Banach space theory

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March 4–9, 2012

The workshop was largely motivated by the recent extraordinary work of Argyros and Haydon [AH] (discussed below) which, following on the fundamental work of Gowers and Maurey in the 1990's, continued the discovery of the incredible variety of possible Banach space structure. [AH] is connected strongly with operator theory and Banach algebras. The last couple of years were very fruitful for researchers and many stunning results were presented at the workshop and new consequences were observed as well. The testimonials of the participants, which included numeruous young researchers and graduate students, attest to the great success of the workshop. The following presents a brief overview in which it is possible to cite only a selection of highlights. Much more can be found in the files and the videos and we expect more developments as these are studied.

Scalar plus compact and Invariant subspace problems

A key theme of the workshop and one underlying a number of presentations, dates back to the remarkable construction in 1980 of J. Bourgain and F. Delbaen [BD]. They constructed a Banach space X with X^* isomorphic to ℓ_1, yet c_0 does not embed into X. The example struck researchers as quite special and too limited to be useful in solving other open problems. After almost 20 years some researchers [A], [H] began to think otherwise. Two years ago, S.A. Argyros and R. Haydon used the BD-construction to solve a famous problem in Banach spaces.

Given a specific classical example of a Banach space, it is usually quite easy to construct many nontrivial bounded linear operators $T \in \mathcal{L}(X)$. But just given that X is separable and infinite dimensional, this is not at all clear. Over 35 years ago Lindenstrauss [L] asked if such an X existed so that $\mathcal{L}(X) = \{\lambda I + K : \lambda$ is a scalar and K is a compact operator$\}$. Here I is the identity operator on X. In their remarkable example of a space X not containing an unconditional basic sequence W.T. Gowers and B. Maurey [GM] proved that for their space all operators had the form $\lambda I + S$, where S is strictly singular (i.e., not an isomorphism restricted to any infinite dimensional subspace). But the "scalar plus compact" problem remained open. Then Argyros and Haydon [AH] constructed a space X with the "scalar plus compact" property. X is formed using the BD technique and thus X^* is isomorphic to ℓ_1. Shortly after that D. Freeman, E. Odell and Th. Schlumprecht [FOS] proved that if X^* is separable then X embeds into an isomorphic predual of ℓ_1. The proof, again, adopted the BD construction. It is amazing that the class of preduals of ℓ_1 is so large and yet small and that confining oneself to this class, the "scalar plus compact" problem could be solved.

Spaces X satisfying the "scalar plus compact" property are also of interest to operator theorists in that every operator $T \in \mathcal{L}(X)$ must admit a nontrivial invariant subspace. Furthermore $\mathcal{L}(X)$ is separable, and from the construction, is amenable as a Banach algebra.

Richard Haydon gave the first talk, outlining the BD-construction. In particular he presented a new way of focusing on families of compact subsets of $[\mathbb{N}]^{<\omega}$, during the construction, to consolidate numerous previous technical arguments such as appear in [AH] and [FOS]. He also mentioned a new example of a space X, with the "scalar plus compact" property, that contains an isomorph of ℓ_1. **Dan Freeman** followed with a discussion of [FOS] and then a newer result [AFHORSZ1]. The latter is that any separable superreflexive space can be embedded into an isomorphic predual X of ℓ_1 with the "scalar plus compact" property. X shares the properties of [AH]. Namely all $T \in \mathcal{L}(X)$ admit nontrivial invariant subspaces, $\mathcal{L}(X)$ is separable and amenable. Furthermore X is somewhat reflexive (every infinite dimensional subspace of X contains an infinite dimensional reflexive subspace). He also mentioned a more general but much more technically difficult result, by the same seven authors ([AFHORSZ2], in preparation). If X^* is separable and does not contain ℓ_1 as a complemented subspace, then X embeds into an ℓ_1 predual with the "scalar plus compact" property.

