constructions of string models of low-scale inflation exist.

2.4 Arthur Hebecker: Outlook/Discussion

The logical structure of my presentation was strongly inspired by the review part of [18]. After a brief reminder of the basic definitions and the phenomenological requirements for the QCD axion, I emphasised that one may distinguish two different fundamental origins for this pseudoscalar:

One the one hand, its origin could be purely 4d field-theoretic, i.e. it could be the phase of a 4d complex scalar. In this case, a significant amount of model building is required to avoid the quality problem, which in general will be severe (see e.g. [43] for a classic reference and [44] for a recent example).

On the other hand, the axion could originate in higher-dimensional p-forms, integrated over a cycle of the compactification space [2, 3]. This is very natural in string theory and it can easily avoid the quality problem if the relevant cycle is large enough.

In this 'p-form axion' case, a rather general prediction is that the axion decay constant f obeys $f \sim 1/\sqrt{\mathcal{V}}$, where $\mathcal{V}$ is the volume of the Calabi-Yau orientifold measured in 10d Planck units. The phenomenologically preferred regime of $f \ll M_P$ than points towards compactifications with large volume, in particular the Large Volume Scenario (LVS).

Avoiding the overclosure of the Universe by axion dark matter and respecting the strong observational bounds on isocurvature perturbations imposes significant constraints on the volume and, crucially, on the scale of inflation:

$$\mathcal{V} \gtrsim 10^7 \quad , \qquad H_I \lesssim 10^9 \, \mathrm{GeV} \, / \, \mathcal{V}^{5/24} \, . \tag{2.4}$$

As a side remark, this deep LVS regime is well-known to suffer from a significant dark radiation problem [45, 46], which however can in many cases be very naturally overcome if the SUSY breaking scale is high [18].

The restriction to relatively low inflation scales is problematic in string constructions. The only one of the established models I am aware of that realises such a low scale without an excessive (and presumably not anthropically justifiable) fine-tuning is blow-up inflation [47]. But loop corrections unavoidably spoil this construction, turning it into loop blowup inflation [48], with very different pheno characteristics and, crucially, a much higher scale. Thus, a key open problem is to better understand and overcome the apparent clash between a string-axion resolution of the strong CP problem and stringy expectations for the scale of inflation.

Finally, while much progress has been reported on explicit string axion model building (see [13, 49] and subsequent work), the task of moduli stabilisation and uplifting to a de Sitter vacuum is not automatically resolved in these constructions. Since the leading stringy de Sitter models have recently come under considerable pressure (see e.g. [50–54]), this is a key problem: It is crucial to determine how the purely geometrical and statistical predictions for the QCD axion will be affected once additional constraints imposed by realising de Sitter are properly taken into account.

2.5 Additional Contributions

Nicole Righi: String Axions: The Hot and the Fuzzy [summarised by N. Gendler] The central question is whethe string theory can accommodate models of fuzzy axion dark matter consistent with current bounds on fuzzy dark matter relic abundance. This talk presented two complementary approaches to studying this problem: first, by studying concrete examples of models with one or two axions and all Kähler moduli explicitly stabilised, and second, by scanning over models with up to 7 axions, carefully analysing the cosmological history, but ignoring the effects of moduli stabilisation. In the latter, it was discussed how to engineer a cosmology such that there exists a fuzzy dark matter candidate, while simultaneously not overproducing axion dark matter.

3 Axion Phenomenology

3.1 Joerg Jaeckel: Axions and ALPs Beyond Discovery

Axions [55–58] and more general axion-like particles (ALPs) [59, 60] typically feature $1/f_a$ suppressed interactions with a range of Standard Model (SM) particles summarised in the effective Lagrangian, e.g. [61–63],

$$\mathcal{L} = \frac{1}{2}(\partial_\mu a)^2 + \frac{1}{2}m^2 a^2 + \frac{1}{4}g_{a\gamma\gamma}a(F^{\mu\nu})^2 + \frac{1}{2}g_{agg}a(G^{\mu\nu})^2 + \sum_f g_{aff}a\bar{f}\gamma^5 f. \tag{3.1}$$

In this equation a is the axion/ALP, F, G are the SM photons and gluons, and f the SM fermions. The coupling coefficients are related to the symmetries and particles in the underlying UV complete model. For example, for a QCD axion we have for the photon coupling and mass [64],

$$g_{a\gamma\gamma} = \frac{\alpha}{2\pi f_a}\left(\frac{E}{N} - 1.92(4)\right), \qquad 5.70(7)\mu\text{eV}\left(\frac{10^{12}\,\text{GeV}}{f_a}\right). \tag{3.2}$$

Here, E and N are the electromagnetic and gluon anomaly coefficients and the second term corresponds to the low energy QCD contribution and its uncertainty. The salient point is that measuring the photon coupling $g_{a\gamma\gamma}$ and the mass would allow to determine both f_a and E/N. It therefore allows to glean information on the scale of spontaneous symmetry breaking f_a and some information on the particle content at this scale. Having information on additional couplings can then allow to gain more detailed information on the underlying model and/or to reduce assumptions such as the QCD axion nature. For example measuring the gluon coupling and checking whether it fits the predicted unique relation to the mass for a standard QCD axion, can answer whether we are indeed dealing with a QCD axion solving the strong CP problem.

