Dark Side of Extra Dimensions

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The idea that the spacetime may have more than four dimensions is very old. Starting with works of Kaluza [1] and Klein [2], higher dimensional models were used to unify gravity with other fields. In more recent time, it was demonstrated that the string theory, which is often called a theory of everything, requires higher dimensions for its consistency. Models with the spacetime with large extra dimensions were recently proposed in order to solve the hierarchy problem, that is to explain why the gravitational coupling constant is much smaller than the coupling constants of other physical interactions. In such models, our 4-dimensional spacetime is described by a 4-dimensional brane (submanifold) embedded into a higher dimensional (bulk) space. Particles and fields (except gravity) propagate within the brane, while the gravity can propagate in the bulk space. These new concepts of higher dimensional physics have a number of interesting applications in modern cosmology and theory of gravity. At the same time they require developments of the theoretical and mathematical tools to address many new important questions. At the "Dark Side" workshop new results and open questions in this fast developing field were discussed.

One of the most important questions is to analyze how the gravitational theory is modified in the presence of extra dimensions. In the study of the Einstein equations in the 4-dimensional spacetime several powerful mathematical tools were developed, based on the spacetime symmetry, algebraical structure of spacetime, internal symmetry and solution generation technique, global analysis, and so on. At our workshop there was discussion and concrete proposal, how to develop some of these methods to higher dimensional spacetime.

Many exact solutions of the Einstein equations in 4-dimensional case were obtained by algebraic methods based on the Petrov classification. At the workshop it was proposed and discussed the generalization of the Petrov classification to higher dimensional case. It was demonstrated that the robust classification into Petrov classes can be done in arbitrary number of dimensions [3]. At the same time, the number of different degenerate subclasses within each of the Petrov class depends on the number of spacetime dimensions. To classify these subclasses in higher dimensions is much more sophisticated problem than in 4-dimensional case.

Another problem which was discussed at the workshop is an existence and properties of different "black objects" in higher dimensions. These objects are generalization of 4-dimensional black hole solutions. According to the definition, a black hole is an (asymptotically flat) spacetime with non-trivial causal structure. Black hole boundary is an event horizon, a 3-dimensional surface which separates a spacetime region which can be "seen" from infinity from an "invisible" region. Under physically reasonable conditions, in 4 dimensions the horizon has the topology of $S^2 \times R^1$. Moreover, "uniqueness theorems" were proved, which guarantee that for given value of global parameters (mass, angular momentum, and charge) the stationary solutions describing black holes are unique. Recently it was demonstrated that the uniqueness theorems are not valid if the number of spacetime dimensions is greater than 4 [4]. Higher dimensions open room for a variety of dark objects, which are natural generalizations of 4-dimensional black holes. Main difference between these dark objects is the topology of their horizons.

One of the problems which was discussed at the workshop is stability of higher dimensional dark objects and possible transitions between them. Gregory and Laflamme [6] described a particular mechanism of the instability of higher dimensional string, but what is a final state of a decaying dark string is still an open question. At the workshop there were presented results of the numerical simulations of decaying dark strings [7]. Unfortunately these results do not allow one to resolve dynamics and final state of this process. There exist evidences in favor that the black-string–black-hoole transitions may be similar to critical phenomena and the very transition from a black string to a black hole phase may have similarity with the critical gravitational collapse phenomena [8]. Another important connected problem which was discussed at the workshop is possible instability of rapidly rotating black holes and black rings [5]. This area (stability of higher dimensional dark objects and possible transitions between them) is developing very fast and for its progress developed mathematical tools are required.

Another subject which was in the focus of the workshop was study of exact solutions of higher dimensional Einstein equations. Two new families of solutions were presented and discussed at the workshop. One of them is a generalization of Mayers-Perry metrics for higher dimensional black holes to the case when there a non-vanishing cosmological constant [9]. Another new set of solutions describes the gravitational field of spinning relativistic objects (gyratons) in a spacetime with arbitrary number of dimensions [10]. An interesting property of the latter solutions, that the non-linear system of Einstein equations is effectively reduces to two linear set of equations in a flat spacetime. By solving these linear equations, one can generate a solutions of the non-linear problem.

One of the reasons why the higher dimensional theories becomes so popular recently is a possibility that in the presence of extra dimensions one can expect creation of mini black holes in future collider and cosmic ray experiments. At the workshop there was given a detailed overview of the corresponding results and were formulated concrete physical problems which are to be solved for better understanding of such processes [11].

To summarize, the workshop gave very nice view of the state of art in the higher dimensions physics and mathematics of dark objects. It has very enthusiastic support and many of participants proposed to organize again a workshop on a similar subject in future.

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