Report on the BIRS workshop

Quadratic Forms, Algebraic Groups, and Galois Cohomology

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The algebraic theory of quadratic forms began with the seminal paper of E. Witt [31] in 1937, where what are now called "Witt's Theorem" and the "Witt ring" first appeared. But it was not until a remarkable series of papers [19] by A.Pfister in the mid-1960s that the theory was transformed into a significant field in its own right. The period 1965 to 1980 can be considered the "first phase" of the algebraic theory of quadratic forms and is well documented in the books of T.-Y.Lam [13] (1973) and W.Scharlau [25] (1985). Lam's book itself came at a critical time and greatly influenced the development and popularity of the field.

In 1981 another phase began with the first use of sophisticated techniques from outside the field—in this case algebraic geometry—by A.S.Merkurjev, who proved a long-standing conjecture of A.A.Albert on a presentation for the exponent two subgroup of the Brauer group. This answered the first open case of the Milnor conjecture of 1970 [17] on the relationship between algebraic K-theory, the Brauer group and the Witt ring. As the preeminent open problem for many years in the algebraic theory of quadratic forms, the Milnor conjecture exerted a profound influence on the subject. This work was extended shortly thereafter (1982) in a paper by Merkurjev and A.A.Suslin [15], showing that when the field F contains the n^{th} roots of unity, the n-torsion in the Brauer group of F is isomorphic to the algebraic K-group $\mathrm{K}_2(F)/n\mathrm{K}_2(F)$.

During this time, quadratic form theory also widened its involvement with other areas of mathematics. T.A.Springer's use of Galois cohomology in 1959 [26] in recasting some of the classical invariants of quadratic forms in terms of the Galois cohomology of the orthogonal group was one of the first applications of Galois cohomology in algebraic groups, following A.Weil's classification of algebraic groups of classical type. And the strong approximation theorem of M.Kneser [10] for orthogonal groups led to a generalization of that theorem in the theory of semisimple algebraic groups. Siegel's work [24] on the representations of one quadratic form by another led to the use of adèles in algebraic groups by T.Tamagawa [28], in particular to the notion of Tamagawa numbers in this wider context.

This relationship of Galois cohomology and algebraic groups to quadratic form theory continues to grow in importance as this workshop has strikingly demonstrated. These fields have become increasingly sophisticated in recent years, mainly through further incursions by algebraic geometry and, in particular, by motivic methods. Voevodsky's work, for which he received the Fields Medal in 2002 [7], has been very influential, not least because of his complete (positive) solution of Milnor's Conjecture. A highlight of our workshop was his proof of the final link in the confirmation of the Bloch-Kato Conjecture, which can be viewed as the most general form of Milnor's Conjecture.

Thus the primary emphasis in the meeting dealt with the impact of motivic methods on the subjects of the workshop. Many new and startling results in the algebraic theory of quadratic forms have been proved by these methods. There were as well many interesting and very important talks on a variety of other topics which do not fit within a coherent group or groupings.

Therefore we begin this report with a description of the talks on motivic methods, followed by a section on "miscellaneous" results.

Motivic Methods

V. Voevodsky. In the late nineties V. Voevodsky developed an algebraic homotopy theory in algebraic geometry similar to that in algebraic topology. He defined the (stable) motivic homotopy category and certain spectra that give rise to interesting cohomology theories such as motivic cohomology, K-theory and algebraic cobordism. Note that before the work of Voevodsky, the only comprehensive cohomology theory available in algebra was the algebraic K-theory (defined by D. Quillen by means of algebraic topology). In the eighties, A. Beilinson [3] and S. Lichtenbaum [14] predicted the existence of motivic cohomology and conjectured relationships between it and étale motivic cohomology theories. This conjecture (known as the Beilinson-Lichtenbaum Conjecture) has been one of the central conjectures in algebraic geometry. A particular case of the Beilinson-Lichtenbaum Conjecture, the

Bloch-Kato Conjecture [4], asserts that the norm residue homomorphism

$$\mathrm{K}_n(F)/p\mathrm{K}_n(F) \to H^n_{\text{ét}}(F, \mu_p^{\otimes n})$$

is an isomorphism for every field F, positive integer n and prime $p \neq \operatorname{char} F$. But A. Suslin and V. Voevodsky [27] proved that in fact the Bloch-Kato Conjecture is equivalent to the Beilinson-Lichtenbaum Conjecture.

The Bloch-Kato Conjecture has its origins in Milnor's Conjecture, which is the case of $p = 2$. As mentioned earlier in the introduction, the first steps in the proof of that special case were made in a paper of Merkurjev who verified it for $n = 2$, and then shortly thereafter, Merkurjev and Suslin gave a proof the general case of $n = 2$, and then a few years ago, V. Voevodsky provided a proof for the general case of Milnor's Conjecture.

The highlight of this workshop was the announcement by Voevodsky in his conference talk of a full solution of the Bloch-Kato Conjecture [30]. Thus, the Beilinson-Lichtenbaum Conjecture is now proven in full generality!