It is therefore of interest to see what information on the parameters in Eq. (3.1) can be obtained from experiments and observations. In the following we will discuss a few example cases that show that after a discovery more details may be learned from axion experiments.

Axion Couplings from Helioscopes: The International Axion Observatory (IAXO) [65, 66] uses the helioscope technique [67] to search for axions produced in the sun. By now the flux calculations as a function of the axion photon coupling are good enough to allow distinguishing [68] between different KSVZ [69, 70] models as well as excluding some of those simple models. Depending on the parameter region IAXO or a somewhat enhanced version of it would acquire enough data to determine both mass and coupling [68, 71, 72]. Indeed, it may even be possible to determine the electron coupling as well [71], making it possible to even more clearly delineate between KSVZ [69, 70] and DSFZ [73, 74] type models. This could give very interesting hints also for possible stringy realisations.

Axion Dark Matter, Density and Coupling: It is likely that a discovery of axions arises from an axion dark matter experiment, e.g. an axion haloscope [67], a dish antenna [75] or dielectric haloscope [76, 77]. Some examples of these types of experiments are ADMX [78, 79], BREAD [80], CAPP [81] and MADMAX [76] (more can be found in the nice compilation [82]). In this case there is a degeneracy between the local dark matter density ρ and the coupling g typically, signal $\sim g^2\rho$. Resolving this degeneracy would allow to answer two crucial questions: 1) Have we discovered the dark matter or just a small fraction of it?[5] 2) The coupling again can give us model information.

Let us give two examples of potential ways to do this. One is to hope for the crossing of an axion minicluster through the haloscope [83]. In this case the spectral resolution of the haloscope may be sufficient to resolve the gravitational binding energy when crossing the cluster as a function of the location inside the cluster. Via the Poisson equation this gives direct information on the cluster density independent of the coupling to photons. The power then allows to determine the coupling [83]. Alternatively one could built an experiment independent of the coupling, e.g. a light-shining-through-walls experiment [84]. Information from the dark matter discovery measurements can significantly improve the feasibility of building a dedicated experiment [84].

To conclude. Beyond an initial discovery, axion experiments can often yield significant extra information that can help us better understand the cosmology as well as the detailed underlying, potentially stringy, fundamental model.

3.2 David J. E. Marsh: Ultralight Axions in Astrophysics and Cosmology

3.2.1 Introduction

If axions are ultralight, which for the purposes of this discussion we define as $m_a \lesssim 10^{-18}$ eV, and production in the early Universe proceeds via the vacuum realignment mechanism, then

[5] As an interesting speculation. If the density is high compared to the one expected from the estimated scale f_a (this could be inferred from the mass, Eq. (3.2)) one could take this as an indication for a post-inflationary scenario with a high contribution from axion strings.

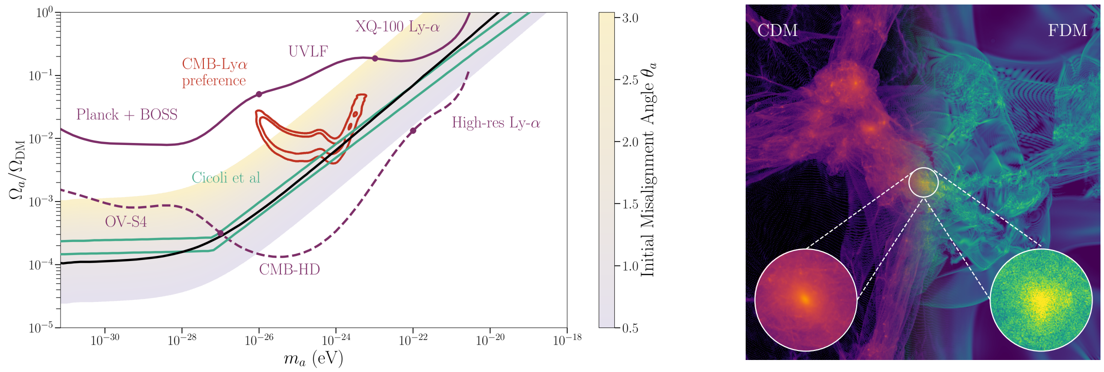

Figure 2: *Left panel:* observational constraints and forecasts for fuzzy axions, compared to the predictions of a single model with $h^{1,1} = 7$, demonstrating the discovery potential of future missions. *Right panel:* mixed DM simulation for cold DM and a fuzzy axion with $m_a \approx 10^{-24}$ eV composing 10% of the relic density. (Images reproduced from Refs. [20, 85])

the clustering of such a dark matter (DM) component is observationally distinguishable from standard cold DM. We refer to such DM as "fuzzy" (following Ref. [86]). Fuzzy DM is a collection of phenomena where wavelike effects manifest in astrophysics. It is a lamppost under which we can look for the ultralight axions predicted by string theory, and of which our understanding has progressed significantly in the last 15 years.

The consistency of (most) cosmic observables with the predictions of CDM leads to constraints on the allowed relic density, $\Omega_a h^2 < 0.12$. The precise limit is axion mass dependent, with different masses also being constrained by different observables. The basic physical principle that distinguishes such ultralight axions from CDM is the large de Broglie wavelength. Thus, heavier axions show differences to CDM on smaller length scales than lighter axions, since $\lambda \propto 1/m_a$. This in turn implies that lighter axions have tighter constraints on $\Omega_a h^2$ than heavier ones do because (i) large, linear scales in cosmology are more well measured (ii) for a departure from CDM on large scales, more total modes k are affected compared to a departure on small scales.

