Inverse Transport Theory and Tomography

Guillaume Bal (Columbia University),
Plamen Stefanov (Purdue University),
Gunther Uhlmann (University of Washington)

May 16–21, 2010

Activity Report

This workshop brought together experts in inverse problems with interest in the broadly defined field of transport theory. The workshop balanced research in theoretical and computational inverse transport and in experimental atmospheric science and biomedical imaging with four speakers (and a few more in the audience) coming from the Engineering and Applied Science communities and the rest of the speakers coming from the applied analysis and applied mathematics communities. Some of the main objectives of this workshop was to provide a cohesive summary of the very active research activities performed over the past five years in the field of inverse problems and to identify potential areas of research where collaborations between mathematicians and engineers and both necessary and fruitful.

Integral geometry

Inverse transport theory may be separated broadly into two categories. The first category involves propagation in the absence of scattering and is closely related to the broad field of integral geometry. An important problem consists of inverting attenuated ray transforms as their appear in, e.g., medical and geophysical imaging. Mikko Salo presented recently obtained uniqueness results with Gunther Uhlmann for the attenuated geodesic ray transform of functions and 1-forms on simple 2D Riemannian manifolds with an arbitrary known absorption coefficient. The approach is constructive, as well.

Alexandre Bukhgeim considered the problem of the reconstruction of both the source term and the absorption coefficient in Euclidean geometry using the tools of A-analytic functions that he and collaborators had introduced in the past. This remains an open problem.

Nick Hoell (graduate student) showed explicit reconstruction formulas for the attenuated integral of functions along the integrals of specific vector fields in the analytic category, whereby generalizing earlier results obtained for the hyperbolic geometry.

Sean Holman analyzed the problem of polarization tomography on a Riemannian manifold and generalized to this setting results of generic reconstructions that were obtained by, e.g., Stefanov and Uhlmann, in the setting of the reconstruction of functions from their geodesic ray transform.

Discretization aspects

The above formulas apply in the presence of a continuum of data. The practical problem of reconstructions with sparse and limited data was considered by Matti Lassas. His presentation analyzes the interplay between noise and discretization effects and which numerical algorithms should be used to perform inversions in X-ray tomography and more generally in all linear inverse problems of the form $m = Af + e$ with e random noise.

In the presence of extremely noisy data as they appear in, e.g., detection of low emission radioactive sources, detailed statistical models need to be introduced. Peter Kuchment reported on recent results in the reconstruction of sources whose intensity can be as weak as one part in a thousand of the noise level.

Inverse transport theory

Many important recent results were reported in inverse transport theory when scattering is taken into account. From the theoretical viewpoint, Alexandru Tamasan presented joint results with McDowall and Stefanov about full characterizations of non-uniqueness (gauge equivalence) results in transport theory when the absorption coefficient depends on the velocity variable, with important applications for reconstructions in anisotropic media. A notable result is that the reconstruction of anisotropic absorption coefficients can be made stable on the support of the scattering coefficient and not elsewhere.

Alexandre Jollivet reviewed recent stability analyses of inverse transport reconstructions in the different regimes that appear in applications. Which coefficients may be reconstructed from available data and with which stability properties strongly depends on the available measurements, for instance whether time-dependent measurements or angularly resolved measurements are accessible.

Vadim Markel presented explicit reconstructions formulas when only single scattering is taken into account. Neglecting multiple scattering allows one to obtain stable and explicit reconstructions of the scattering and absorption coefficients in geometries of practical interest.

Numerical aspects of inverse transport

Several presentations reported on recent results in the numerical simulations of forward and inverse transport problems. Although many numerical methods have been proposed to solve transport equations, the solution of inverse transport problems requires specific treatment. A novel forward transport solver based on a few multigrid method with optical molecular imaging applications was presented by Hongkai Zhao.

Nonlinear kinetic models also find important applications of inverse transport theory. The reconstruction of doping profiles in semi-conductors may be accurately described by a Boltzmann-Poisson system of equations. Heuristic arguments then show that the reconstruction of the doping profile is a severely ill-posed problem. Numerical algorithms presented by Kui Ren show that limited of practically useful information can nonetheless be reconstructed from available measurements of current-voltage curves. Moreover, these studies show that less accurate forward descriptions than the Boltzmann-Poisson system such as drift-diffusion models fail to adequately reconstruct this information.

Stability of inverse transport problems as they appear in Jollivéts talk comes from the fact that singularities in the object we wish to reconstruct propagate to the available data. In transport, such singularities are singularities in the angular and spatial variables. It is notoriously difficulty to capture such singularities numerically. Based on a spectrally accurate algorithm to rotate domains of interest, Francois Monard (graduate students) presented a new discretization of the transport equation that allows one to accurately capture all relevant transport singularities and obtain stable numerical reconstructions of the optical parameters in cases of limited amounts of scattering.