In [AH] the authors also show that their space X can be constructed to be HI (hereditarily indecomposable). This means that for all $Y \in X$ if $Y = Z \oplus W$ then Z or W must be finite dimensional. **Matthew Tarbard**, a student at Oxford, discussed his construction of some new remarkable spaces. He was motivated by the question as to whether any HI isomorphic predual of ℓ_1, with the "scalar plus strictly singular" property must have the "scalar plus compact" property. He showed this to be false. Indeed one can obtain such spaces with the Calkin algebra, $\mathcal{L}(X)/K(X)$ having any finite dimension. Here $K(X)$ denotes the ideal of compact operators on X. It was pointed out by W.B. Johnson during Tabbard's talk that his examples solved a longstanding open problem concerning which $T \in \mathcal{L}(X)$ are commutators, i.e., $T = UV - VU$ for some $U, V \in \mathcal{L}(X)$. The problem solved is that there is a space X and $T \in \mathcal{L}(X)$ with T lying in a proper ideal (here the ideal of strictly singular operators) that is not a commutator.

Another major breakthrough was presented by **Spiros Argyros** [AM]. He gave an example of an HI reflexive space X so that every $T \in \mathcal{L}(X)$ admits a nontrivial invariant subspace. Moreover this holds for all $T \in \mathcal{L}(Y)$, $Y \subseteq X$. This construction is not of the BD type. Nor does X have the "scalar plus compact" property. The strictly singular operators on every subspace of X form a nonseparable ideal. Furthermore the only spreading models of X are c_0 and ℓ_1, and such exist in all subspaces. This then solves another open problem in that the spreading models are stabilized within $\{\ell_p : 1 \leq p < \infty\} \cup \{c_0\}$ and yet do not form an "interval." Also every $T \in \mathcal{L}(X)$ either commutes with a non-zero compact operator or else $T^3 = 0$. All in all, quite a remarkable space. The construction uses Tsirelson ideas under constraints, motivated by earlier constructions in [OS1], [OS2].

Despoina Zisimopoulou showed how to adapt the BD construction to obtain certain $\mathcal{L}_\infty$ sums of Banach spaces. In particular, she constructs Z_p, a $\mathcal{L}_\infty$ sum of ℓ_p, so that $Z_p \sim Z_p \oplus \ell_p$ and every $T \in \mathcal{L}(Z_p)$ satisfies, for some λ, $T - \lambda I$ is horizontally compact, i.e., the restriction to every horizontally block subspace is compact.

Ioannis Gasparis showed that the BD constructions can be performed within $C(K)$ spaces as well. Using (a dual version of) the BD construction, he showed an interesting example of an ℓ_1-predual which is isomorphic to a subspace of $C(\omega^\omega)$ but it is neither isomorphic to c_0 nor contains a copy of $C(\omega^\omega)$. It follows from the literature that this space is not isomorphic to a complemented subspace of a $C(K)$ space.

As mentioned it can be difficult to construct nontrivial operators on Banach spaces. **Antonis Monoussakis** [M,P-B] and **Anna Pelczar-Barwacz** [KM,P-B] discussed two papers in which strictly singular noncompact operators are constructed in two settings. One is certain mixed Tsirelson (and HI) spaces and the other is in certain asymptotic ℓ_p spaces.

Thomas Schlumprecht discussed a recent paper with Daws, Haydon, and White [DHSW] which explored the Banach algebra structure of $\ell_1(\mathbb{Z})$. The latter is a Banach algebra under convolution:

$$(f * g)(n) = \sum_{k \in \mathbb{Z}} f(k) g(n-k) \ .$$

$\ell_1(\mathbb{Z})$ is, as a Banach space, the dual of uncountably many nonisomorphic Banach spaces. It is said to be a dual Banach algebra if it is realized as the dual of X so that the product is separately ω^*-continuous. They consider preduals where the bilateral shift is ω^*-continuous (equivalently the above natural convolution is separately ω^*-continuous) and produce an uncountable number of such preduals. They use Banach space theory (Szlenk indices) to show that, as Banach spaces, the preduals are all isomorphic to c_0 and go on to construct many other nonisomorphic preduals.

Operator ideals and Commutators

The workshop also featured the dissemination of new and significant results in two related topics that have become very active in Banach spaces theory during the last few years: the study of the lattice of closed ideals in the Banach algebra of bounded linear operators on a Banach space, and the study of commutators on Banach spaces.