If the axion decay constant is close to $f_a = 10^{16}$ GeV, then the predicted relic abundance across the entire range 10^{-33} eV $\lesssim m_a \lesssim 10^{-18}$ eV turns out to be just outside present precision cosmology bounds [87–89], but just within reach of future surveys [90–92], offering an opportunity of discovery [4]. It was recently shown in Ref. [20] that models of fuzzy axion DM with large f_a in this discovery window can be found within explicit string theory compactifications on Calabi-Yau orientifold hypersurfaces in toric varieties [34] constructed from the Kreuzer-Skarke database [8], so long as the total number of axions (Hodge number $h^{1,1}$) is relatively small, $h^{1,1} \lesssim 10$. Thus, the discovery prospects of cosmology are intimately linked to cohomology.

Existing observational limits (solid) and forecasts (dashed) are summarised in Fig. 2 (left panel), along with a particular model constructed in Ref. [20] with $h^{1,1} = 7$ (black line and shaded band). This demonstrates the possibility for discovery, as well as the ability for string models to explain the "Lyman-α tension" [93] (red contours). The journey from the ideas originally outlined in Ref. [4] to future discovery has been facilitated by increasingly detailed modelling and understanding of the effects of ULAs on cosmic structure. The next leap will be guided by numerical simulations, such as those shown in Fig. 2 (right panel), which incorporate wave effects inside the cosmic web in mixed DM models.

Ref. [20] considered axions arising from dimensional reduction of the four form field C_4 from the closed string sector of type IIB string theory on Calabi-Yau orientifolds with $h^{1,1}_{-} = 0$, such that these are the only axions present. Such extra dimensional axions are by necessity in the "pre-inflation" cosmological scenario [94]. At generic points in moduli space, all the axions have comparable decay constants f_a (more precisely, they are log-normal distributed with $\sigma \approx 1$ [95]). Thus, in a model with fuzzy axions, heavier axions will also contribute significantly to the relic density. In particular, the QCD axion (which is necessarily present in such models) with $f_a = 10^{16}$ GeV requires a fine tuned initial misalignment angle in order not to produce too much DM. Minimising the overall fine tuning measure across all axions thus leads to the expectation that the DM has an almost equal mix of heavy, cold axions, along with a sub-leading fraction of the lighter fuzzy axions. Thus in the following we assume there to be a fuzzy axion with $\Omega_a h^2 < 0.12$ and a CDM component composed of the QCD axion and any other heavier, stable axions with $\Omega_c h^2 < 0.12$, with the total $\Omega_d h^2 = \Omega_c h^2 + \Omega_a h^2 = 0.12$, and in general $\Omega_a h^2 < \Omega_c h^2$.

3.2.2 Some observational signatures

The earliest example of limiting the relative fraction of fuzzy DM to CDM using effects described here can be found in Ref. [96], and notable advances were made in Refs. [97, 98]. Cosmic observables are affected gravitationally by two distinct features of fuzzy DM. The first is encoded in the background expansion rate, and the second from the growth of structure. Coupling between fuzzy DM and electromagnetism induces additional effects.

The expansion rate and the CMB. The distinctive effect of fuzzy DM on the expansion rate is caused by the transition at $t_{\rm osc}$, when the energy density goes from being approximately constant with $w = -1$ (where $w = P/\rho$ is the equation of state defined by the canonical energy momentum tensor) to behaving like DM with $w = 0$. The difference to CDM ($w = \text{const.} = 0$), a cosmological constant ($w = \text{const.} = -1$) or hot neutrinos ($w = 1/3$ transitioning to $w = 0$) makes the effect distinct to any component in ΛCDM. A change in w manifests in a change to the background expansion rate, which in turn impacts the CMB. A transition prior to decoupling manifests in a modified diffusion scale, affecting the heights of the CMB acoustic peaks (Silk damping). The transition also affects the time dependence of gravitational potentials (Sachs-Wolfe effect), which manifests dominantly in the large angle CMB for transitions occurring after decoupling. These effects were first described in detail in Ref. [98], and are the foundation of the strongest limits at low axion mass shown in Fig. 2 (left panel) [87]. These effects become negligible at very low mass,

$m_a \lesssim 10^{-33}$ eV, where axions are indistinguishable from a cosmological constant [6], and for masses with $t_{\rm osc} \gg t_{\rm eq} \Rightarrow m_a \gtrsim 10^{-25}$ eV, where the effects on the expansion rate of all components with $w < 1/3$ become increasingly negligible.

