Non-Newtonian Flows in Porous Media

Ian Frigaard (University Of British Columbia),
Arezoo Ardekani (Purdue University),
Sujit Datta (Princeton University),
Alex Hansen (NTNU, Norway),
Laurent Talon (CNRS, Univerist-Paris-Saclay)

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1 Overview of the Field

Porous media abound in natural and industrial settings: rocks, soil, fractures, biological tissues, cements, bricks and ceramics, to name a few. Flow through the pore space of these media is of classical interest in hydrogeology and reservoir engineering, but also in many newer application fields e.g. filtration, acoustics, energy, environmental science, and biotechnology. The perennial challenge of porous media flow is that of scale. While pore scale flows can be effectively computed individually, the scales of interest for applications are typically much larger. Either one works directly with characterization of the large-scale flows (e.g. Darcys law), or one works first with the pore scale followed by some form of upscaling or homogenization. The first approach is largely empirical, useful for computations and approximations of well-characterised media, but has limitations in building deeper understanding and generalizing to unusual situations.

While the second approach is viable for some simple pore geometries (tube bundles, channels, packed spheres), the real pore structure can be more complex as the pore shape and size are usually spatially heterogeneous. This heterogeneity presents a significant degree of randomness, which can be very extreme such as fractal geometries (in the case of fractures for instance). For simple Newtonian fluids (e.g water, oil, etc.), this has led to a rich diversity of mathematical, statistical, physical and computational techniques, both for characterizing the pore space and understanding the associated flows. At one extreme, actual pore space of a rock sample is imaged (e.g. via MRI), digitized and meshed in 3D, followed by solving the linear Stokes equations computationally to determine the flow law. Alternatively, highly randomized pore-throat network models are developed to capture statistics of the flow. Other natural settings contain a mix of permeable continuous phase punctuated by fissures and channels along which liquid flows more freely. Depending on the application, one might wish to estimate transport in both structural components, particularly if heat or contaminants are also of concern, e.g. geothermal energy applications.

The above paragraphs describe porous media flows of simple Newtonian fluids which are characterised by a linear response to the applied pressure gradients. There are however many rheologically complex materials that lie somewhere between Newtonian fluids and solids in their mechanical responses. These non-Newtonian fluids include polymer solutions, slurries, suspensions, foams, pastes, etc. These materials have an internal structure that strongly influences their mechanical behavior in response to applied stresses. Mechanical behaviors observed include nonlinearity, time dependency, memory or deformation history effects. Descriptors such as: shear-thinning, shear-thickening, thixotropic, yield stress/visco-plastic and viscoelastic encompass these behaviors and have been used to develop mathematical descriptions of the materials (constitutive laws). One of the main difficulty, but also the richness, of studying this type of flows lies in the understanding of the coupling between the heterogeneity of the medium and the non-linear rheology of the material.

These more complex constitutive laws can produce a wide range of flow dynamics that are not fully understood, even in simple geometries. Moving beyond simple generalizations of Darcys law and addressing porous media geometries remains a formidable challenge, to be faced in this workshop. Even without complex fluids, porous media can produce flows which themselves might be described as non-Newtonian. For example, this can arise in two-fluid (displacement) flows from capillary pressure and/or wetting effects. Coupling of fluid behavior with that of a deformable or compressible porous media can also lead to nonlinearity in the flow law and a limiting pressure gradients, required to “open pathways”.

Lastly, there arise many modelling problems of scale. Hydrogels, suspensions, foams, viscoelastic polymers etc., all have inherent structural length-scales and dependent on the pore-scale our understanding of the complex fluids as continua becomes limited. For example, colloidal micron-sized clay fines are common in many mineral suspensions with yield stress, but for pore-scales ≤ 10 diameters individual particle behaviour becomes increasingly relevant. Gel structures can be micron-scale or smaller. Foam bubbles are sheared in porous media flow. Polymer chain length-scales are typically smaller. Thus, different multi-phase flows along micro-fluidic channels have become relevant to understanding the limits of continuum descriptions.