Until a few years ago, the only Banach spaces X for which the lattice of closed ideals was completely described were the spaces ℓ_p, with $1 \leq p < \infty$, in which case the ideal of compact operators $\mathcal{K}(X)$ was the only non-trivial ideal in $\mathcal{L}(X)$. Laustsen, Loy and Read [LLR] and Laustsen, Schlumprecht and Zsak [LSZ] added two new members to the family of Banach spaces for which the lattice of closed ideals in $\mathcal{L}(X)$ is completely understood: $X = (\oplus_n l_2^n)_{c_0}$ and $X = (\oplus_n l_2^n)_{\ell_1}$, respectively. In both cases it was shown that there exist only one other non-trivial closed ideal besides the ideal of compact operators: the ideal of operators on X that factor through c_0 (respectively ℓ_1). These results prompted a renewed interest in this topic and the workshop presented a good forum for communicating interesting new results in this direction:

- we have already described above the talks of **R. Haydon** and **M. Tabard** who presented some remarkable constructions of Banach spaces whose Calkin algebra can have any given finite dimension; therefore the lattice of closed ideals in the algebra of bounded linear operators on these spaces can have any given finite cardinality.

- there were two talks, by **Andras Zsak** and **Denny Leung**, that discussed the (unique) maximal closed ideals in the algebra of bounded linear operators on some Banach spaces which share a similar structure of the complemented subspaces with the spaces from [LLR], [LSZ]: $X = (\oplus_n \ell_1^n)_{c_0}$ and $X = (\oplus_n \ell_\infty^n)_{\ell_1}$, respectively. Namely, these spaces are known to have only two different isomorphic types of complemented subspaces, the whole space X or c_0 (respectively ℓ_1), and these makes them good candidates for investigation in this direction of research. In his talk, **A. Zsak** presented a more general approach, starting with spaces of the form $X = (\oplus_n E_n)_{c_0}$ where E_n's are finite dimensional spaces. The results obtained are in the form of interesting dichotomies about sequences of operators into L_1, including a dichotomy theorem for random matrices. Similarly, **D. Leung** discussed the general case of spaces of the form $X = (\oplus_n E_n)_{\ell_1}$.

- **Bentuo Zheng** [LSZh] discussed results regarding the structure of the closed ideals in the algebra of bounded linear operators on a class of p-regular Orlicz sequence spaces ℓ_M which are close (in a certain sense) to ℓ_p. After obtaining some structural results about these spaces, it was shown that the immediate successor of the ideal of compact operator $\mathcal{K}(\ell_M)$ in $\mathcal{L}(\ell_M)$ is the closed ideal generated by the formal identity from ℓ_M into ℓ_p, and that the maximal closed ideal in $\mathcal{L}(\ell_M)$ is of the form $M = \{T : \ell_M \to \ell_M \mid Id_{\ell_M} \text{ does not factor through } T\}$. It should be noted that there are several other known examples of Banach spaces X for which $M_X = \{T : X \to X \mid Id_X \text{ does not factor through } T\}$ is the unique maximal closed ideal in $\mathcal{L}(X)$.

- It is also the case for $C[0, \omega_1]$, where ω_1 is the first uncountable ordinal. This new result was part of **Niels Laustsen**'s talk, which also contained some other interesting results regarding the closed ideals in $\mathcal{L}(C[0, \omega_1])$. A maximal closed deal in $\mathcal{L}(C[0, \omega_1])$ was identified before in the literature, using a representation of operators on $C[0, \omega_1]$ as scalar-valued $[0, \omega_1] \times [0, \omega_1]$-matrices. The new result discussed in the workshop provides a matrix-free characterization of this ideal and, in the same time, implies that there is no other maximal ideal in $\mathcal{L}(C[0, \omega_1])$. The talk provided also a list of equivalent conditions describing the strictly smaller ideal of operators with separable range.

The workshop also featured remarkable new results on the important and difficult problem of classifying the commutators in the algebra of bounded linear operators on a Banach space X. When studying derivations on a general Banach algebra $\mathcal{A}$, a natural problem that arises is to classify the commutators in the algebra, that is, elements of the form $AB - BA$. This problem is hard to tackle on general Banach algebras. Nevertheless one has a better chance in the special case of the algebra of bounded linear operators on a Banach space X since there is hope that the underlying structure of the space X could provide useful information regarding the operators acting on X. For a Banach space X for which there is a unique maximal closed ideal in $\mathcal{L}(X)$, one can hope to obtain a complete classification of the commutators on the space. The natural conjecture is that the only operators on X that are not commutators are the ones of the form $\lambda I + S$, where S belongs to the unique maximal ideal in $\mathcal{L}(X)$ and $\lambda \neq 0$ ([DJ]). This was known to be true for the spaces ℓ_p, when $1 < p < \infty$ (by an old result of Apostol), and there has been some remarkable progress in this direction during the last three years, with the conjecture being verified for the spaces ℓ_1, ℓ_∞, L_p $(1 \leq p < \infty)$ ([D], [DJ], [DJS]).

