Synergy of the Formulation of Atmospheric Convection Parameterization

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April 23, 2025–May 9, 2025

1 Backgrounds

The team meeting focused on the so–called mass–flux convection parameterization, whose basis was laid down by Arakawa and Schubert (1974), and this formulation is adopted in majority of the operational forecast models as well as the climate models today, yet not without problems.

The need for this team meeting emerged through informal discussions between the two senior scientists, BK and JIY, who had been engaged on the investigation of this problem as that of basic mathematical formulations, but different perspectives. BK's PhD student, EL, arrived at the concept of segmentally–constant approximation (SCA) independent of JIY (Yano 2014), was also asked to participate by further promoting his initiatives.

2 Overview of the Meeting

The team meeting was organized around the two fundamental issues of the mass–flux convection parameterization: the closure and gray–zone problems. The discussion of the closure problem was focused on the prognostic version of closure originally sketched out by Arakawa and Schubert (1974), and subsequently pursued by Pan and Randall (1998), Yano and Plant (2012a, b), and Khouider and Leclerc (2019). All of them focus on the problem of the convective energy cycle, so was the team discussion as the adopted header below suggests.

The gray–zone refers to the question of the convection parameterization formulation when convection is almost resolved, but yet still needs to be "parameterized" in a certain manner. The ultimate answer to this question is clear from the analysis of Yano (2014) that we must adopt a representation of the subgrid–scale processes obtained by the application of the segmentally–constant approximation (SCA) to the cloud–resolving model (nonhydrostatic–anelastic model, NAM), but without any further approximations nor assumptions. This formulation can be referred as NAM–SCA.

From the operational point of view, this problem is better addressed backwards, and as that of relaxing various constraints in the existing operational operational schemes. The team discussion was organized with this latter perspective, and we addressed various issues from this perspective. Those issues are reported under several headings in the following.

The following report focuses on the synergies that has emerged from the team meeting as well as important perspectives for the further investigations, rather than trying to report every item that has been discussed.

3 Convective Energy Cycle and the Closure

Arakawa and Schubert (1974) propose the convective energy–cycle system as a basis for developing the closure for the mass–flux convection parameterization formulation. This system consists of the prognostic equations for the cloud work function, A, and the convective kinetic energy, K, with the additional unknown, the convective mass flux, M_B, defined at the convection base. Thus, an additional condition is required to solve this system in a closed manner.

The most formal approach to address this question is to write down a formal relation between the convective kinetic energy, K, and the mass flux, M in a formal manner:

$$K = \frac{M^2}{2\sigma} \tag{1a}$$

where σ is the convection fraction. The mass flux, M, is related to its convedtion base value, M_B, by

$$M = \eta M_b, \tag{1b}$$

where η is a normalized vertical profile of the mass flux, which is defined by prescribed entrainment and detrainment rates. Thus,

$$K = \frac{\eta^2 M_B^2}{2\sigma} \tag{1c}$$

Obviously, the problem is even less well posed by introducing another unknown, σ.

Khoudier and Leclerc (2019) attempt to solve this problem (Eq. 1c) by introducing the transform equation for σ based on a phenomenological formulation for the transformations between the clouds types based on a Markovian formulation. This approach relies on stochastic lattice model from statistical mechanics, where various cloud types interact with each other and with the environment based on conditional probability rules motivated by observations (Khouider *et al.* 2010; Khouider 2014). In the mean field limit, the probabilistic model leads to a system of deterministic equations for the evolution of σ. When the variations of σ, in the case of single cloud type, were incorporated to the energy cycle equations together with the evolution equation of the cloud work functions, Khouider and Leclerc found a menagerie of rich dynamics including damped oscillations, limit cycles, and chaos which overshadows the quasi-equilibrium assumption which is the basis for closing the mass-flux formulation by Arakawa and Schubert (1974) and many others. It is worth noting that an earlier study by Pan and Randall (1998) suggested that when σ is kept constant, the energy cycle undergoes damped oscillations.

A simpler approach to indirectly take into account the variation in sigma, from a purely theoretical point of view, is to set the relation between K and M_B in a more phenomenological manner, by setting:

$$K = \alpha M_B^{\nu} \tag{2}$$

where α is a constant depending on the convection types, and ν is a power to be specified. Here, to keep the formula (2) analytical, we focus on the two possibilities, $\nu = 1$ and 2. We can show that for $\nu > 2$, the equation for K becomes singular as $M_B \to 0$. Yano and Plant (2012a) show that the choice of $\nu = 1$ is favored against that of $\nu = 2$ by reviewing the existing CRM (cloud–resolving model) results.

