44th Cascade Topology Seminar

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The Cascade Topology Seminar was started in the early 90s by Steven Bleiler (Portland State Univ.), Douglas Ravanel (Univ. of Washington), and Dale Rolfsen (Univ. of British Columbia). It has continued since that time as a very successful biannual topology meeting for the Pacific Northwest region, although the original geographic terminology representing the Cascade Mountains of Oregon, Washington, and British Columbia has been broadened to include meetings in California, Nevada, Idaho, and Alberta. The CTS has also been run under the auspices of PIMS for the past half dozen years or so. The current meeting was the second to be held in Alberta, both taking place at BIRS. Very aptly, one can look out of the rooms at BIRS and have a striking view of nearby Cascade Mountain.

The influence of the original three founders was quite apparent at this meeting. Steven Bleiler was present and one of the talks, as well as a talk given a day before the meeting at the Univ. of Calgary (by Stanislav Jabuka), was related to his work. Dale Rolfsen has attended most of the meetings but this time he was well represented by his former graduate student Adam Clay, who gave the second lecture. Doug Ravanel, the third founder, moved to Rochester a number of years ago but his recent work with Hopkins and Hill was represented by the final talk of the meeting, given by Mike Hill.

Since the list of talks and participants is available to BIRS, we will just give a brief demographical summary in this paragraph. In total there were 26 participants. The 12 from the US came from California, Illinois, Indiana, Michigan, Nevada, New York, New Mexico, Oregon and Virginia. The 14 from Canada included 8 from Alberta, 4 from British Columbia, and 2 from Ontario. We have not listed the individual universities, but it is worth noting that for the first time a topologist (Ryan Budney) from Univ. of Victoria participated, a very welcome addition to the list of universities in the area that are active in topology. Indeed, the 45th Cascade Topology Seminar is already planned for Spring 2011 in Victoria. In total, there were 16 faculty members, 4 postdoctoral fellows and 6 graduate students in attendance.

Before turning to a brief description of the material covered in the lectures, here is a quote from an email of one of the participants, Rustam Sadykov, who is from Moscow State University, then spent two years in Japan, and currently is a postdoc at the Univ. of Toronto.

"The conference was very enjoyable and the organization was superb. To begin with the talks were interesting and well presented. I think the speakers did a superb job. Of course I was happy to finally meet you (the conference was a nice opportunity) and it was nice that we had time for a discussion. I hope that we will continue our cooperation.

As for the place of the meeting, it's perfect! Nature, hiking, nice food - all these also create a friendly atmosphere. I hope to participate in subsequent meetings as well. I also hope to have a chance to present my work at one of these wonderful meetings."

To obtain some perspective on the content of the lectures, let us go back to the CRG (Collaborative Research Group) that took place under the auspices of PIMS in 2003-2005. This CRG was divided into two

halves, the first year concentrating on low dimensional topology and the second year on geometric topology and homotopy theory. Of course topology has grown during the 20th Century into an enormous subject, but it is probably safe to say that these two concentrations of the CRG represent the most important current trends and developments. One can easily make a strong case that topology has experienced the most significant growth of any branch of mathematics in the first decade of the 21st century, due to the solution of two outstanding conjectures in the subject. The first is the Poincaré Conjecture, which has received the most publicity (as one of the seven Millenium Problems). The second and more recent is the solution of the Kervaire Conjecture (also called the Arf Invariant One Conjecture), by Mike Hill, Mike Hopkins, and Doug Ravanel. In the author's view this is even more important than the Poincaré Conjecture, since it is not only basic to our understanding of space through the theory of manifolds, but is also fundamental to the understanding of stable homotopy theory (of course not everyone will agree with this view). In any case, having Mike Hill speak on this breakthrough as the final talk of the meeting was a very exciting and fitting conclusion to the meeting.

Indeed, each of the six talks represented significant new progress in various areas of topology as well as the closely related area of algebraic K-theory. The first two talks, given by Matthew Hedden (Michigan State Univ.) and Adam Clay (Univ. of British Columbia), were in the general area of low dimensional topology. Both of these talks were related to knot theory, but to different aspects of it. Hedden's talk dealt with the knot concordance group, the smooth version being denoted $\mathcal{C}$ and the topological version $\mathcal{C}_{top}$. There are some subtle differences between the two and the talk described significant progress in understanding this. Clay's talk dealt with a subject very dear to his supervisor, ordered groups. The first half of the talk gave a clear introduction to this topic and dealt mainly with algebraic questions. In the second half interesting topological applications were made, in particular to knot theory.

The next two talks, given by Eric Malm and Ralph Cohen (both from Stanford Univ.) came under the general heading of string topology. Cohen is an acclaimed authority in this area, and the main organizer of the meeting, Veronique Godin, was a student of his. The first talk emphasized some of the algebraic aspects of string topology arising from the celebrated Chas-Sullivan Theorem, with close connexions and analogies to Hochschild homology and non-commutative geometry. The second talk went more into some of the categorical aspects and the connexions with topological field theory.

The second last talk, by Teena Gerhardt of Indiana Univ., was mainly about algebraic K-theory as mentioned above. Computations in this subject are few and notoriously difficult, but she has succeeded in a few interesting examples. As one example,

$$K_{2i-1}(\mathbb{Z}[X]/(X^m),\ X) \approx \mathbb{Z}^{m-1},$$

while K_{2i} is not fully computed here but is known to have finite order $(mi)!(i!)^{m-2}$. We have already discussed the final talk by Mike Hill.

All these talks were presented using chalk and blackboard only, a most impressive achievement for the speakers and a most pleasant experience for the listeners, I believe, in this day and age of power point presentations that are nearly always so rapid as to be incomprehensible and often hardly even worth attending. To conclude, many thanks to the main organizer Veronique Godin, and the other organizers were Kristine Bauer, Jens von Bergmann, and myself, all from the Univ. of Calgary.