MATHEMATICAL THEORY OF RESONANCES: CONFERENCE REPORT

1. Research at the conference

The conference gathered a diverse group of experts working on different aspects of mathematical theory of resonances.

Certain topics were subject to particularly lively discussions:

- Lower bounds on resonances using several complex variable methods; C Guillarmou and F Naud started the investigation of these methods (used by T Christiansen and P Hislop) in the context of geometric scattering.
- D Jakobson and F Naud started a collaboration on lower bounds for resonances for Schottky quotients which resulted in the preprint

  `http://www.math.mcgill.ca/jakobson/papers/resonance-lowbd.pdf`

  (with the BIRS conference acknowledged).
- N Burq's talk led to some discoveries about the work of H Christianson and the discussions resulted in an ongoing collaboration.
- D Bindel provided M Zworski with new codes for computing resonances relevant to the latter's collaboration with experimental physicists (H Manoharan's group at Stanford).
- S Nonnenmacher and M Zworski made progress in the ongoing project on resonances for chaotic scattering with topologically one dimensional trapped sets.
- W Müller and A Vasy discussed resonances for locally symmetric spaces.
- D Borthwick, T Christiansen, P Hislop, and P Perry made progress on a project involving the distribution of resonances for perturbations of infinite volume hyperbolic manifolds.

2. Titles and abstracts

I. Alexandrova

Title: The Structure of the Scattering Amplitude for Schrodinger Operators with a Strong Magnetic Field

Abstract: We study the microlocal structure of the semi-classical scattering amplitude for Schrodinger operators with a strong magnetic field at non-trapping energies. We prove that, up to any order, the scattering amplitude can be approximated by a semi-classical pseudodifferential-operator-valued Fourier integral operator.

D. Bindel

Title: Numerical methods for resonance calculations

Abstract: In this talk, I revisit the problem of computing resonances for one-dimensional Schroedinger-type operators in two ways: using perfectly matched absorbing layers (a variant on complex scaling) and by converting the problem to a nonlinear eigenvalue problem. For each formulation, I describe both the numerical methods and error estimates for the computation.

If time permits, I will also talk about an application of resonances in micro-mechanical components from next-generation cell phone designs, and say how we used perfectly matched layers to compute resonances in this problem.

D. Borthwick

Title: The Poisson formula for resonances on manifolds hyperbolic near infinity.

Abstract: We present some very developments on the distribution of resonances for conformally compact manifolds which are hyperbolic near infinity. These include (1) the optimal upper bound on the resonance (or scattering pole) counting function, (2) the Poisson formula for the regularized wave trace, (3) optimal lower bounds for resonances, and (4) Weyl asymptotics for the relative scattering phase defined by a compact perturbation.

V. Bruneau

Title: Regularized determinant and semi-classical resonances

Abstract: Based on a joint work with J.M. Bouclet I will show that the resonances can be view as zeroes of some regularized determinants. It works for long range perturbations of the Laplacian. Some “Breit-Wigner” approximation of regularized spectral shift function can be deduced.

Nicolas Burq

Title: Probabilistic Sobolev embeddings and behaviour of solutions to wave equations

Colin Guillarmou

Title: Resonances in hyperbolic geometry

Peter Hislop

Title: Resonances for Schrödinger operators with compactly supported potentials

Abstract: The resonance counting function for Schrödinger operators $H_V = -\Delta + V$ on $L^2(\mathbb{R}^d)$, for $d \geq 1$, with compactly-supported, real- or complex-valued potentials V is known to be bounded above by $C_V(r^d+1)$. The main result is that for a dense G_δ-set of such potentials, the resonance counting functions have the maximal order of growth d. For the even dimensional case, it is shown that the resonance counting functions have maximal order of growth on each sheet Λ_m, $m \in \mathbb{Z}\backslash\{0\}$, of the logarithmic Riemann surface. This results is obtained by constructing a certain plurisubharmonic function from the determinant of a modified S-matrix and proving that the order of growth of the counting function can be recovered from a suitable estimate on this function. An example is constructed in each even dimension of a potential having a resonance counting function bounded below by $C_m r^d$ on each sheet.

