Applications of Multivariate Hawkes Processes in Finance, Insurance and Epidemiology

Anatoliy Swishchuk (University of Calgary, AB, Canada),
Tom Bielecki (IIT, Chicago, IL, USA),
Jacek Yakubowski (University of Calgary,AB, Canada),
Mariusz Neiweglowski (Warsaw University of Technology, Poland)

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1 Overview of the Field

Research in Team workshop Applications of Multivariate Hawkes Processes in Finance, Insurance and Epidemiology was devoted to the research activities associated with applications of multivariate Hawkes processes in finance (multidimensional models for stock prices), insurance (modelling of claims arrival and risk processes) and epidemiology (modelling of spread of epidemic diseases accounting for various types of cases and SIR-Hawkes modelling). This very interesting and important class of stochastic processes was introduced by Alan Hawkes [11]-[14]. These processes, called now Hawkes processes, are meant to model self-exciting and mutually-exciting random phenomena that evolve in time. The self-exciting phenomena are modelled as univariate Hawkes processes, and the mutually-exciting phenomena are modelled as multivariate Hawkes processes (MHP). Hawkes processes belong to the family of marked point processes, and, of course, a univariate Hawkes process is just a special case of the multivariate one. There are numerous applications of the multivariate Hawkes processes. Therefore, there is an interest and demand in studying applications of MHP in finance, insurance, and epidemiology. For other applications, we refer to Bielecki et al. [3].

2 Recent Developments and Open Problems

There are many applications of the Hawkes processes. Here we present some objectives for possible applications of MHP in insurance, epidemiology and finance. Some ideas on potential applications of multivariate general compound Hawkes processes and generalized multivariate Hawkes processes in finance, insurance and epidemiology may be found in [4, 7, 8, 9, 10]. Hawkes processes have found important applications in finance over the past two decades. We refer to [13] for a relevant survey. In a series of papers [1, 2, 5] Bacry et al. introduced a multidimensional model for stock prices driven by (multivariate) Hawkes processes. The model for stock prices is formulated in [2] via a marked point process N. In [2] it is assumed that a d-dimensional vector of assets prices S = (S1,...,Sd) is based on N via the representation of Sti via the difference between an upward jumps of the i-th asset and an downward jump of i-th asset. Bacry et.al. [2] showed that within such framework some stylised facts about high frequency data, such as microstructure noise and the Epps effect, are reproduced. Using the GMHPs we can easily generalize their model in several directions. In particular, a model of stock price movements driven by a generalized multivariate Hawkes process N allows for common jumps in upward and/or downward direction. Including possibility of embedding co-jumps of the prices of various stocks in the book in the common excitation mechanism, may turn out to be important in modelling the book evolution in general, and in pricing of spread and basket options in particular [7, 21]. As it was also observed, clustering and self- exciting arrivals of claims is an inherent feature of claims arrival processes that is faced by the insurance companies. See e.g. [6] or [16]]. It was observed that theory of GMHPs [23] can potentially be applied to enrich that analysis of insurance risk management allowing for accounting for mutual and simultaneous excitation between claims streams corresponding to differ- ent lines of insurance, as well as allowing for accounting for various types/severities of claims arriving within the insurance lines. If we consider an insurance company with d major lines of insurance and having N as a generalized multivariate Hawkes process, then it can be models for the claims arrivals. Specifically, we can interpret the coordinate Ni as a model for claims arrival process corresponding the the i-th line of insurance. This model generalizes the compound Hawkes surplus model originally introduced in [19]. It was already observed by Hawkes in [11] that Hawkes processes may find applications in epidemiology for modeling spread of epidemic diseases accounting for various types of cases, such as children or adults, that can be taken as marks. This insight has been val-idated over the years in numerous studies. We refer for example to [15, 17, 18, 20] and the references therein. It is important to account for the temporal and spatial aspects in the modeling of spread and intensity of epidemic, endemic and pandemic diseases, such as COVID-19. The variant of the generalized multivariate Hawkes process that was described at the end of Section 12.4.1 in [3] offered a valuable tool in this regard.