2 Recent Developments

The objective of this meeting was to bring together experts in different fields: complex fluids mechanics, porous media, statistical physics, as well as different disciplinary approaches (experimental, computational, analytical modelling), to share their different approaches and share advances in specific applications. As applications arise in a vast range of geophysical, engineering, ecological, environmental and biological settings, the workshop attendees were widely interdisciplinary. Both the porous media and the rheology/non-Newtonian fluid mechanics communities are themselves highly interdisciplinary, but have not interacted extensively. This was evident in the presentations at 1st workshop, held in Fortaleza, Brazil, June 28-30, 2022. Here, we had strong representations from both communities and the presentations went deeply into a number of specific directions, grouped loosely as follows.

1. Underlying aim of porous media: exploring the pore scale to be able to upscale and make quantitative prediction on the (Darcy-law) macro-scale.
2. Viscoelasticity dominated flows at the pore scale: from linear relaxation, retardation to nonlinear viscoelasticty; elastic turbulence, understanding its relevance.
3. Yield stress dominated flows: pore-scale dead zones/flowing area, open pathways.
4. Multi-phase flows and other constitutive models in porous media.
5. Computational methodologies from continuum modelling of pore-scale flows through to increasingly exotic pore-throat network models.

In addition, as the workshop also served to bring researchers in closely aligned areas into this field, there were many presentations driven by applications, or exploring interesting physicochemical, biological and mathematical flows and effects.

Moving back 20 years, a typical approach to dealing with non-Newtonian effects in porous media was to effectively impose analogous mathematical constructs onto a modified Darcy law. For example, taking linear elasticity effects and including relaxation and/or retardation terms into a linear Darcy law. Also common was to see e.g. Darcy’s law changed algebraically to a nonlinear filtration with power law behaviour matching that of the supposed power law fluid. Yield stress effects were modelled as a limiting pressure gradient flow. Many other models used an ad hoc representation of the effective viscosity, substituted into Darcy's law. Sadly, it must be admitted that many of these approaches were motivated by mathematical convenience, leading to new PDE problems and analyses, but with little concern for physical validity. A number of trends have corrected the above misdirection and are leading to new directions.

In viscoelasticity, constitutive models such as the FENE family, Giesekus, White-Metzner and PTT families have become widely accepted and used for polymeric liquids, moving firmly away from linear viscoelasticity. These models are reliably coded in open source software codes. Numerical high Weissenberg number effects, that led to often spurious results and model limitations, have been alleviated via the log-conformation tensor approach (and similar), so that there is increased confidence that computed effects are predominantly physical. This has led to computational studies in increasingly complex geometries, e.g. expansion-contraction, of relevance to porous media. Elastic turbulence, occurring at low Reynolds numbers and triggered by streamline curvature effects, has received much attention. The relevance to porous media flows, with tortuous pathways, has become increasingly evident and many researchers have begun to explore these flows: either directly in porous media or via more regular micro-fluidic channels.

For generalised Newtonian fluids, the ability to compute pore scale flows came earlier and there have been numerous flows computed through e.g. packed beds, tube bundles, etc., with the main effect that Darcy law-scale closure laws can be much better informed than earlier. At higher (laminar) flow rates and pressure gradients, these nonlinear filtration laws tend to mimic the nonlinearity of the underlying constitutive model at high shear, as can also be argued theoretically. However, the pore-scale flows are more complex and particularly as the flow develops from zero with increasing pressure gradients. Parts of the pore space are either very slowly moving, recirculating or static dead zones. This is particularly true with yield stress fluids, where it is found that the flowing region of the pore space self-selects and is itself a function of the flow rate (pressure gradient). Fundamentally, this changes the validity of specifying flow laws with only simple geometric metrics such as the porosity. Moving away from the pore scale and studying larger regions of porous media, the dynamics of path opening has also been recognised to strongly affect the flow law dynamics. More clearly, not only is there a nonlinear effect of flow along a pathway through the pore space, but also the number of pathways open to flow, changes dynamically as the pressure gradient is increased. These phenomena have been studied increasingly using pore-throat network approaches.