Ph. Gille. Another look at the Bloch-Kato Conjecture was given by Ph. Gille in his talk. He considered an "elementary" approach involving Severi-Brauer varieties instead of general splitting varieties, and Tate's continuous Galois cohomology instead of motivic cohomology. Gille gave an equivalent reformulation of the Bloch-Kato Conjecture in this setting.

A. Vishik. There has been striking progress in the algebraic theory of quadratic forms since Voevodsky introduced his motivic methods. In particular the long-standing problem on the relation between Milnor's K-theory and the graded Witt ring was solved by D. Orlov, A. Vishik and V. Voevodsky [18]. More precisely, they proved that the canonical homomorphism

$$\mathrm{K}_n/2\mathrm{K}_n(F) \to I^nF/I^{n+1}F,$$

where IF is the fundamental ideal of the Witt ring of a field F, is an isomorphism. Another important conjecture on the description of the first Witt index of quadratic forms has been solved by N. Karpenko. In the proof he used the algebraic Steenrod operations invented by Voevodsky and described (on the Chow groups of algebraic varieties) by P. Brosnan. In his talk Vishik discussed other operations that arise on the level of the cobordism groups of algebraic varieties. He is now using these operations in an attempt to describe the so-called generic discrete invariant of quadratic forms. This invariant includes all discrete invariants such as splitting patterns of quadratic forms and dimensions of quadratic forms in $I^n F$.

N. Karpenko. Karpenko announced a solution of the following problem: If q is an anisotropic quadratic form in I^n of dimension less than 2^{n+1}, then

$$\dim q = 2^n + 2^{n-1} + 2^{n-2} + \ldots + 2^k$$

for some $k = 1, 2, \ldots, n$. In the proof he uses the whole spectrum of modern "elementary" techniques—the Steenrod operations of P. Brosnan and the motivic decomposition of quadratic forms in the category of Chow motives (developed in works of A.Vishik [29]). A solution of this problem had been announced earlier by Vishik, who used different and more involved techniques.

P. Brosnan. Brosnan presented an alternative "elementary" proof of Rost's Nilpotence Theorem [22]:

Let X be a projective quadric over a field F, with motive $M(X_F)$ in the category of Chow motives. Then for every field extension L/F the kernel of the canonical ring homomorphism

$$\mathrm{End}\, M(X_F) \to \mathrm{End}\, M(X_L)$$

consists of nilpotent elements.

The Nilpotence Theorem is an essential ingredient of the basis of the motivic theory of quadratic forms. Brosnan's proof avoids the use theory of cyclic modules involved in the original proof of M. Rost.

V. Chernousov. Chernousov reported on the generalization of Rost's Nilpotence Theorem to the whole class of projective homogeneous varieties. The main ingredient of the proof is a motivic decomposition of isotropic projective homogeneous varieties into a direct sum of motives of twisted anisotropic projective homogeneous varieties.

F. Morel. Morel discussed the construction and computation of stable cohomology operations in the cohomology theory on simplicial smooth schemes $\mathcal{X}$ given by $H^*_{Nis}(\mathcal{X}; k_*)$, the Nisnevich cohomology of $\mathcal{X}$ with coefficients in the unramified mod ℓ Milnor K-theory sheaves k_* (where ℓ is a prime different from char k). He showed how the Bloch-Kato conjecture at ℓ predicts the structure of the algebra of all stable cohomology operations and that conversely, the knowledge of that algebra almost implies formally the Bloch-Kato conjecture. This explains the difficulty of computing that algebra, as opposed to the computation of the Steenrod algebra in mod ℓ-motivic cohomology which is "easy" by comparison.

He also explained how just the existence of some operations have nontrivial consequences for Milnor K-theory; for example in the case $\ell = 2$ the existence of the operation Sq^2 is "close" to proving the Milnor conjecture on quadratic forms, and an example of the construction of an explicit "extended power operation", which might prove useful in this regard, was given.

Miscellaneous Talks

J. Arason, B. Jacob. The study of quadratic forms over fields in characteristic two has a different flavor than that in other characteristics. Results are also often different. One outstanding example has been the computation of the Witt group of quadratic forms over a rational function field when the base field has characteristic two. Two talks were given on this topic, each independently solving this problem. The basis of such a computation is the local case.

Arason provided a presentation for the Witt group in characteristic two and used careful computations among the generators and relations to determine the Witt group of a Laurent series field. He then showed how to determine the Witt group of a rational function field (in one variable) over a field of characteristic two as a corollary.

Jacob, in collaboration with R. Aravire, also determined the local case and hence the rational function field case. They also obtained a reciprocity law, at least if the base field is perfect (a condition they are currently attempting to remove).