3 Presentation Highlights

The presentations during the workshop were devoted to reviewing and discussing the relevant papers and books in finance [1, 2, 4, 5, 7]-[12], some papers and books in insurance [6, 10, 16, 19] and papers in epidemiology [15, 18, 20, 25].

The social program included hiking to Tunnel Mountain and other places around BIRS, Banff.

4 Scientific Progress Made

During the RiT workshop, we prepared two research papers, in finance and insurance, produced some initial results for the third paper and initiated and discussed some ideas for the 4th paper.

1). The first paper is "Functional Limit Theorems for Marked Hawkes Processes and Compound Marked Hawkes Processes". In this paper, FCLTs were proved for marked Hawkes processes and compound marked Hawkes process. A progress has been made in the area of functional limit theorems for marked Hawkes processes and for compound marked Hawkes processes with applications to finance and insurance. Specifically, marked Hawkes processes with marks taking values in a Lusin space have been studied subject to conditions that the explosion times are infinite. Appropriate structural assumptions have been imposed on the related Hawkes kernel. This set-up allows for accommodating a special case of a generalized multivariate Hawkes process that has been previously introduced and studied by the workshop participants. Mathematical results have been obtained laying ground for proving functional laws of large numbers and functional central limit theorems for such processes, as well as for related compound marked Hawkes processes. The resulting functional limit theorems for marked Hawkes processes and compound marked Hawkes processes generalize several results obtained previously in the literature. An interesting application that has been worked out regards analysis of time asymptotic of the ruin process that arises in insurance. In particular, the law of large numbers and the central limit theorem are obtained for a ruin process that is modelled as a compound marked Hawkes process. The latter result is a conduit to a diffusion approximation of an appropriately standardized ruin process. Such approximation facilitates computations of various functionals of the ruin process. And that was considered in details in our second paper.

2). The second paper is "Risk model based on marked Hawkes process with applications to Merton problem and ruin probabilities". In this paper we consider risk model based on marked Hawkes process (RMMHP), and solve Merton optimization problem and find finite-horizon and infinite-horizon ruin probabilities for that model. We proved Law of Large Numbers (LLN) and Functional Central Limit Theorem (FCLT) for the RMMHP, and apply those results to the mentioned above problems. Numerical examples are presented using the real data. The novelty of the paper associated with the new LLN and FCLT, and solutions of Merton problem and finding ruin probabilities for the RMMHP;

These two papers are required some polishing and almost ready for submissions.

3). Third working paper "A model for cyber risk based on Hawkes process and its application to insurance", as it says, is devoted to modelling of cyber risk using marked Hawkes process and applying this model in insurance [22, 10]. We considered Hawkes total claim amount process (HTCAP) and presented several results associated with this process: 1) Expected Value Principle for HTCAP; 2) Standard Deviation Principle for HTCAP; 3) Exponential Principle for HTCAP; 4) Value-at-Risk for HTCAP; 5) Average Value-at-Risk for HTCAP. The methodology is based on the results rom [23, 10].

4). The 4th working paper (under discussion and in progress) "$SEIHR_H$ Model based on Multivariate Hawkes Process" is devoted to the study of SIR ("susceptible", "infective", "removed") model in epidemiology that contains mode components, namely, "exposed" and "hospitalized" (thus, SEIHR), [24], and the event rate function in SEIHR model is described by multivariable Hawkes process (therefore, $SEIHR_H$) [25]. The idea is to consider simpler model based on diffusion approximation of Hawkes-based model [23].

5 Outcome of the Meeting

The workshop have brought together academics people to share their ideas, to share their research experience and to learn more about the applications of Hawkes processes. This workshop also accomplished its aim to produce some papers associated with the topics. As a result, we almost finished two research papers, produced some initial results for the third paper and initiated and discussed some ideas for the 4th paper.

Several other research projects and working papers were discussed and created based on the workshops topics.

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