Other rheological trends of the past 20 years have included extensive interest in suspension flows, both colloidal and non-colloidal. Aside from flows of suspensions through porous media, applications of relevance to porous media include those of sealing/filling a pore-space with a suspension, e.g. proppant transport in hydraulic fracturing, grouting of foundations and tunnels, squeeze cementing. In these flows the solid phase eventually packs the cavity and the solute then flows through the bed, i.e. we have a transition from suspension flow with interphase momentum coupling to a porous media flow. The study of material exhibiting aging or (non-elastic) structural evolution has led to a plethora of thixotropic models, often coupled to viscoelasticity, e.g. the BMP family of models, the VCM model for micellar fluids, etc.. The inclusion of viscoelasticity into yield stress models has sparked a trend of developing and studying EVP (elasto-visco-plastic) models, e.g. the Saramito models, the IKH and the KDR. Foam dynamics interest has been driven by application, but also individual bubble, particle and droplet flows through porous media channels are being studied. Related to these, bacterial low Re swimming is popular, also used to probe the structures of gels and porous media as active tracers. Lastly, there are many fluid-fluid displacement applications in small cavities and along pores, e.g. associated with hydrocarbon extraction and sealing of wells, which lead to non-Newtonian porous media flows, either due the fluids themselves or to capillary/wetting effects.

3 Presentations and participation

In total 44 technical talks were given, including a number of online presentations in hybrid format. This was supplemented by an overview talk regarding Porelabs and by various ad-hoc summing up discussions during the week. On the initial Sunday night after dinner reception, participants presenting later in the week were invited to give a 5 minute chalk talk in the lounge, to break the ice and encourage equal discussion. In total the workshop had 88 registered participants, many listening on line and some observers attending at Banff. The technical presentations given were as follows.

- Arezoo Ardekani (Purdue University): Stability and dispersion of viscoelastic flows through porous media
- Vedad Dzanic (Queensland University of Technology): Numerical characterisation and control of viscoelastic instabilities through porous media
- Emad Chaparian (University of Strathclyde): Percolation of yield-stress fluids in porous media: yield limit and a general Darcy-law
- Seyed Mohammad Taghavi (Universit Laval): Rheological and geometric interactions in viscoplastic fluid transport through grooved channel
- Paul Grassia (Eindhoven University of Technology): Three Bubbles Good, Two Bubbles Better: A Foam Hadron Collider
- Miguel Beneitez (University of Cambridge): Instabilities in rectilinear flows and their link to viscoelastic turbulence with and without inertia
- Pedro Ponte Castaeda (University of Pennsylvania): Variational Linear Comparison Estimates for the Flow of Yield-Stress Fluids through Porous Media
- Federico Lanza (University of Oslo): Dynamic instability of a temperature-dependent viscous fluid in a Hele-Shaw Cell
- Vivek Narsimhan (Purdue University): Microhydrodynamics of spheroids in weakly viscoelastic fluids - lift forces, orientation dynamics, and effective stress
- Christopher Bowers (North Carolina State University): Modeling generalized Newtonian fluid flow in porous media at the macroscale: New models based on averaging theory
- Ilaria Beechey-Newman (NTNU, Norway): Hierarchical Drying Patterns of Colloidal Suspensions Under Confinement
- Rob Poole (University of Liverpool): Viscoelastic fluid flow in microporous media
- Alex Hansen(NTNU, Norway): A New Kind of Thermodynamics for Two-Phase Flow in Porous Media
- Emily Chen (Princeton University): Viscoelastic flow instabilities in porous media: insights from pore-scale flow fields
- Davide Picchi (University of Brescia): Decoupling rheological and geometry effects on the description of flow in confined environments
- Sarah Hormozi (Cornell University): Motility of Bacteria in elastoviscoplastic biological fluids
- Marco Edoardo Rosti (Okinawa Institute of Science and Technology): The effect of elasticity and plasticity on model porous media flows
- Holger Stark (Technische Universitt Berlin): Swimming and Rheology of Active Suspensions in Non-Newtonian Fluids
- Laurent Talon (CNRS, Univerist-Paris-Saclay): On the statistical properties of yield stress fluid in porous media
- Isaac Pincus (Massachusetts Institute of Technology): Rheology and linear dichroism of dilute solutions of flexible and semiflexible polymers in shear flow
- Giovanniantonio Natale (University of Calgary): Dynamics and microstructures of Pseudomonous aeruginosa biofilms at fluid-fluid interfaces
- Duncan Hewitt (University of Cambridge): Simple yield-stress fluids in slots and channels: clogging and channelling