The Jeans scale and the power spectrum. The scalar field Jeans scale [102] is caused by gradient energy in the linearised Klein-Gordon equation opposing the effective negative mass squared term induced by gravitational potentials. In the effective fluid equations, this manifests as a contest between the gravitational growing mode and pressure perturbations and has an intuitive interpretation in terms of the de Broglie wavelength. The effective fluid equation for sub-horizon density perturbations, δ, in a scalar field DM dominated Universe takes the form:

$$\ddot{\delta} + 2H\dot{\delta} + (k^2 c_s^2 - 4\pi G_N \bar{\rho})\delta = 0\,, \tag{3.3}$$

where $\bar{\rho}$ is the background density which satisfies the Friedmann equation and scales as a^{-3} with a the scale factor, and $c_s^2 \approx k^2/m_a^2 a^2$ is the effective soundspeed arising from gradients in the Klein-Gordon equation in Fourier space (hence its scaling with k^2). The Jeans scale is defined as the scale k_J for which the term in brackets multiplying δ is equal to zero: modes with $k < k_J$ undergo growth, while those with $k > k_J$ oscillate and do not grown. Intuitively, scales above the de Broglie wavelength at approximately the Hubble flow appear particle-like and gravitationally cluster, while those below it are wavelike and (statistically) homogeneous.

The effect of the Jeans scale on cosmic observables can be seen in the matter power spectrum, $P(k)$ (the Fourier transform of the matter overdensity two-point correlations). The power spectrum on large scales resembles CDM, but for k modes larger than the Jeans scale evaluated at matter radiation equality (the time when CDM perturbations begin faster-than-logarithmic growth) the power spectrum has reduced amplitude [86]. The scale where suppression begins is determined by the axion mass m_a and scales like $m_a^0.5$, while the amplitude of suppression, S, relative to pure CDM at redshift zero increases with the axion relic density (see Refs. [4, 97, 103] for fits).

Estimators of the power spectrum are largely consistent with CDM. On the largest cosmic scales, the BOSS galaxy power spectrum measurements are complementary to and strengthen precision constraints derived from the CMB, while the EFT of large scale structure allows constraints to extend into the mildly non-linear regime up to $m_a \approx 10^{-24}$ eV [87]. The high resolution XQ-100 Lyman-alpha forest flux power spectrum on very small scales and constrains the fraction in the heaviest fuzzy axions [89]. Suppressed clustering also leads to fewer DM halos relative to CDM [104]. Fewer DM halos at high-z leads to lower UV luminosity. Hubble space telescope measurements currently provide the best measurement of the high-z luminosity function and lead to precise constraints on fuzzy axions in an intermediate mass range [88].

[6] Recent reported evidence for dynamical dark energy by DESI can be interpreted as caused by an axion of mass $\log_{10}(m_a/\text{eV}) = -32.6$ [99, 100]. The required decay constant, $\log_{10}(f_a/M_{pl}) = -0.22$ appears too large to achieve in the string theory constructions of Ref. [20] with modestly large numbers of axions, and would be even more difficult with large numbers of axions. For discussion of quintessence and axions in string theory see e.g. Ref. [101].

Birefringence. We do not have space here to discuss all the myriad effects of axion-matter couplings. We focus on just one: isotropic birefringence, for which there are some observational hints [105, 106]. The axion photon coupling:

$$\mathcal{L} = -\frac{g}{4}\phi F_{\mu\nu}\tilde{F}^{\mu\nu}\,, \tag{3.4}$$

leads to a rotation of polarisation of light moving through a medium in which the axion field is changing in time, as if the vacuum were an active optical medium [107]. [7] The angle of rotation, β is given by the integral of $g\phi$ along the line of sight. If the axion begins to roll only after the CMB is formed, $m < H_{\rm CMB}$ then this integral is approximated by $g\theta_i/f_a$ where θ_i is the initial misalignment angle. If furthermore such an axion has an unsuppressed coupling to electromagnetism, i.e. $g \approx \alpha/2\pi f_a$, then the amount of rotation measured in radians is simply $\beta \approx \theta_i\alpha/2\pi \approx 2 \times 10^{-3}\theta_i$. The observational hint for this signal arises from E to B-mode polarisation rotation of the CMB anisotropies, and is of order the value predicted in the axion model. Unsuppressed couplings to electromagnetism arise for C_4 axions in type IIB only if there is no unification (i.e. the Standard Model gauge group remains a product at the KK scale), and when $\alpha(M_{\rm KK}) \approx 1/40$, precisely the value expected in the Standard Model with high scale SUSY [14].

Non-linearities. In the non-relativistic regime axion DM is described by the classical mean field Schrödinger-Poisson equations, and thus departs from a pure CDM (collisionless Vlasov-Poisson) description at scales of order the de Broglie wavelength. Recent years have seen significant advances in simulating non-linear cosmic structure formation using these equations, rather than standard N-body methods (e.g Refs. [5, 109, 110]). Inside virialised DM halos, the axion field has a turbulent outer region and a coherent inner core. The core, also known as a soliton or axion star, is a distinct feature of scalar DM with a wide range of phenomenological consequences that can be used to limit the existence of, or search for, axions (e.g. Refs. [5, 111–119]). The turbulent region ofthe outer halo has an averaged density profile close to the CDM Navarro-Frenk-White [120] profile. The key property of the turbulent region is two-body relaxation caused by scattering of stellar (or other) tracers by the de Broglie sized transient "quasiparticles" caused by wave interference [39, 121].

Future work on understanding string axion models in the non-linear should focus on mixed DM simulations (as shown in Fig. 2, right) and on the inclusion of Active Galactic Nucleus (AGN) feedback. Understanding the mixed DM scenario in the non-linear regime and AGN feedback is needed to properly calibrate models for the power spectrum and extract precision constraints, particularly on small scales and at late cosmic times (see e.g. Refs. [122–124] for model calibrations in the absence of feedback).