Deterministic numerical transport solutions are expensive and have difficulties handling complex geometries. Many solvers have thus been developed to use the probabilistic representation of transport solutions and devise Monte-Carlo statistical algorithms. The main difficulty with such algorithms is their sometimes large variance, which results in expensive numerical simulations to combat statistical noise. Ian Langmore presented recent results of variance reduction techniques based on deterministic adjoint calculations and showed how variance could be significantly reduced in numerical simulations of transport equations as they arise, e.g., in remote sensing.

Large scattering limits and diffusion models

In many applications of forward and inverse transport, the transport mean free path (the main distance between successive interactions of the particles with the underlying medium) is so small that more macroscopic models such as the diffusion equation are more appropriate. Several presentations were devoted to the theoretical and numerical analysis of inverse diffusion problems. A major drawback of inverse diffusion problems of course is their severe ill-posedness. In order to stabilize the reconstructions, either more data need to be acquired or prior information needs to be included. Pedro González presented recent results where diffuse optical tomography (an inverse diffusion problem) can be somewhat stabilized by the acquisition of spectral data, since the absorption and scattering coefficients behave differently as a function of the color (wavelength) of the probing light. His talk gave numerical evidence that spectral information allows us to better distinguish between healthy and non-healthy tissues.

Arnold Kim introduced prior information by assuming a simple two-layer half space geometry of the problem and by reconstructing point-like absorbers from boundary measurements of back-scattered light. Such formulations are useful to devise at which stage cancer development can be detected in epithelial tissues.

Tanja Tarvainen presented the Bayesian method as a versatile statistical mean to introduce prior information into the reconstructions. The framework was then used to incorporate pre-computed errors between the accurate transport model of photon propagation in optical tomography and its diffusion approximation. The methodology allows one to obtain reconstructions with the accuracy of the transport solution at the cost of the much less expensive diffusion model.

Gen Nakamura considered the framework of time-dependent measurements to mitigate the ill-posedness of diffuse optical tomography and in applications of heat diffusion. He presented theoretical and numerical evidence of improvements of reconstructions in this setting.

Theoretical results in connected areas

We have seen that the transport of particles was often modeled using kinetic equations or their diffusive limits. Similar equations are used to model many fields of applied science and share similar difficulties as far as inverse problems are concerned.

One such problem is the inverse spectral problem for weighted Laplacians on Riemannian manifolds with singularities in one of its dimensions leading to n-1 dimensional Riemannian orbifolds. Using the Gromov-Hausdorff metric that is adapted to the reconstruction of geometric objects (defined independent of re-parameterization), Yaroslav Kurylev presented results of stability of the reconstruction of the geometry of such Riemannian manifolds from boundary spectral information.

We have already mentioned that inverse diffusion problems were severely ill-posed. It turns out that singular diffusion tensors may sometimes not be visible from outside measurements, with applications in the cloaking of objects. Since invisibility requires that one constructs un-physical singular conductivities, there has been a lot of recent activity to try and understand how accurate cloaking might be for less singular conductivities as they can be engineered in practice. Hongyu Liu presented recent results on approximate cloaking when wave propagation is no longer necessary a low-frequency approximation of diffusion type but takes the more general form of acoustic and electromagnetic systems of equations.

Such systems of equations are more difficult to analyze mathematically if only because the standard complex geometric solutions of the form $e^{\rho\cdot x}$ with ρ a complex vector are much more involved for systems of equations as they are for scalar equations. Ting Zhou presented results on the use of such complicated complex geometric optics solutions to reconstruct obstacles in a system of Maxwell equations by using the so-called enclosure method.

Applications of inverse transport in biomedical imaging and atmospheric science

Transport of particles finds many applications in medical and geophysical imaging. An important application in medical imaging is optical tomography. Several theoretical and numerical results presented during this workshop have been mentioned already. Andreas Hielscher reported on recent applications of optical tomography in small animal and human imaging. Optical tomography is an important modality as the optical properties of healthy and non-healthy tissues are quite different. Specifically, tumors absorb near infra red light much more so than healthy tissues and may be characterized by detailed reconstructions of oxy- and deoxy-hemoglobin concentrations. The presentation reviewed potential strengths and limitations of the practical implementation of optical tomography and gave examples of applications encountered in clinical and pre-clinical imaging such as monitoring of tumor growth and regression, effects of anti-angiogenic drugs in pediatric cancer treatment, breast cancer screening, and detection of arthritis.

Transport theory is also important in remote sensing as it is applied, e.g., to reconstruct cloud and aerosol properties in the Earth atmosphere. Quantifying their optical properties remains a hard problem in the global climate models used, e.g., to understand global warming. At present, very crude models for the cloud geometry are used in practical implementation. Anthony Davis surveyed in this presentation the recent steps that have been taken toward fully three dimensional atmospheric tomography and covered the analogies that can be made between inverse transport in atmospheric imaging and in medical imaging. A fully integrated three dimensional inverse transport setting in atmospheric tomography is still very much in the making. Once the technical challenge and observational resources are understood, the Earth's particulate atmosphere may inspire new applications for advanced methods in inverse transport theory as well as in physics-based tomography.