- As mentioned earlier, it was pointed out by W. B. Johnson during the workshop that, as a byproduct of his results, **M. Tabard** has produced an example of a Banach space X that admits an operator T in the maximal closed ideal of $\mathcal{L}(X)$ which is not a commutator, thus providing a negative answer to the conjecture mentioned above.

- **Detelin Dosev** presented some recent progress on the problem of verifying the above conjecture for the spaces $C(K)$. The results obtained rely on a theorem of Kalton about decomposition of Borel measures on an infinite compact metric space.

- In his talk, **Gideon Schechtman** presented some interesting results regarding the converse of the following well-known fact: if an $n \times n$ matrix A is a commutator ($A = BC - CB$, where B, C are also $n \times n$ matrices) then the trace of A is zero, in which case it is clear that $\|A\| \leq 2\|B\|\|C\|$. The question discussed in the talk is the following: assuming that A is an $n \times n$ matrix with trace zero, is it possible to find $n \times n$ matrices B, C such that $A = BC - CB$ and $\|B\|\|C\| \leq K\|A\|$ for some absolute constant K? The talk provided a weaker estimate. Namely, the above holds for $K = K_\epsilon n^\epsilon$ for every $\epsilon > 0$, where K_ϵ depends only on ϵ.

Gowers' classification program

After proving his famous dichotomy theorem that every (separable) Banach space contains either a hereditarily indecomposable subspace or a subspace with an unconditional basis, Gowers proposed a classification program for separable Banach spaces. The aim of this program is to produce a list of classes of infinite dimensional Banach spaces such that

(a) the classes are hereditary, i.e., stable under taking subspaces (or block subspaces),

(b) the classes are inevitable, i.e., every infinite dimensional Banach space contains a subspace in one of the classes,

(c) the classes are mutually disjoint,

(d) belonging to one class gives some information about the operators that may be defined on the space or on its subspaces.

Gowers also proved a second dichotomy theorem which asserts that every Banach space contains either a quasi-minimal subspace or a subspace with an unconditional basis such that disjointly supported block subspaces are totally incomparable (i.e., neither embeds into the other). Recall that a space X is minimal if it embeds into all of its subspaces, and quasi-minimal if it has no pair of totally incomparable subspaces. Gowers deduced from these dichotomies a list of four inevitable classes of Banach spaces; HI spaces, no disjointly supported subspaces are isomorphic, strictly quasi-minimal (i.e., quasi-minimal with no minimal subspace) with an unconditional basis, and minimal spaces. In a series of papers, Ferenczi and Rosendal further refined this to a list of six inevitable classes;

1. HI and tight by range,
2. HI, tight, and sequentially minimal,
3. Tight by support,
4. Has unconditional basis, tight by range, and quasi-minimal.
5. Has unconditional basis, tight, and sequentially minimal
6. Has unconditional basis and minimal.

We will refer to **Valentin Ferenczi**'s talk for the notion of tightness, and for a nice survey of the Gowers' classification program. The above list is obtained via some dichotomy theorems, and it wasn't clear if there are examples of spaces in class 2 and in class 4. Ferenczi presented a recent result (joint with Th. Schlumprecht) showing that a subspace of a variant of the original Gowers-Maurey space is indeed a class 2 space. No example of class 4 is yet known.