Future observables that depend on such effects include small scale weak lensing from *Euclid* and CMB-HD, and the Ostriker-Vishniac effect in CMB secondaries. It is also an open question as to whether wavelike effects might be important in modifying existing or future

[7] This effect, known as the Kerr rotation in condensed matter, was recently used to discover the "axion quasiparticle" in magnetic topological insulators [108].

constraints from the Lyman-alpha forest flux power spectrum (where axions are currently modeled using only the N-body approximation [89, 125]). As we can see in Fig. 2 and shown in detail in Ref. [20], the future observations will reach sensitivity to probe the predictions of the KS axiverse at low Hodge number, and a detection would place strict limits on our possible location in the landscape.

3.2.3 Recent advances in reconstructing fuzzy structures

This discussion closes with an account of two recent advances in which the author participated [126, 127]. Both allow for fuzzy DM wavefunction reconstruction in cosmic environments (either real or simulated) where the gravitational potential is known. The method builds on previous work of Refs. [128, 129] and solves the Schrödinger-Poisson equations self-consistently to arrive at the wavefunction given the potential. Compared to previous work, the numerical method is significantly faster, and the method has been deployed beyond spherical symmetry.

Utility: systems that cannot be simulated with current methods. Current cosmological simulations of pure fuzzy DM are not able to reach the resolution to simulate the Milky Way and its satellite galaxy population at large fuzzy DM masses (recently a Milky Way sized halo was simulated in a pure DM model for $m_a = 2 \times 10^{-23}$ eV) [130]). Wave function reconstruction of consistent halos in Ref. [126] allows study of systems that cannot currently be resolved in full cosmological simulations. Application of this method to mixed DM models to set limits on the fuzzy DM fraction from dwarf galaxies is underway. Other applications of this method could be to provide initial conditions for simulations of the Milky Way halo for direct detection, or formation of tidal streams and their perturbation by subhalos.

Utility: "painting" existing CDM simulations. Most large scale cosmic simulations, especially those incorporating the most advanced gas physics for example, are rightly performed for CDM, which is reasonable since it is established that the dominant form of DM on cosmic scales is effectively cold. However, this limits what can be said precisely about DM physics beyond CDM and for sub-components.It is possible that the interference patterns constructed in Ref. [127] could be "painted on" to CDM-only simulations in certain instances (perhaps evolving for a short period with full fuzzy, cold, and gas dynamics to monitor stability). This may allow for new searches for fuzzy DM with large surveys, where the interplay of data and simulation can be highly important.

Utility: new physics insights. Ref. [127] provided the first semi-analytical insight into the nature of wave interference in cosmic filaments, as visible in Fig. 2, and a striking feature of fuzzy DM simulations since the pioneering work of Schive et al [5]. This new understanding allowed the identification of the distinct feature of filament interference patterns in the power spectrum, constructed following a generalisation of the halo model for large scale structure. The insight in terms of the physics of filament interference could be likened to the insight of Ref. [39] into the heating effect of halo interference on stellar populations. This physical imprint of interference may allow for new searches for fuzzy DM using filament surveys [131], and move from limit setting to discovery.

3.3 Anirudh Prabhu and Christopher Dessert: Discussion

3.3.1 Some Open Questions in Axiverse Phenomenology

The ongoing program of axion direct (laboratory) and indirect (astrophysical) detection is poised to provide powerful constraints on axiverse constructions, as first proposed in [4]. The approach to axion detection relies heavily on the expected coupling between the axion sector and the Standard Model (SM). Even the minimal scenario, in which axions couple only gravitationally to the SM, possesses rich phenomenology such as black hole superradiance [132].

Another phenomenological entry point is the axion-photon coupling. For the QCD axion, the following axion-photon coupling arises from the chiral anomaly in the presence of fermions charged under both the QED and QCD gauge groups,

$$\mathcal{L} \supset -\frac{1}{4} g_{a\gamma\gamma} a(x) F_{\mu\nu} \tilde{F}^{\mu\nu}, \quad g_{a\gamma\gamma} = \frac{\alpha_{\rm EM}}{2\pi f_a} \left(\frac{E}{N} - 1.92 \right), \tag{3.5}$$

where $g_{a\gamma\gamma}$ is the axion-photon coupling constant, $a(x)$ is the axion field with corresponding decay constant f_a, $F_{\mu\nu}$ is the electromagnetic tensor, $\tilde{F}^{\mu\nu} = \varepsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}/2$ is its dual (ε is the Levi-Civita symbol), $\alpha_{\rm EM}$ is the electromagnetic fine-structure constant, and E and N are the QED and QCD anomaly coefficients, respectively. For the QCD axion, there is a defined relationship between the axion mass and decay constant, $m_a f_a \sim \Lambda_{\rm QCD}^2$.

In string theory, couplings like (3.5) arise in low-energy effective field theories of axions that arise from the compactification of p-forms on Calabi-Yau manifolds. Axion-photon couplings were computed for a large number of compactifications in type IIB String Theory, and span a wide range [14]. Therefore, the common phenomenological approach is to derive model-independent constraints by treating $g_{a\gamma\gamma}$ and m_a as independent free parameters.