- Hossein Hassanzadeh (Universit Laval): What happens when a jet meets viscoplasticity? Insights from mixing to fingering and fracturing regimes
- Hossein Rahmani (University of British Columbia): Network modeling of yield stress fluids in porous media: a Hele-Shaw cell flow analogy
- Hadi Mohammadigoushki (Florida State University): Magnetophoresis of Newtonian Metal Salt Fluids in Porous Media
- Yohan Davit (CNRS): Localized stress patterns control viscoelastic flows in porous media
- Chandi Sasmal (Indian Institute of Technology Ropar): Preferential paths and birefringent strands formation in viscoelastic fluid flow through porous media
- Sibani Lisa Biswal (Rice University): Bubble Trapping in Flowing Foam in Porous Media
- Amir Pahlavan (Yale University): Bacterial chemotaxis and dispersion in porous media
- Marcel Moura (University of Oslo): Linking anomalous diffusion and rheology for a droplet spreading in a corner: A rheometer for power-law liquids
- Outi Tammisola (KTH Royal Institute of Technology): Elastoviscoplastic fluid flow through randomized porous media
- Ian Frigaard (University of British Columbia): Let me tell you about my problems
- James Hewett (University of Canterbury Christchurch): Modelling and estimating the viscoelastic properties of blood clots
- Claudio Fonte (University of Manchester): Network modelling of viscoplastic fluid flow in randomly disordered porous media
- Prabir Daripa (Texas A&M University): Some aspects of modeling chemical enhanced oil recovery and fracturing instability in complex fluids
- Vinicius Gustavo Poletto (Federal University of Technology - Parana): Simulation of generalized Newtonian Fluid Flow in porous media applied to drilling fluid hydraulics and lost circulation control
- Santanu Sinha (NTNU, Norway): Non-linear growth of viscous fingers in two-phase flow of immiscible Newtonian fluids in porous media
- Erika Eiser (NTNU, Norway): Microrheology of Hydrogels
- Mahdi Izadi (University of British Columbia): Dynamics and Stoppage Mechanisms in Viscoplastic Suspensions: Modelling Capillary Forces, Pore Blockage, and Yield Stress Effects
- Parisa Mirbod (University of Illinois at Chicago): Viscoelastic Flow Stability overlying Porous Media: Insights from Linear Stability Analysis
- Gwynn Elfring (University of British Columbia): Viscoelastic propulsion due to rotating bodies
- Jos Soares de Andrade Jr. (Universidade Federal do Cear): Flow through three-dimensional self-affine fractures
- J. Esteban Lpez-Aguilar (UNAM): Non-Newtonian Flow Modelling in Fractal Porous Media
- Quirine Krol (NTNU, Norway): Microscale fluid fluctuations during drainage and imbibition in porous media measured with rapid NMR profiling: The non-linear relation between macroscopic flow rate and local flow dynamics

See Fig. 1

Figure 1: Participants at BIRS 2024

4 Scientific progress and outlook

Overall, the workshop was a great success in fostering connections between quantitative researchers interested in both porous media flows and complex fluids. It takes time to understand our differences and commonalities, particularly when coming from different research communities that speak a different language. In this sense, in person workshop-style meetings are an indispensable part of any new scientific direction.

4.1 Underlying aim of porous media: from the pore scale to the (Darcy-law) macroscale

A central problem in porous media flows is to find effective constitutive flow equations at the Darcy scale, i.e. the scale at which the porous medium appears to be a continuum, but before large-scale heterogeneities set in. A number of attendees (Bowers, Hansen, Lpez-Aguilar, Picchi, Ponte Castaeda) presented work that touched on different aspect of this key problem.

At the pore level, one may be dealing with a single non-Newtonian fluid flowing, a mixture of Newtonian fluids that are immiscible and develop non-Newtonian behaviour, or fluids mix and the mixture may have non-Newtonian characteristics. After upscaling to the Darcy scale, one will be dealing with one effective fluid that has characteristics that are different from the pore-level fluids. This is e.g. observed when dealing with a mixture of two-immiscible Newtonian fluids at the pore scale, transforming into a single fluid where the flow velocity depends on the pressure gradient to a power different from one. During the workshop, different approaches to this problem were presented.