PAT and TAT

As was mentioned several times above, optical tomography and more generally inverse transport theory in highly scattering environments is a severely ill-posed problem. This prevents the method to be used as a stand alone imaging technique for human beings when millimeter or sub-millimeter resolution is required. Yet, the good discrimination properties of optical waves provides an important diagnostic for the presence of, e.g., malign tumors. Ultrasound tomography somehow suffers from opposite defects. Healthy and non-healthy tissues display very similar sound speeds at least in early stage tumors. In spite of sub-millimeter resolution capabilities, ultrasounds are therefore difficult to use in this context. Several recent imaging modalities have been proposed recently that aim to combine the good discrimination properties of light with the high resolution capabilities of acoustic waves. One such modality is photo-acoustic tomography (PAT). A similar modality combining microwave radiation (to obtain good discrimination properties) with acoustic waves (to obtain good resolution) is called thermo-acoustic tomography (TAT).

Several experts on PAT, TAT, and ultrasound tomography presented their recent research at the workshop. Lihong Wang is a pioneer in the experimental aspects of PAT and TAT. His group has developed photo-acoustic imaging technologies for in vivo early-cancer detection and functional imaging by physically combining non-ionizing electromagnetic and ultrasonic waves. The hybrid imaging modality provides relatively deep penetration at high ultrasonic resolution and yield speckle-free images with high electromagnetic contrast. With this technique, optical contrast can be used to quantify the concentration of total hemoglobin, the oxygen saturation of hemoglobin, and the concentration of melanin. Melanoma and other tumors have been imaged in vivo in small animals.

Sarah Patch was a pioneer in the development of reconstruction algorithms in TAT. Her talk presented the device used at the university of Wisconsin-Milwaukee to perform TAT experiments and underlined the many difficulties associated with generating non-resonant electromagnetic signal excitations.

Reconstructions in photo-acoustics and thermo-acoustics can be separated into two steps. The first step consists of reconstructing the amount of absorbed radiation by the tissues from measurements of ultrasounds at the boundary of the domain. The latter quantity is, however, a functional of the optical parameters of the tissues, which depends on unknown solutions to a partial differential equation. The second step consists then of reconstructing the optical parameters from the now known absorbed radiation. This second step is called quantitative photo-acoustics. Roger Zemp addressed this issue and presented results of simultaneous reconstructions of both the absorption and the scattering coefficients from knowledge of absorbed radiation corresponding to several illuminations of the sample. Quantitative photo-acoustic is known in some cases to correspond to a well-posed problem (more precisely very mildly ill-posed) unlike optical tomography. This was confirmed by spatially accurate and robust numerical reconstructions.

Most photo-acoustic and thermo-acoustic reconstructions ignore acoustic wave absorption. However, high frequencies are significantly attenuated causing difficulties to achieve sub-millimeter resolution at a depth of several centimeters in human tissues. Otmar Scherzer presented recent results on the modeling of absorption and its impact on thermo-acoustic reconstructions. Absorption modeling is rendered extremely complicated by the fact that different frequencies are attenuated differently. Causality must be preserved, which generates well-recognized difficulties. The talk detailed a specific attenuation law used in other contexts and analyzed its effects on reconstructions in thermo-acoustics.

Although radiation transport is an important step in photo-acoustics, a central step is the reconstruction of amount of absorbed radiation from measured ultrasounds. Mathematically, this takes the form of an inverse wave problem where the initial condition is sought from boundary measurements. This problem belongs to the broad family of inverse source problems which can be tacked by time reversion since the wave propagation solution operator is a unitary operator. Alison Malcolm analyzed the time reversal methodology to address the underground sequestering of CO2 and determine whether changes in the underlying medium (such as cracks) is important or not and possibly reconstruct such changes. More specifically, the shape and frequency of correlations of the coda of multiply scattered waves were exploited to obtain localization of such potential changes.

Where is the field going?

The workshop was successful in bringing together mathematicians, applied scientists and engineers who specialize in the analysis and applications of forward and inverse transport theory. Although many inverse problems remain to be addressed in this broad field, significant, practically relevant, theoretical results have been obtained in recent years. This workshop allowed the participants to obtain an up-to-date cross section of the field. The workshop also presented several applications of inverse transport as they are studied in the applied sciences and engineering disciplines. This generated discussions between the communities represented at the workshop, which was an important organizational objective.

The workshop also helped identify some areas in which mathematical analysis may very well be useful for practitioners. Let us give one such example in the field of photo- and thermo-acoustics. Although the wave inversion problem is well understood for propagation in smoothly varying media with sufficient information at the domain's boundary, significant difficulties arise when the refractive effects of, e.g., the skull are taken into account. Understanding this problem is one of the major roadblocks to brain imaging using thermo-acoustic tomography.

The problem of simultaneously recovery of the absorption and the source in the attenuated X-ray transform remains a challenging open theoretical problem.