The case with $\nu = 1$ is mathematically more fascinating, because the equation for K becomes fundamentally nonlinear, consistent with the findings of Khouider and Leclerc (2019). A weakly nonlinear formulation has been developed, and nonlinear interactions matrices, that characterized the evolution of the system, have been identified. These interactions matrices can be evaluated from the data from the observational arrays organized during field campaigns, which are expected to provide insights on the basic nature of the convection interactions.

4 Ensemble Plume Dynamics with SCA

In proposing the modification of the existing formulation for convection parameterizations towards the gray zone, we must keep in mind that the inherently conservative mentality of the operational community. For this reason, the proposed modifications must be stepwise to be acceptable for the operational purposes. Unfortunately, such an approach also inherently interferes withe mathematical attitude of the strictness that tends to avoid to accept any *ad hoc* assumptions. However, here, we need to seek a certain compromise.

Under the notations introduced in considering the convective energy cycle, the gray–zone problem is more formally stated as the formulation for representing the subgrid–scale convection asymptotically as $\sigma \to 1$. Note that in this asymptotic limit, the convection is no longer "parameterized" in its proposer sense: see a good discussion on the notion of "parameterization" in Ooyama (1982). Convection must be *almost* explicitly introduced, but not quite. And in the final limit of $\sigma = 1$, an explicit convection must fill out the given grid box. Thus, the gray–zone representation of convection must be designed in such a manner that it continuously transits into explicit convection as $\sigma \to 1$.

An obvious and critical first step toward the gray–zone formulation is to remove one of the key assumptions in the standard mass–flux formulation, *i.e.*, the steady–plume hypothesis. Here, by following the principle of the stepwise modification, we adopt exactly the same set of governing equations for the subgrid–scale convection variables, but merely re–introduce the time dependencies into those governing equations. More specifically, we choose not to explicitly introduce the equation for the convective vertical velocity, which is not included in this standard set.

Here, more strictly, most of the operational models introduce an equation for the convective vertical velocity in one way or another for practical purposes, as explained in Yano (2014), but rather in *ad hoc* manner. Introducing the equation for the convective vertical velocity properly is of its own efforts to be discussed later. Thus, this strategy contradicts with the principle of the stepwise modification introduced above. Instead, we adopt the prognostic kinetic energy to predict the evolution of the convection–base mass flux, M_B.

A major difficulty in proceeding in this manner is the partitioning of the mass–flux profile, $\eta(z)$, into the part, $\eta_\sigma(z)$, due to the convection fraction, σ, and $\eta_w(z)$, due to the convective vertical velocity, w_c. Thus, $\eta(z) = \eta_\sigma(z)\eta_w(z)$. This partitioning remains arbitrary with no clear principle to follow. Another difficulty is to predict the temporal evolution of σ with time. It must be emphasized that being SCA an artificial subdivision of the grid–box domain, again, there is no objective principle to predict its evolution.

To avoid both difficulties together, we simply set σ to be constant both in time and height. From a point of view of the standard mass–flux formulation as formulated by Arakawa and Schubert (1974), this rather artificial assumption has no consequence nor of serious concern, because it is formulated with the asymptotic limit of $\sigma \to 0$, thus the value of σ plays no part in its formulation, not to mention its time evolution. From a point of view of reproducing the entraining–plume dynamics, neglect of the height dependence of σ may appear to be a serious drawback. However, for the purpose of representing atmospheric convection as a subgrid–scale process, this is probably the best choice is to make it constant with height, because this profile is much closer to observed convection than the profile of the entraining plume. For this reason, in many operational implementations, σ is set constant with height, or close to it, as discussed in Sec. 7 of Ch. 10 (de Rooy *et al.* 2015) of the monograph by Plant and Yano (2015).

The final formulation is rather lengthy, and to be presented elsewhere soon with associated numerical results, that we plan to pursue.

5 Convective Pressure Problem

As mentioned above, the current operational mass–flux parameterizations introduce the governing equation for the convective vertical velocity for the practical purposes. However, the adopted formulation is based on a drastically simplified momentum equation based on the drag formulation. In short, the pressure term is neglected in the formulation: development of a solver for the convection pressure is a crucial operational need for this reason.

Lack of the proper formulation for the pressure term is also a major obstacle proceeding to a fully prognostic formulation for the subgrid–scale convection representation, say, based on SCA. The present team meeting made a great deal of progress with this convective pressure problem.

Technical notes (Yano 2015a, b) provided a basis for the discussions. As a result of these discussions, various shortcoming of the formulations in these notes are pointed out, and a way for developing a mathematically more rigorous formulation was outlined. It is also likely to drastically simplify the formulation. The full formulation is still under development, and will be reported elsewhere.

6 Fully Prognostic Formulation for the Subgrid–Scale Convection Representation

The final goal of our team meeting has been to develop a formulation for the fully prognostic formulation for the subgrid–scale convection representation based on SCA. For this reason, we had extensive discussions on this subject.

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