One such approach was based on the TCAT (Thermodynamically Constrained Averaging Theory) approach, which essentially consists of converting averages over gradients into gradients of averages. This was used to consider non-Newtonian fluids at the pore level. Another approach was based on an adaption of statistical mechanics, i.e. the method that derives thermodynamics from an atomistic picture. Here, this approach was used to derive effective constitutive equations at the Darcy scale when the pore scale immiscible two-fluid mixture was Newtonian. More direct approaches consisted of assuming fractal scaling properties of the topology of the porous medium, making it possible to calculate the effective behavior of the non-Newtonian fluids at the Darcy scale.

Other work used a variational formulation to make estimates of nonlinear permeability (effectively bounds), using the comparisons with Newtonian results. Others considered more classical applied mathematics methods, studying channels and fractal distributions of tubes. In more specific studies (viscoelastic and visco-plastic, summarized below), other researchers developed upscaled constitutive flow laws from detailed study.

As a core problem of porous media flows, there remain unanswered questions. A goal for the next workshop is to see if we can apply these different formal methods to wider results, e.g. from the many 2D pore scale simulations that were showcased. With the new insights into unstable transient viscoelastic fluid flows, there were also some question raised about whether closure expressions (flow law) will need to be time-dependent or not¿

4.2 Viscoelasticity dominated flows at the pore scale

Viscoelastic fluids in porous media were the topic of many talks (Ardekani, Beneitez, Chen, Davit, Dzanic, Mirbod, Pincus, Poole, Samsal). There has been an explosion of activity in elastic turbulence in microfluidic geometries over the past 5-10 years and this is now translating to porous media flows. The attractiveness of instigating elastic turbulence at the pore scale for applications is twofold: (i) that the turbulence increases the flow resistance, which may be useful in displacement type flows, e.g. enhanced oil recovery; (ii) the resultant mixing will enhance dispersive transport processes while keeping at low Re.

The various presentations showed clear advances on understanding of pore scale flows through idealized cylinder arrays. The phenomena identified and explored included: stress localization, channelization, onset of instabilities, dispersion, mixing and disorder. While each of these was nicely illustrated in one or more talks, it is likely that this type of study will continue to be popular for many years. Tying elastic turbulence to the intricacies of the pore geometry is very complicated, so that progress is being made primarily by studying well-defined model geometries that may give the building blocks for later upscaling to give flow laws. Elastic turbulence onset was studied and related back to instability instigated by streamline curvature effects (of Pakdel-McKinley origin). Thus, any form of model closure that hopes to include elastic turbulence effects needs to approximate the onset criterion, including curvature effects e.g. via tortuosity. Most of the results presented were computational, but there were also high quality microscale experiments presented.

While a wide range of results were presented, elastic turbulence required a leap forward in comprehension for those coming from a conventional Newtonian porous media background. Thus looking ahead, we need to continue the engagement to make sure that the advances are adopted in porous media models in a way that is pragmatic. Perhaps the weight of exciting new results with viscoelastic instabilities hides practical questions such as: does every porous media flow have these instabilities? Secondly, in attempting to upscale from the pore scale for the Darcy-law scale, do we need special methods for these fluid flows? In particular, must we develop models that are also dynamic or is the time-averaged response adequate for most applications?

Some cautionary discussion occurred. Firstly, it was noted that the community is reliant on a small subset of constitutive models for the computational simulations, without necessarily understanding the nuanced differences between models and how these might affect the flow observations. Thus broader study is needed. Secondly, there is an over-reliance on particular open source codes for generating results, and hence on the specific numerical implementations. Although reliable, study of benchmark flows needs to be performed systematically to ensure that these codes are adequately performing, i.e. slow down! Hopefully the future will also see a sharing of experimental methodologies and benchmark data, as well as more experiments conducted.

