Retreat for Young Researchers in Stochastics

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1 Overview

This was the fourth annual meeting of the PIMS Postdoctoral Training Centre in Stochastics (PTCS). The Retreat offers an opportunity for young researchers in pure or applied probability from Western Canada and Washington state to interact, communicate their recent results and ongoing research programs and initiate new collaborations. Eight of the nine postdoctoral fellows affiliated with PTCS spoke at the meeting. The 25 participants included postdoctoral fellows from U. Washington, U. of Alberta and UBC, Ph.D. students from U. Saskatchewan, U. Calgary, U. Alberta and UBC, and faculty from U. Calgary, U. Alberta, U. Victoria, U. Regina, and UBC.

The response from the participants after the retreat was hugely positive.

2 Presentation Highlights

Moumanti Podder (UW) spoke about her joint work [1] with Ander Holroyd, Avi levy and Joel Spencer on second order logic and random trees. A sample problem is given by the logical complexity of the statement that a tree is finite, or infinite. Infiniteness can be expressed by an 'existential monadic second order (EMSO) sentence', but finiteness cannot. This logical result does not preclude the possibility that for a class of random trees, there exists an EMSO sentence that with probability one does detect finiteness. Podder shows this is not possible for Galton-Watson trees with a Poisson offspring law. The proof proceeds by an elegant mapping of the problem into a set of games. The result holds for more general offspring mechanisms but the proofs require the offspring law to have full support, although it is expected to be true much more generally. After a question of Yinon Spinka, a discussion ensued on whether or not there is an EMSO sentence that detects finiteness with positive probability.

Sarai Hernandez (UBC) spoke about her ongoing joint work with Omer Angel, David Croyden and Daisuke Shiraishi on scaling limits of Uniform Spanning Trees (UST) in 3 dimensions. In dimension 2 the scaling limit is SLE(8) and in high dimensions it is believed to be super-Brownian motion, but in intermediate dimensions (like 3) the non-Gaussian limits that are anticipated are notoriously difficult to find, let alone study. For Loop Erased Walk, Kozma [2] had shown that along dyadics there is a scaling limit in 3 dimension as random sets with the Hausdorff metric but little is know about this non-Gaussian limit. Given Wilson's algorithm, there is a close connection here with limits of UST's. The striking result of Sarai and her co-authors is that rescaled UST's in 3 dimensions converge in the space of paths (and so also in the space of random sets) to a limiting object. This ancestral structure gives us some additional insight into this limiting object. They are currently working on extending this groundbreaking result to convergence in the space of metric spaces equipped with the Hausdorff-Gromov topology.

Liping Xu (UW) presented some very recent work with Zhenqing Chen on pathwise uniqueness for finite-dimensional stochastic differential equations (SDE's) with multiplicative noise and non-Lipschitz drifts driven by a Lévy process. The classical results here for Brownian motion with additive noise were resolved some forty years ago by Zvonkin [4] and Veretennikov [3]. The analogue of Zvonkin's original transformation is still the tool being used but obtaining the required regularity of this map for integral operators when dealing with multiplicative noise is a much more difficult problem. Xu first presented recent work of Chen, Zhang and Zhao which establishes pathwise uniqueness for "α-stable-like" Lévy processes and β-Hölder continuous drift for $\beta > 1 - \alpha$. The latter is sharp as classical results of Tanaka, Tsuchiya and Watanabe showed. However, the rather specific nature of the noise should not be needed here. Dr. Xu's recent result extends this work by allowing one to add much more general Lévy processes which have an α-stable-like component. The result is intuitive but establishing the analogue of Zvonkin's transformation in this non local setting when $\alpha < 1$ is quite interesting.

Yinon Spinka (UBC) spoke on the problem of determining whether or not a given translation-invariant random field on $\mathbb{Z}^d$ is a finitary factor of an i.i.d. process (ffiid). Spinka presented various sufficient conditions for a translation invariant process on $\mathbb{Z}^d$ to be ffiid. The conditions include a spatial mixing properties for Markov random fields, monotonicity and uniqueness of the Gibbs measure. Quantitative results on the tails of the coding radius were also presented (exponential or polynomial decay of tails). Here stronger mixing properties of the Markov random fields lead to faster decay of tails. Finally, he presented applications to various well-known models from statistical physics such as the Potts model, proper colorings and the random cluster model.

The lively Open Problems Session featured 5 problems from math finance and insurance, population genetics, and coding theory. Noah Freeman asked for the distribution of the ordered excursion length of the sticky Brownian bridge–note that the conditioning here is not singular as . Yinon Spinka asked Noah about considering the Brownian path up to the time when the local time hits 1 (instead). Noah pointed out this law in the non-sticky case is different. Ed Perkins discussed it with Noah hiking up Tunnel Mountain and felt Yinon's model may be more relevant to the sticky case than the bridge case and they agreed that conditional on the local time at 0, one should get a scaled Poisson-Dirichlet law and so it reduces to the law of this local time which looks quite accessible. Spinka asked a follow up question from his lecture on finitary coding. Tony Ware asked about 'Hawaiian options': this is an option which is traded in energy markets, where on exercise of the option (call or put) the holder receives an option of the other type, put or call. The infinite time horizon causes some difficulty in valuation, and there was a lively discussion on approaches to the problem.

3 Outcome of the Meeting

The level of talks at this Retreat was extremely high in terms of content and presentation. Five of the ten lectures were given by outstanding young female probabilists from U. Alberta, U. Calgary, U. British Columbia and U. Washington. The Open Problems Session was also particularly successful. For example, the day after his presentation Tony Ware announced that as a result of the discussion he had made significant progress on the Hawaiian options problem.

A number of the participants wrote after the meeting, all expressing thanks for a stimulating meeting. Yaozhong Hu (CRCI at U. Alberta) pointed to new connections made with faculty and young researchers at U. Calgary.

During the meeting it was agreed that after the funding for the Postdoctoral Training Centre for Stochastics stops, we should continue these annual meetings featuring young researchers in Probability from PIMS sites. It was felt that a 3-day meeting might be better to offer more time for informal discussion. With recent hires in Probability at U. Alberta and U. Victoria, both those sites were discussed. Tony Ware suggested we meet for a day or two at U. Calgary and then move on to BIRS for another two days.

References

[1] A. Holroyd, A. Levy, M. Podder, J. Spencer. Existential monadic second order logic on random rooted trees. To appear in *Discrete Mathematics*, Volume 342, Issue 1, January 2019, Pages 152-167.

[2] G. Kozma. The scaling limit of loop-erased random walk in three dimensions, *Acta Mathematica*, Volume 199, 2007, 29-152.

[3] A. Veretennikov. On the strong solutions of stochastic differential equations. *Theory. Probab. Appl.* Volume 24, 1979, 354-366.

[4] A. Zvonkin. A transformation of the phase space of a diffusion process that removes the drift. *Mat. Sbornik* Volume 93, 1974, 129-149.