Fig. 3 shows the current status of axion searches overlaid with axiverse predictions. Constraints in the figure fall into one of two categories: those that rely on axions making up dark matter (DM) and those that are agnostic to the relationship between axions and DM. Note that for large Hodge number $h_{1,1}$ compactifications, axions with masses $m_a \gtrsim$ keV may be readily observable through their decays to two photons, even if they are only produced via freeze-in [133]. Axiverse axions may have additional decay channels to lighter axions and to dark gauge groups so that the electromagnetic decay signatures are weaker than anticipated. Overproduction of DM through *e.g.* misalignment in the axiverse is, however, a generic concern.

3.3.2 Phenomenology of Multiple Axions

A key question for future phenomenological studies is how the single-axion constraints in Fig. 3 are modified in the presence of multiple axions. Current approaches to axion detection typically derive phenomenological consequences assuming a single axion, treated in a model-independent manner by allowing its mass and photon coupling to vary as free parameters. When extended to an axiverse with multiple axions, existing constraints are often applied to a single linear combination of axions. An experiment which is searching for an axiverse with $h_{1,1}$ individual axions ϕ_i and photon couplings $g_{\gamma,i}$ is sensitive to the effective single-axion coupling

$$g_{a\gamma\gamma}^{\mathrm{eff}} = \left(\sum_i g_{\gamma,i}^2\right)^{1/2}$$

where the sum runs over any axions which are able to participate in the interaction, *e.g.* due to kinematic restrictions. However, this is merely the statement that each individual axion state has some interaction with the SM. On the other hand, the presence of multiple axions can fundamentally alter single-axion phenomenology, as highlighted by the examples below.

For instance, as discussed in [134, 135], the theory above can be rotated into the electromagnetic basis, where only a single axion couples directly to electromagnetism, $a_\gamma = \sum_i g_{\gamma,i}\phi_i/g_{a\gamma\gamma}^{\mathrm{eff}}$. In this basis, the theory acquires an additional mixing term, $\propto M_{ij}\phi^i\phi^j$, which couples different axion mass states. This mixing allows the electromagnetic axion, once produced, to oscillate into electromagnetically sterile states, thereby weakening detection probabilities at experiments which require multiple axion interactions, such as CAST and IAXO. Constraints from these helioscopes are reduced by a factor $\sim 3\,(h_{1,1}/30)^{1/4}$. However, competitive limits from stellar cooling and other single-conversion probes are unaffected by mass mixing.

In the single-axion scenario, natural cosmological production mechanisms, such as the vacuum misalignment mechanism, are able to produce axions in the correct abundance to explain DM. The dynamics of axion DM can be significantly modified in an axiverse with dense mass spacing. For example, axions sharing a joint potential can resonantly exchange energy, which allows the misalignment-produced axion to dump energy into an axion with similar mass, but very different coupling, which can have significant implications for axion DM direct detection experiments [136], including substantial structure on small scales [137]. In general, QCD axion mass mixing with a sterile species can deform DM abundance expectations from standard misalignment [138, 139]. For example, Ref. [140] present an analytical study of the cosmological evolution of multi-axion systems due to adiabatic and non-adiabatic resonant conversion from one axion state into another during the misalignment process using the Landau-Zener formalism.

A qualitatively different production mechanism for axions arises in the 'post-inflationary' scenario, which involves the formation of axion strings. Open- and closed-string realisations of this scenario were constructed in [141, 142], and could lead to small-scale structure consistent with that of a 4D field-theory QCD axion whose PQ scale is below the reheating temperature T_{RH}.

Early-universe production mechanisms of axions as dark radiation are also modified. Each dimension-5 axion-SM operator will freeze-in one particular linear combination of the ϕ_i (*e.g.* a_γ for the axion-electromagnetic operator), while dimension-6 operators freeze-in every axion state under generic assumptions. For sufficiently high T_{RH}, N_{eff} may be vastly overproduced [143]. Existing ACT data allows only 6 axions to ever have been in thermal equilibrium with the SM, severely constraining the possible couplings of axions to the SM.

A systematic re-evaluation of axion constraints across different compactifications is therefore necessary to properly account for these effects.

3.3.3 Coupling to Fermions

Low energy axion effective field theories are expected to contain axion couplings to SM fermion fields of the form

$$\mathcal{L} \supset \frac{c_{ij}^A}{2f_a}(\partial_\mu a)\bar{f}_i\gamma^\mu\gamma_5 f_j + \frac{c_{ij}^V}{2f_a}(\partial_\mu a)\bar{f}_i\gamma^\mu f_j, \tag{3.6}$$

where f_i is a fermion field of generation i, and c_{ij}^A (c_{ij}^V) is a dimensionless coupling constant to the axial (vector) current between generation i and j. Note that flavor-conserving vector couplings c_{ii}^V have no physical effects.

Many astrophysical constraints shown in Fig. 3 are on the combination of $|g_{aff} \times g_{a\gamma\gamma}|$, where $g_{aff} \equiv c_f/f_a$ is a flavor-conserving coupling. Translating such probes to constraints on $g_{a\gamma\gamma}$ requires making model-dependent assumptions on the expected size of c_f. For instance, in DFSZ-type QCD axion models, $c_f \sim \mathcal{O}(1)$, whereas in KSVZ-type models the fermion coupling is generated radiatively by gauge field loops, giving rise to suppressed couplings (see, e.g., [64]). General arguments relying on supersymmetry suggest that in the case of extra-dimensional axions, the fermion coupling is one-loop suppressed [38, 94, 144, 145].