In terms of the 2D porous media geometries studied so far, mostly these have been cylinder arrays. We need to explore these in increasingly random and complex configurations. We need to study and include the effects of angular particles in these arrays, sometimes observed to result in stress singularities in processing flows. There were very few other geometries studied: e.g. affine fractures, fissures, smooth and rough uneven channels are all relevant for different porous media flows. Although 3D computations are expensive to execute, full comprehension requires that the community find better and faster ways to compute these. There are questions of whether the dynamic effects of elastic turbulence can be approximated to be included in reduced order models such as pore-throat network models and Hele-Shaw models. Results on the calculation of stress using physics informed reduced models of viscoelastic flows were also presented. Future research will focus more on data driven approaches as well as physics information models to obtain flow and viscoelastic stresses.

4.3 Yield stress dominated flows

A number of participants presented results that were associated with predominantly yield stress fluid flows and/or effects (Chaparian, Fonte, Frigaard, Hassanzadeh, Hewitt, Hormozi, Izadi, Ponte Castenada, Rahmani, Rosti, Taghavi, Talon, Tammisola). This included forays into elasto-visco-plastic fluid flows in porous media. The main tools used here were computational, based on pore throat network models, or fully 2D flows through arrays of obstacles, or flow along narrow fissures using Hele-Shaw type models.

The main phenomena, of path blockage/opening, dead zones within the pore space and more globally on the Darcy-flow scale, appear well established and observable in analogous flows computed using different underlying models. There is qualitative and sometimes quantitative agreement. There was some exploration of numerical methods, similarities and differences. It is fair to say that adaptive meshing methods for Stokes flow computations in 2D are well evolved and used effectively. Here we saw new advances in how one might optimally generate pore-throat networks from the underlying pore geometry and comparisons with 2D computations. There was work on a different 2D flow: the Hele-Shaw cell, exploring the analogy directly with 2D limiting pressure gradient flows and then with network models.

Some of the above contributed to advances in generating more effective Darcy-scale closures. Homogenization methods and scaling arguments were both used to generate estimates of the critical pressure gradient and bounds for the flow laws at high pressure gradients. A Monte-Carlo approach was used to explore the statistics of flowing areas in pore-throat networks. Thus, broadly we have had good success in comparing 2D flows, pore throat network results and upscaling. There reman however, relatively few detailed 3D Stokes flow studies of pore-scale flows. We assume that the phenomenology will be similar to 2D. 3D computations are time consuming: perhaps too expensive still to explore randomized 3D porous media effectively. However, 3D pore-throat networks should be feasible. There are also 3D structures, such as fissures and fractures, that might be studied partially using Hele-Shaw approaches and for which 3D computations may be feasible due to the narrow dimensions.

We saw some of the first EVP model results presented for porous media type flows. This led to a mix of enthusiasm and pessimism! EVP simulation and experimental results were compared favourably for expansion-contraction flows. In the porous media simulation context we saw the passage from fluids dominated by yield stress to increasingly important viscoelastic effects as the Weissenberg number was increased. Thus we observed initial local inhomogeneity and channelization on the larger scale, giving way to elastic instabilities eventually, i.e. similar to those observed in viscoelasticity dominated flows. There was some discussion of whether we really see porous media flows with fluids that bridge the range of properties from yield stress dominance to viscoelasticity dominated flows. Experimentally, this variation is being studied by mixes of PEO with Carbopol. Another future direction for the EVP studies is to extend to progressively more EVP constitutive models.

In applications, many viscoplastic fluids are suspensions at the microscale. Some presentations addressed the effects of particle size on flow/no flow and pore blockage. It is clear that in the future we need to begin modelling individual particle transport once diameters are comparable to the pore scale and in particular how these contribute to blockage, stopping and jamming phenomena that we want to model for larger scale dynamics. Other applications were specifically 2-phase or 2 fluid, e.g. critical pressures from capillary/wetting effects, flows of bubbles through yield stress fluids on the pore scale, squeeze cementing displacement/invasion flows. Nearly always these are transient flows. We have not seen many detailed 2D/3D simulations of these fluid-fluid scenarios at the pore-scale. There could be some room for advances in modelling these evolving flows using pore-throat approaches, e.g. discretizing also along the throats to capture the flow evolution.

Aside from these, we saw applications involving yield stress fluids and jets, grooved channels and bacterial motility.