M. Hitrik

Title: Non-elliptic quadratic forms and operators with double characteristics

Abstract: This is a report on a work in progress together with Karel Pravda-Starov. For a class of non-selfadjoint pseudodifferential operators with double characteristics, we study semiclassical resolvent bounds and estimates for low lying eigenvalues. Specifically, assuming that the quadratic approximations along the characteristics enjoy an ellipticity property along a suitable symplectic subspace of the phase space, we establish semiclassical hypoelliptic a priori estimates with a loss of a full power of the semiclassical parameter. We also compute the (discrete) spectrum of the associated quadratic operators and describe the large time behavior for the corresponding heat semigroups.

D. Jakobson

Title: Estimates from below for the spectral function and for the remainder in Weyl's law on negatively-curved surfaces

Abstract: We obtain asymptotic lower bounds for the spectral function of the Laplacian on compact manifolds. In the negatively curved case, thermodynamic formalism for hyperbolic flows is applied to improve the estimates. Our results can be considered pointwise versions (on a general manifold) of lower bounds (due to Hardy and Landau) for the error term in the Gauss circle problem. We next discuss lower bounds for the remainder in Weyl's law on negatively-curved surfaces. Our approach works in variable negative curvature, and is based on wave trace asymptotics for long times, thermodynamic formalism for hyperbolic flows, and small-scale microlocalization. This is joint work with I. Polterovich and J. Toth.

A. Martinez

Title: Resonances for non-analytic potentials (joint work with T. Ramond and J. Sjöstrand)

Abstract: We consider semiclassical Schrödinger operators on $\mathbb{R}^n$, with C^∞ potentials decaying polynomially at infinity. The usual theories of resonances do not apply in such a non-analytic framework. Under some additional conditions, we show that resonances are invariantly defined up to any power of their imaginary part. The theory is based on resolvent estimates for families of approximating distorted operators with potentials that are holomorphic in narrow complex sectors around $\mathbb{R}^n$.

Marco Merkli

Title: Time-dependent Resonance Theory for Open Quantum Systems

Abstract: We consider interacting quantum systems of the type S+R, where S is a system of interest and R is a reservoir. While S has finitely many degrees of freedom, R is a spatially infinitely extended dissipative system. The central problem in the theory of open quantum systems is to describe the reduced dynamics of S, which is induced by the interaction with R. I present a theory of quantum resonances which allows for a detailed description of the dynamics of the reduced denisty matrix of S, valid for all times and for a small, fixed interaction strength between S and R. In the present setting, resonances come about by perturbation of embedded eigenvalues of the generator of dynamics, and they are described by analytic spectral deformation. I illustrate the theory with applications to quantum computing. In this situation, S is a chain of N spins 1/2, representing an N-qubit quantum register. I will give decoherence rates and show how they behave as functions of the complexity N. This is joint work with G.P. Berman and I.M. Sigal.

L. Michel

Title: Semiclassical analysis of a random walk on a manifold

W. Müller

Title: Distribution of resonances on locally symmetric spaces of finite volume

S. Nakamura

Title: Time-dependent scattering theory for Schrödinger operators on scattering manifold (joint work with K. Ito)

Abstract: We propose a time-dependent approach to the scattering theory for Schrödinger operators on manifolds with asymptotically conic structure. We use a simple comparison system and the 2-space scattering theory of Kato to define wave operators and the scattering matrix.

S. Nonnenmacher

Title: Quantum resonances in presence of a fractal trapped set (collab. with M.Zworski)

Abstract: We investigate the resonance spectrum close to the real axis, for a certain type of semiclassical Schrodinger operators. The smooth potential (or curved part of the metric) is taken of compact support, while the metric is Euclidean near infinity. Furthermore, we

assume that, near a certain positive energy, the dynamics generated by the corresponding classical Hamiltonian admits a hyperbolic trapped set, which is generally "fractal". We show that, provided this trapped set is "thin enough", resonances near this energy cannot be "too close" to the real axis. More precisely, we show that the corresponding quantum decay rates are uniformly bounded from below by the topological pressure, a quantity characterizing the classical flow on the trapped set. This generalizes similar results by Ikawa for Euclidean obstacle scattering. We also prove a resolvent estimate analogous with the case of a single trapped orbit. Finally, we partially describe the phase space structure of the "resonant eigenstates": they are microlocalized along the unstable manifold of the trapped set.

