"Alberta Number Theory Days XVI" 25w2030 (Banff)

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Website: https://www.birs.ca/events/2025/2-day-workshops/25w2030

1 Overview of the Field

Number theory is a central and broad area of research with an immense amount of applications to other areas of mathematics and science. It is also an area of research that is extremely active and diverse. In recent years, there have been significant advances especially in the areas of algebraic and analytic number theory. The subject of number theory may be divided into several sub-disciplines that range from pure mathematics to more applied areas such as computational number theory and mathematical physics. Some of these sub-disciplines are algebraic number theory, arithmetic geometry, analytic number theory, automorphic forms and representation theory, computational number theory and cryptography. All of these fields are well represented among the Albertan number theorists from Calgary, Edmonton and Lethbridge and other institutions in the province.

2 Scientific Highlights

Alberta number theorists are working in a wide range of subdisciplines, including analytic number theory, arithmetic geometry, modular forms and L-functions, the Langlands programme, computational number theory, arithmetic statistics, algebraic dynamics and diophantine geometry, and the interactions of number theory with mathematical physics and cryptography.

There were a total of 14 presentations given: four 50-minute plenary talks, and ten 25-minutes talks primarily given by postdoctoral fellows and graduate students. All the talks were delivered in person in TCPL 201 at BIRS. Out of all presentations, 12 video recordings were shared to the public. We aimed to actively include several early-career researchers, post-doctoral fellows, and graduate students, in our speaker list. We also asked speakers for their talks to be graduate-student-friendly. We are proud to say that graduate students and postdoctoral fellows account for 64% of the speakers (9 out of 14) at this event.

2.1 Plenary Lectures

We invited two speakers from outside of Alberta and two speakers from within Alberta to give presentations on their research. These speakers were carefully selected: it was important to ensure that the research presented fell under the umbrella of research topics supported by Alberta institutions. The 4 plenary speakers (and approximate research area) were Alia Hamieh (Modular Forms, Moments of L-functions, and Rankin-Selberg Convolutions), Manish Patnaik (Representation Theory, Automorphic Forms, and Loop Groups), Shabnam Akhtari (Number Theory, Geometry of Numbers, and Diophantine Analysis) and Habiba Kadiri (Analytic Number Theory). The titles and abstracts of their talks are included below:

- **Alia Hamieh (University of Northern British Columbia)**
  Title: *Moments of Rankin-Selberg L-functions in the prime-power level aspect*

Abstract: In this talk we discuss joint work with Fatma iek on establishing asymptotic formulae with power saving error term for first and second moments of Rankin-Selberg L-functions $L(1/2+i\mu, f\otimes g)$ where f varies over newforms of conductor p^n with p being a fixed prime and $n \to \infty$.

- **Manish Patnaik (University of Alberta)**
  Title: *Borel-Serre type constructions for Loop Groups*

  Abstract: (Joint work with Punya Satpathy) For a reductive group G, Borel and Serre introduced a compactification of a large class of arithmetic quotients of the symmetric space attached to G. After reviewing some aspects of their construction, we explain how to generalize it to the case when G is replaced by an infinite-dimensional analogue LG, the loop group of G. Along the way, we describe a partition of an arithmetic quotient of LG, inspired by the work of P.-H. Chaudouard for GL_n and related to earlier constructions of Harder-Narasimhan and Behrend.

- **Shabnam Akhtari (Penn State University)**
  Title: *Index Form Equations and Monogenized Orders in Quartic Number Fields*

  Abstract: An order O in a number field is called monogenic if it can be generated by one element over the integers, that is $O = \mathbb{Z}[\alpha]$. In this case we call α a monogenizer of O. Since $\mathbb{Z}[\alpha] = \mathbb{Z}[\pm\alpha + c]$, for any integer c, we call two algebraic integers α and α' equivalent if $\alpha+\alpha'$ or $\alpha-\alpha'$ is a rational integer. By a monogenization of O, we mean an equivalence class of monogenizers of O. Gyry has shown that there are finitely many monogenizations for a given order. An interesting (and open) problem is to count the number of monogenizations of a given monogenic order. First, we will observe, for a given order O, that $O = \mathbb{Z}[\alpha]$ in α, is indeed a Diophantine equation, namely an index form equation. Then we will modify some algorithmic approaches, due to Gal, Peth and Pohst for finding solutions of index form equations in quartic number fields to obtain new and improved upper bounds for the number of monogenizations of a quartic order. If time permits, we will discuss how the resolution of an index form equation can give precise information on the number of distinct monogenized orders of a given index.

- **Habiba Kadiri (University of Lethbridge)**
  Title: *An explicit version of Chebotarevs Density Theorem*

  Abstract: This talk will first provide a survey of explicit results on zero-free regions and zero densities of the Riemann zeta function and their relationship to error terms in the prime number theorem. We will extend this discussion to Dirichlet L-functions and Dedekind zeta functions, exploring estimates for error terms in prime counting functions across various contexts, with specific attention to number fields. Chebotarev's density theorem establishes that prime ideals are equidistributed among conjugacy classes of the Galois group in any normal extension of number fields. While Lagarias and Odlyzko first established an effective version of this theorem in 1977, we present an explicit refinement of their result. This research represents joint work with Sourabh Das and Nathan Ng.