We are unaware of any works discussing the prospects for detecting an axiverse through flavor-violating (FV) couplings to fermions. FV processes in the SM are loop-, Yukawa-, or GIM-suppressed, while the axions may couple to FV at tree-level, so FV can be a powerful probe of axion physics. Indeed, assuming natural values of Wilson coefficients $c \sim \mathcal{O}(1)$, searches for FV decays to missing energy provide leading constraints on the (single) axion decay constant [146], $f_a \gtrsim 10^{12}$ GeV.

However, a comprehensive compilation of fermion couplings for extra-dimensional axions, similar to [14], has yet to be assembled. Such a compilation is essential for refining our search for axions through their interactions with fermions.

3.4 Additional Contributions

Gray Rybka: Detecting Axions [summarised by D. J. E. Marsh]. Axion DM in string theory naively can span the whole range of conceivable DM particle masses, for example from the lowest fuzzy DM range, 10^{-22} eV up to more SUSY-inspired masses of TeV or more. However, focusing on the QCD axion restricts the range to the neV to meV range predicted by decay constants in the range 10^{16} GeV to 10^{10} GeV, correlated string model properties as in Ref. [147]. The interaction Lagrangian is [148]:

$$\mathcal{L}_{\rm int} = -\frac{1}{4} g_{a\gamma\gamma} a F_{\mu\nu}\tilde{F}^{\mu\nu} - \frac{i}{2} g_d a \sigma^{\mu\nu}\bar{N}\gamma_5 N F_{\mu\nu} + g_{aNN}(\partial_\mu a)\bar{N}\gamma^\mu\gamma_5 N + g_{aee}(\partial_\mu a)\bar{e}\gamma^\mu\gamma_5 e\,. \quad (3.7)$$

The first operator provides a clean experimental target with mature experiments in existence ripe for application of quantum sensors, while the other operators are promising, with experiments under development. Focusing on the coupling $g_{a\gamma\gamma}$, we can consider the whole range of probes reviewed at [82]. There is presently a program in place [149] to search the entire QCD axion parameter space. Near term experiments in the US include ADMX [79] and DMRadio [150], with a future generation of experiments in a research phase.

For axion dark matter direct detection, the key distinct feature is to model axions as a macroscopically occupied wave, $a = a_0 \sin\omega_a t$, using the language of power, frequency, and so on familiar from radio technology. The lineshape of the axion in frequency space, ω, is determined by the halo model of the Milky Way (e.g. Refs. [151, 152]). For detection schemes, it is important to consider, within the context of this model, the Compton, de Broglie, and coherence timescles relative to the size of the apparatus and the measurement times accessible. For haloscopes like ADMX (based on the haloscope principle of Sikivie [67]) and DMRadio (based on the circuit principle of Ref. [153]), the signal via $g_{a\gamma\gamma}$ can be undertsood via the modification to Maxwell's equations, considering the axion DM field as a source of effective charge/current density.

ADMX operates as a cavity whose resonant frequency is tuned using mechanical motion of rods inside. Axion detection is typically sought using the TM_{010} mode, with quality factor $Q \approx 10^4$ leading to signal power of order a yocto Watt. Frequencies are tuned every 100 seconds, which sets the minimum value of the coupling constant $g_{a\gamma\gamma}$ that can be probed given the noise in the experiment operating at mK temperatures. Details of the design and operation can be found in Ref. [154] (details of analysis for the related HAYSTAC experiment are in Ref. [155]).

Experiments like ADMX aim principally to reach the QCD axion field theory benchmarks of KSVZ and DFSZ coupling strengths, $g_{a\gamma\gamma} = \mathcal{O}(1)\alpha/2\pi f_a$, assuming the axion is all of the local DM. However, there is a long term goal to eventually revisit frequencies reached at DFSZ sensitivity (the lower value of the coupling) and rescan to lower fractions of the local DM density (historically, this was done returning to frequencies excluded at KSVZ sensitivity). This may be motivated by the existence of miniclusters, which reduce the local density to some 10% of its average value [156], or more broadly by the landscape of models for mixed DM and lower couplings in string theory and field theory, and uncertainties in the halo model that are not marginalised over. However, the present priority is to scan wide, since constraints on the coupling scale like the fourth root of the integration time. There are theory motivations in the "post-inflation" scenario to search at higher QCD axion masses [157], but this is not the generic expectation for "extra dimensional" axions (such as those descending from C_4) [94]:

addressing this scenario within string theory was identified as an important issue. [8]

Cliff Burgess: A Dark Horse for the Dark Sector (Naturally) [summarised by D. J. E. Marsh] This contribution explored ideas from Refs. [158–165] about the value of the cosmological constant and the existence of dynamical dark energy from the UV perspective, in particular focusing on the axion-dilaton two-field system.

In addition to these, Masha Baryakhtar (University of Wahington) presented "Pi in the Sky: Compact Objects and Exceptionally Light QCD Axions.", and Joshua Foster (Fermilab) presented about axion direct and indirect detection.