P. Perry

Title: Scattering Theory for Complex Manifolds with CR-Boundary

Abstract: This talk concerns joint work with Peter Hislop and Siu-Hung Tang. A pseudoconvex domain in $\mathbb{C}^n$ such as the complex unit ball is a model of a complex manifold with CR-boundary. When equipped with a Kahler metric of Bergmann type, the interior becomes a complete Riemannian manifold and one can study scattering theory for the Laplacian. We'll show that certain poles of the scattering operator determine CR-covariant differential operators analogous to the GJMS operators that occur as poles of the scattering operator for the conformally compact Einstein manifolds studied by Graham and Zworski; we'll also give a geometric condition under which this construction works for complex manifolds with strictly pseudoconvex boundary. A key ingredient in the construction is a global approximate solution of the complex Monge-Ampere equation.

Vesselin Petkov

Title: Analytic continuation of the resolvent of the Laplacian and the dynamical zeta function for open billiards

A. Vasy

Title: High energy estimates on the analytic continuation of the resolvent and wave propagation on the De Sitter-Schwarzschild space

Abstract: In this joint work with Richard Melrose and Antonio Sa Barreto we construct a high energy parametrix for the analytic continuation of the resolvent of the Laplacian on asymptotically hyperbolic spaces which are close to hyperbolic space; this parametrix is valid uniformly in a strip beyond the real axis. We use this and cutoff resolvent estimates of Bony and Haefner to obtain resolvent estimates on weighted spaces for the spatial "Laplacian" on De Sitter-Schwarzschild space (which, near infinity, is close to the hyperbolic Laplacian). This is then used to derive the asymptotics of solutions of the wave equation. Some similar results on wave asymptotics were obtained by Dafermos and Rodinianski.

Georgi Vodev

Title: Distribution of the resonances for the ellasticity system with dissipative boundary conditions

J. Weir

Title: A non-self-adjoint differential operator with purely real spectrum

Abstract: There are few examples of non-self-adjoint operators whose spectra are purely real, other than those which are similar to self-adjoint operators. We consider a highly non-self-adjoint differential operator arising in fluid mechanics which depends upon a small parameter ϵ. This operator is not similar to any self-adjoint operator, yet we identify a self-adjoint Sturm-Liouville operator which has the same spectrum, thus proving that the spectrum is real. By studying this self-adjoint operator, we then determine the asymptotic distribution of the eigenvalues and the behaviour of the spectrum as $\epsilon \to 0$.

3. Participants

Alexandrova, Ivana, East Carolina University

Bindel, David, Courant Institute of Math. Sciences (New York University)

Borthwick, David, Emory University

Bruneau, Vincent, Universite de Bordeaux I (France)

Burq, Nicolas, Universite de Paris XI (France)

Christiansen, Tanya, University of Missouri

Christianson, Hans, Massachusetts Institute of Technology

Datchev, Kiril, University of California, Berkeley

Froese, Richard, University of British Columbia

Guillarmou, Colin, Université de Nice

Hislop, Peterm, University of Kentucky

Hitrik, Michael, University of California, Los Angeles

Jakobson, Dmitry, McGill University

Martinez, André, Università di Bologna

Merkli, Marco, Memorial University,

Michel, Laurent, Université de Nice,

Müller, Werner, Universität Bonn,

Nakamura, Shu, University of Tokyo,

Naud, Frederic, Université d'Avignon,

Nedelec, Laurence, Stanford University

Nonnenmacher, Stephane, Commissariat à l'Energie Atomique, Saclay

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Villegas-Blas, Carlos, UNAM (Mexico),

Vodev, Georgi, Université de Nantes,

Weir, John, Kings College, London,

White, Denis, University of Toledo,

Zworski, Maciej, University of California at Berkeley.