Spreading models

The notion of a spreading model has proven to be a very useful tool in the geometry of Banach spaces. Roughly speaking, a spreading model of a Banach space X is another Banach space with a spreading basis whose norm is obtained by stabilizing at infinity the norm on a non-degenerate sequence of vectors in X. Such a stabilization is possible thanks to Ramsey theory. In general, a spreading model is not a subspace but it is finitely represented in the space in a special way. The initial segments of the basis are $(1+\varepsilon)$-equivalent to a finite subsequence of the generating sequence in X. What type of spreading models must exist on a given Banach space X was a central problem which have been widely investigated since they were first introduced in the 70's. For instance, every space has a spreading model with an unconditional basis. However, Odell and Schlumprecht [OS1] constructed a reflexive space admitting no c_0 or ℓ_p spreading models. One can iterate the process and ask the same question for spreading models of a spreading model of X and so on. Argyros, Kanellopoulos and Tyros [AKT1, AKT2] introduced a generalized notion of spreading models based on sequences indexed by some special families of finite subsets of natural numbers (called $\mathcal{F}$-sequences) which yields, among other things, a natural framework to study the iterated spreading models. They have also constructed a space, generalizing Odell-Schlumprecht's example, so that none of the n-fold iterated spreading models are isomoprhic to c_0 or ℓ_p. In his talk **Kostantinos Tyros** explained the fundamentals of $\mathcal{F}$-spreading models. This notion depends on plegma families, sequences of interlacing subsets, in a regular thin family $\mathcal{F}$ which satisfy strong Ramsey properties. The combinatorics of such families is promising in that essentially the order of $\mathcal{F}$ determines the $\mathcal{F}$-spreading models, and one gets an increasing transfinite hierarchy of spreading models (one for each countable ordinal).

Nonlinear theory

One of the exciting research programs in Banach space theory is the nonlinear theory of Banach spaces. Besides the intrinsic geometric interest, some of the problems are motivated by the applications to theoretical computer science. Parallel to the linear theory, the main focus is the nonlinear classification of Banach spaces. The linear operators are replaced by Lipschitz or uniformly continuous maps. The problems of interest are Lipschitz, uniform or coarse embedding of metric spaces in normed spaces, or such an embedding of a Banach space into another.

Florent Baudier spoke on joint with Fernando Albiac on Lipschitz embeddings between L_p spaces and snowflaked versions. In particular, he presented the joint work with F. Albiac that for $1 \leq p < q$, ℓ_p with the snowflake metric $d_p^{p/q}$ Lipschitz embeds into ℓ_q, and for $0 < p < q \leq 1$, (ℓ_p, d_p) Lipschitz embeds into (ℓ_q, d_q). These are analogs of the function space versions due to Mendel and Naor in which case one gets isometric embeddings.

Alejandro Chavez-Dominguez took on the question of how one could possibly define tensor products of a metric space with a Banach space. In the Banach space setting, there is a close duality relationship between tensor norms and operator ideals. Chavez-Dominguez presented his endeavor of developing a general theory of tensor norms on spaces of Banach-space-valued molecules on metric spaces. Some of the classical results carry over to this new setting. There is a natural notion of a reasonable norm, and among the reasonable norms there is a smallest one and a largest one. A new projective tensor product of a metric space X and a Banach space E was introduced which in the case $E = \mathbb{R}$ coincides with the classical Arens-Eells space. The dual of this tensor product is isometric to the space of Lipschitz mappings from X to E^*. Lipschitz quotient maps between metric spaces induce Lipschitz quotient maps between corresponding projective tensor products. A norm on spaces of molecules can be defined which is a predual for the space of Lipschitz p-summing operators from X to E^*. Moreover, a Hilbertian norm on spaces of molecules can be defined, and it is in duality with the ideal of maps that factor through a subset of Hilbert space.

Denka Kutzarova spoke on joint work with Stephen Dilworth, Gilles Lancien, and N. Lovasoa Randrianarivony on (nonlinear) uniform quotients of Banach spaces [DKLR]. Recently, Lima and Randrianarivony [LR] proved that the uniform quotients of ℓ_p $(1 < p < 2)$ are the same up to isomorphism as the linear quotients of ℓ_p, answering a problem that had been open for over a decade. Their proof made essential use of Property (β) of Rolewicz, which is an asymptotic property of Banach spaces whose definition involves the metric but not the linear structure of the space, and which therefore lends itself nicely to the nonlinear theory.

In this talk it was shown that if $T\colon X \to Y$ is a uniform quotient then the modulus of asymptotic smoothness of Y essentially dominates the β-modulus of X. It follows that certain reflexive spaces, e.g $(\sum \ell_{p_n})_{\ell_2}$, where $p_n \to \infty$, cannot be expressed as uniform quotients of any space with Property (β), and also that ℓ_q is not a subspace of any uniform quotient of an ℓ_p sum of finite-dimensional spaces if $q > p$. The separable spaces that are isomorphic to spaces with Property (β) are precisely the reflexive spaces X such that both X and X^* have Szlenk index equal to ω.