4 Conclusions

The workshop "Prospects for the String Axiverse," held at the Banff International Research Station in January 2025, was a successful conglomeration of experts on axions, string theory, and axions in string theory. The review talks presented comprehensive overviews of the state of the art in axion effective theories arising from string compactifications, as well as in axion phenomenology and detection prospects.

The research talks similarly provided stimulating updates on progress in string theory and phenomenological aspects of axions. On the string theory side, we heard from leaders in the field on using machine learning techniques to more efficiently probe axion physics across the string landscape, as well as on progress in constructing realistic cosmological scenarios involving axions in string theory. On the phenomenological side, we had research talks about fuzzy dark matter as axions, as well as about light scalars from an effective field theory point of view.

A crucial aspect of the workshop was the existence of discussion sessions, both involving the entire set of workshop participants, as well as in smaller breakout groups. The full-group discussions focused on the pressing questions about axions that string theorists should tackle and the prospects for using detection methods to differentiate between different types of axions.

The smaller group breakout sessions focused on narrower, more technical topics involving axions and the string axiverse. Group leaders posed specific questions that the groups were able to make meaningful progress on in the allotted sessions. Breakout session topics included "quantisation of axion couplings," "From Polytopes to Lagrangians: Computing Explicit Stringy Axion EFTs," and "Cosmology of Stringy Axions."

In the last decade, we have simultaneously seen an enormous amount of progress in understanding the physics of axions from string compactifications, and in experimental and observational prospects for detecting axions. The time is therefore right for utilising these two approaches to axion phenomenology in tandem and asking how the fields can inform and reinforce one another. This is precisely the task that "Prospects for the String Axiverse" set out to accomplish. In a week of lectures and discussions, progress towards this goal is

[8] This discussion was motivated by comments from Matt Reece.

now well underway. This workshop marks the start of a fruitful relationship between the more theoretical and observational axion communities, and will hopefully culminate with the world's next detection of physics beyond the Standard Model.

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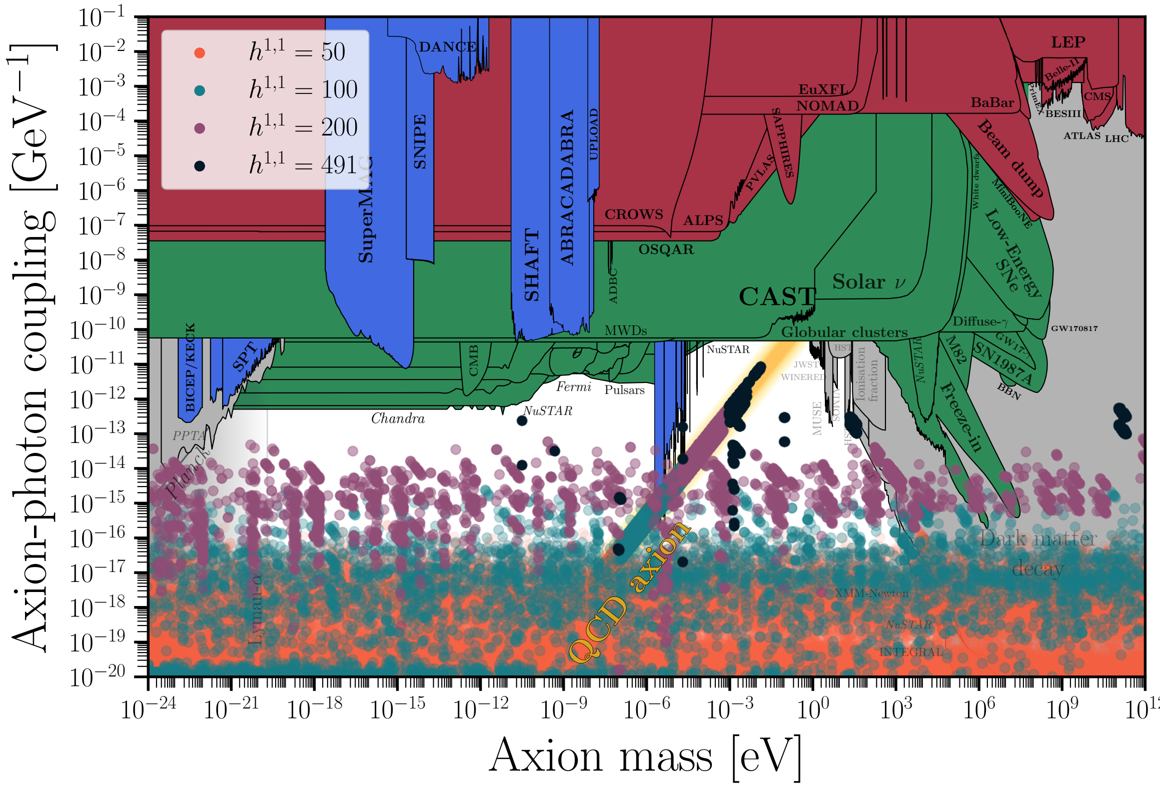

Figure 3: Distribution of axion masses and axion photon coupling, $g_{a\gamma\gamma}$, for different $h^{1,1}$. Also shown are single-axion laboratory, astrophysical, and cosmological constraints. Laboratory (astrophysical) constraints assuming axions make up all of dark matter are shown in blue (light gray), and laboratory (astrophysical) constraints that are independent of the axion's contribution to the dark matter abundance are shown in dark red (green).