2.2 Junior research highlights

We invited junior speakers from institutions within Alberta who had not had the opportunity to share their research in previous ANTD meetings. These talks exhibited the breadth of research being conducted in Alberta: topics included Analytic, Algebraic, and Computational Number Theory, Automorphic Forms, and Representation Theory. We scheduled 10 short talks by junior speakers, including one faculty member, four postdocs, one PhD student, and four master's students. We gave all junior speakers 25 minutes for their talks and 5 minutes for questions.

Talks by **Master's students**:

- **Amir Abbas Asgari (University of Calgary)**
  Title: *Automation of generating explicit formulas for the divisor class arithmetic of hyperelliptic curves*

- **Vincent Macri (University of Calgary)**
  Title: *From hyperelliptic to general Jacobian arithmetic*

- **Tri Nguyen (University of Alberta)**
  Title: *On the semi-stability lattice in higher dimensions*

- **Golnoush Farzanfard (University of Lethbridge)**
  Title: *Explicit zero density for the Riemann zeta function*

Talk by **Ph.D. Student**:

- **Kristaps Balodis (University of Calgary)**
  Title: *Smooth orbits, and representations of p-adic groups*

Talks by **Postdoctoral Fellows**:

- **Antoine Leudiere (University of Calgary)**
  Title: *Computations in zero and positive characteristics*

- **Yanze Chen (University of Alberta)**
  Title: *Whittaker coefficients of metaplectic Eisenstein series and Weyl group multiple Dirichlet series*

- **Emily Quesada-Herrera (University of Lethbridge)**
  Title: *Fourier optimization and quadratic forms*

- **Sumin Leem (University of Calgary)**
  Title: *Fast arithmetic in the divisor class group*

Talk by **Faculty member**:

- **Eric Roettger (Mount Royal University)**
  Title: *A Lucas-Lehmer Primality Test for the Belphegor Numbers*

The question sessions were very active and many continued into the breaks/scheduled social times. Conference participants enjoyed many productive conversations, and left the conference with new ideas for their work.

3 Comments

3.1 Facilitating communication among participants

The structured schedule at BIRS, including coffee breaks and designated social times in the evenings, played a pivotal role in facilitating communication among participants. These intervals provided valuable opportunities for attendees to exchange ideas about their current academic positions, career aspirations, and ongoing research projects. Abbas Maarefparvar, Samprit Ghosh, and Greg Knapp respectively took charge of chairing Saturday morning, Saturday afternoon, and Sunday morning sessions. This responsibility included transferring the talks and facilitating the exchange of questions between speakers and attendees, both in-person and online.

3.2 Fostering a welcoming and inclusive environment at BIRS

In alignment with our sponsors' (BIRS, PIMS, NTF and JNT) statement on Equity, Diversity, and Inclusion (EDI), this edition of Alberta Number Theory Days diligently upheld principles that foster an open, diverse, and inclusive academic community. This year, approximately **43%** of all talks were delivered by female speakers. This includes **75%** of the plenary talks and **33%** of those by postdoctoral researchers and students, highlighting our commitment to supporting emerging female mathematicians.

Our conference included 34 participants: 25 of those participants were supported by BIRS, eight supported themselves through grants, and one participant attended remotely. (The conference format was hybrid, allowing us to include a student from the University of Lethbridge who otherwise would not have been able to attend.) Regarding gender, 11 of the 34 conference participants are women. The composition of senior faculty—predominantly male due to the demographics of Alberta number theorists—affected this statistic. Regarding career stage, there were 11 students, 7 postdocs, and 16 faculty members. Regarding visible minority status, 20 of the 34 conference participants are visible minorities. Our setup not only adhered to but also propelled our vision for a mathematical community enriched by diverse perspectives and equitable opportunities. Through deliberate planning and dedicated effort, ANTD provided a supportive and safe environment, where talented people from underrepresented groups in mathematics were encouraged to showcase their work, thereby supporting those individuals' careers and enriching the quality of the conference.

4 Outcome of the Meeting

The 16th edition of the event continued its tradition of showcasing the emerging talents of Calgary, Edmonton, and Lethbridge. A significant number of presentations were delivered by graduate students and postdoctoral researchers, marking the first formal research talk for several speakers. These presentations were met with enthusiastic engagement, constructive feedback, and supportive questions from an interested and welcoming audience. The sessions fostered a lively exchange of ideas, evidenced by extensive discussions that spilled over from formal talks to informal social gatherings, catalyzing new and ongoing collaborations.

This years program featured four plenary talks by distinguished academics—three associate professors and one professor. While two of these speakers came from outside Alberta, their inclusion was strategic, aimed at extending the local research network and exposing Albertas early-career researchers to seminal work in their respective fields. Maintaining and fostering these academic relationships is crucial for the community's growth, particularly in supporting its newer members.

Among the 14 talks, 12 video recordings from the conference are available to the public, including three of the plenary lectures.