Operators on Banach spaces

Alexey Popov reported on joint work with Laurent Marcoux and Heydar Radjavi on almost invariant subspaces of operators $T \in \mathcal{L}(X)$. A closed subspace $Y \subset X$ is a half-space if Y and X/Y are infinite-dimensional and is *almost invariant* if $T(Y) \subset Y + F$, where F is finite-dimensional. It is an open question as to whether every operator has an almost invariant half-space. It was shown that all polynomially compact operators on reflexive spaces and bi-triangular operators on Hilbert space have almost invariant half-spaces and that if A is a norm-closed subalgebra of $\mathcal{L}(X)$ which admits a complemented almost invariant half-space then A admits an invariant half-space.

Haskell Rosenthal discussed an approach to the Hyperinvariant Subspace Problem. He defined a subspace of $L_2[0, 2\pi]$ consisting of functions whose Fourier coefficients satisfy a certain growth condition. Some properties of the right shift operator T on this space were presented and it was conjectured that T and T^{-1} have no common invariant subspace.

The aim of **Kevin Beanland**'s talk was to use descriptive set theoretic results in order to provide a uniform version of the classical result of Davis-Figiel-Johnson-Pelczynski regarding the factorization of weakly compact operators on Banach spaces. The main result presented is of the following form: if X is a separable Banach space and Y is either a Banach space with a shrinking basis or the space $C(2^{\mathbb{N}})$, then all operators from a Borel (in the strong operator topology) collection $\mathcal{B}$ of weakly compact operators from X to Y factor through the same separable reflexive space $Z_{\mathcal{B}}$.

Isometric theory of Banach spaces

Christian Rosendal spoke on joint work with Valentin Ferenczi on bounded subgroups of the general linear group of X, where X is a separable Banach space [FR]. This work is motivated by the famous *Banach Rotation Problem*: if the isometry group of X is transitive, does it follow that X is linearly isometric (or even isomorphic) to a Hilbert space? It was shown that if X is a complex HI space without a Schauder basis then the isometry group of X acts *nearly trivially* on X, i.e., it restricts to the identity on a closed subspace of finite codimension. This result yields the first known example of a Banach space with no maximal bounded subgroup of its general linear group (equivalently, a space with no equivalent *maximal norm* in the sense of Pełczyński and Rolewicz) and answers a question of Wood raised in the eighties. It also provides the first known example of a complex super-reflexive Banach space which does not admit an equivalent almost transitive norm, answering another longstanding open problem.

Jesus Castillo spoke about spaces U of *universal disposition* for a class $\mathfrak{M}$ of Banach spaces [ACCGM]: i.e., if $A, B \in \mathfrak{M}$ and $u\colon A \to U$ and $i\colon A \to B$ are isometric embeddings then there exists an isometric embedding $u'\colon B \to U$ such that $u = iu'$. It was shown that a space of Kubis obtained from the push-out construction is a space of density character $\aleph_1$ which is of universal disposition for the class of separable Banach spaces (and under CH is unique up to isometry). Several properties of spaces of universal disposition for the classes of separable and of finite-dimensional Banach spaces were presented. A Banach space X is *automorphic* if every isomorphism between subspaces of X can be extended to an automorphism of X. The only known automorphic spaces are $c_0(I)$ and $\ell_2(I)$. It was shown that HI spaces with certain additional properties, if they exist, would be automorphic.

Random Matrices and High-Dimensional Convexity

Nicole Tomczak-Jaegermann spoke about random matrices associated to high-dimensional convex bodies, with applications to the areas of computational geometry, compressed sensing, especially approximate reconstruction problems and the Restricted Isometry Property (RIP), and random matrix theory [ALPT-J1, ALPT-J2]. Consider an $n \times N$ random matrix M with columns $X_1, \dots, X_N$ that are independent copies of an isotropic log-concave vector X. A deviation inequality for the norm of the restriction of M to the collection of k-sparse vectors was presented. This result was used to give an answer to a question of Kannon, Lovász and Simonovits, namely a concentration inequality for the convergence of the empirical covariance matrix $(1/N)\sum_{i=1}^{N} X_i \otimes X_i$ to I_n as $N \to \infty$. It was shown that the spectral properties of M are similar to those of random matrices with independent Gaussian entries and that M has the RIP of order m provided $m \le cn/\log^2(CN/n)$. RIP properties for matrices with independent rows were also presented.

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