4.4 Multi-phase flows

Several presentations addressed the issue of two-phase flow in porous media (Biswal, Grassia, Hansen, Hassanzadeh, Hewitt, Izadi/Frigaard, Narsimhan, Sinha). Even when both phases are Newtonian, the flow of two immiscible fluids through a disordered porous structure exhibits behaviour similar to that of non-Newtonian rheology. This is largely due to the complex interplay between capillary forces and the heterogeneity of the medium.

Foam flow is a striking example of this: the distribution of bubble sizes (referred to as the ’texture’) and the spatial configuration of bubbles lead to non-linear mobility, which can be interpreted as non-Newtonian behaviour. This raises several key questions. Firstly, how are bubble sizes determined? This is determined by processes such as breakup, lamella division and coalescence, all of which are influenced by the heterogeneity of the porous structure and the imposed flow conditions. Once the bubble size distribution has been characterised, the next challenge is to understand bubble mobility. This depends on the local arrangement of bubbles within the pores and their susceptibility to becoming trapped by the porous matrix.

Finally, there is the question of the macroscopic implications of this complex rheology, such as the emergence of nonlinear viscous fingering and other large-scale flow instabilities. A natural extension of these topics is to consider two-phase flows where one of the phases is non-Newtonian itself. This significantly increases the complexity and nonlinearity of the system, posing additional modelling and experimental challenges. While a rheological framework appears promising, several difficulties arise. Notably, if foam texture is sensitive to the heterogeneity of the medium, it is inherently dynamic and evolves over time. This raises the question of whether spatially and temporally varying rheology should be considered to capture the full range of observed behaviours.

Other researchers addressed two-fluid problems, such as the jetting of one fluid into a non-Newtonian fluid of the invasion/displacement flow in porous media. A different direction involved studying the flow of individual bubbles of particles along pore channels. In terms of future directions, having open source codes that are able to deal with such a wide range of old and emerged applications is needed, as well as sharing of experimental results and methods.

4.5 Application areas: new and established

Many presentations were oriented toward specific applications (Beechey-Newman, Daripa, Eiser, Hewett, Hormozi, Krol, Lanza, Mohammadigoushki, Moura, Natale, Poletto, Stark), with a significant number focusing on biological systems. Indeed, numerous biological fluids exhibit non-Newtonian behavior. Mucus and biofilms, for example, possess viscoelastic properties and can also be considered as evolving porous media.

Several topics were addressed, including the locomotion of motile objects (such as bacteria) in viscoelastic media, and the transport of chemical speciessuch as antibioticswithin biofilms. The elasticity of the surrounding fluid significantly affects key swimming characteristics, such as drag and lift. Moreover, the fact that these fluids evolve over time adds a layer of complexity to the modeling and interpretation of these phenomena.

One presentation also explored the modeling of blood clots using a non-Newtonian rheological framework. As is often the case in biological applications, numerical and theoretical studies are strongly dependent on experimental data, which are often challenging to obtain and interpret. It was therefore particularly encouraging to see several presentations showcasing experimental work in this area.

Other application-oriented presentations included the study of the rheology of DNA nanostar solutions. DNA nanostars are multi-branched DNA particles that can undergo a phase transition between a gel and a solution state. Their unique structure and phase behavior make them particularly interesting for understanding the viscoelastic properties of complex fluids at the molecular scale.

Another topic of interest was the use of shear-thinning polymers for enhanced oil recovery. These polymers have been developed recently. potentially improving displacement efficiency in porous geological formations and offering promising strategies for optimizing oil extraction processes.

5 Outcome of the Meeting

This was the second biennial meeting on non-Newtonian flows in porous media, after Fortaleza, Brazil, June 28-30, 2022. We are grateful to BIRS for supporting our workshop, financially and in terms of the fantastic setting in Banff. Aside from the progress made and scientific outlook, as outlined in the previous section, many early career researchers participated fully and built professional relationships in this community. We hope this impact will be felt for many years.

A third workshop in the series is scheduled for May 11-15, 2026, to be held in Cargese, Corsica, France (https://nnfip26.sciencesconf.org/). In this we hope to continue the progress made in Banff.