P. Balmer. Balmer lectured on his joint work with R. Preeti on odd indexed Witt groups of semi-local rings. The setting is an appropriate triangulated category with duality [2] and arose from studies in L-theory ([20], [16]). This category has nice cohomological and topological properties and agrees with

M. Knebusch's Witt groups on an algebraic variety [9] and M. Karoubi's Witt group of an exact category with duality [8]. The main study is that of the odd indexed components W^{2i+1} of the total Witt group over (not necessarily commutative) local and semi-local rings. The factors in the total Witt group have periodicity four. In the general case, Witt cancellation does not hold, and it is conjectured that the odd indexed Witt groups encode information about this failure of Witt cancellation when the ring has an involution. For example, over commutative semi-local rings with involution the identity, cancellation holds and these odd indexed groups are indeed trivial. Balmer and Preeti also determine a decomposition of the odd Witt groups over semi-simple rings with involution, which depends only on the simple factors with involutions of the first kind. They then relate the third Witt group with maximal ideals in the commutative semi-local case. In particular, they show that if R is semi-local and commutative, $W^1 = 0$ and $W^3 = (\mathbf{Z}/2\mathbf{Z})^m$ where m is a computable integer bounded by the number of maximal ideals in R.

G. Berhuy. A most interesting and natural invariant called the *essential dimension* introduced by Z. Reichstein [21] measures the number of parameters needed to describe a given structure up to isomorphism. For example, to describe all quadratic forms of rank n over a field one needs at least n parameters, and n is in fact needed in general. Such definitive answers are rare, but upper and lower bounds have been determined in several interesting cases. In his talk, Berhuy presented his work with G. Favi on cubics over a field, sketching a proof that the essential dimension for the set of cubics in three variables is precisely three (assuming the field contains a cube root of unity and has characteristic not two or three).

S. Gille. One of the primary computations in quadratic form theory is that of the Witt ring of a Laurent series ring, based on work of M. Karoubi and A. Ranicki using L-theory. Geometrically, this can be viewed as the Witt ring of the product of an affine scheme and a punctured affine line. Gille and P. Balmer generalized this result by computing the total Witt ring of the product of a regular finite dimensional scheme with a union of punctured affine spaces (when two is a unit). The usual methodology of reducing to the affine case and the localization sequence is followed by using Koszul complexes instead of coherent Witt theory. This allows globalization of a certain key element of the theorem whose construction they show to be independent of the constructions introduced in the proof. As an application they apply the theorem to an affine hyperbolic space over a regular ring, retrieving a theorem of Karoubi in the case of a field.

D. Hoffmann. Hoffmann presented his work on an algebraic introduction to p-forms. This study mimics known quadratic form theoretic techniques applied to the case that F is a field of positive characteristic p and a p-form is an additive form on a finite dimensional vector space where scalars pull out to the p^{th} power, in other words $a_1X_1^p + a_2X_2^p + \cdots + a_nX_n^p$. Because of the binomial theorem, many of the analogues of quadratic form theory—such as Pfister forms, the Cassels-Pfister theorem, the subform theorem, the Knebusch-Norm theorem—follow. The main reason for this is the fact that the coefficients $a_1, a_2, \ldots, a_n$ generate a field over F^p reflecting important properties of the form.

M. Knus. A classical result of Hurwitz gives the complete list of quadratic composition algebras with identity over a given field F: the field itself, quadratic extensions, quaternions or octonions over F. Thus such a composition is only possible in dimension $1, 2, 4$ or 8. Rost gave a purely tensor categorical proof of this result about the possible dimensions by considering the vector algebra of pure elements inside such a composition algebra. The universal tensorial object associated with a vector algebra can be interpreted as a category of graphs and graph manipulations lead to the equation

$$d(d-1)(d-3)(d-7) = 0$$

for the dimension d, which occurs as a numerical invariant of the category. A complete list of cubic compositions was given by Schafer, using structure theory. According to Rost, graph theoretical computations can also be applied to such compositions: the pure elements (i.e. the elements orthogonal to 1) admit the structure of a "symmetric" composition. Two numerical invariants d and e can be attached to the graph category of a symmetric composition; d is the dimension and e is associated to a "Casimir" element. The possible values are $(d, e) = (0, 0)$, $(1, 1)$, $(2, 0)$, $(4, 4)$, $(8, 0)$ and $(8, 36)$. This gives the possible dimensions 1, 2, 3, 5 and (twice) 9 for the cubic composition. Simple exceptional Jordan algebras of dimension 27 satisfy a generalized notion of cubic composition. In a dissertation in progress (L. Cadorin), a graph theoretical approach is developed to include the case of exceptional Jordan algebras (type D_4).

D. Lewis. Lewis presented his joint work with T. Unger and J. van Geel on the Hasse Principle for Hermitian forms over a quaternion algebra (with the standard involution) over a number field. The Hasse Principle is known to fail in the case of skew hermitian forms. It was unknown whether the Principle fails if weakened by replacing equivalence by similarity (when the form is of odd dimension). Lewis, Unger, and van Geel prove this also fails. In fact they show that there exist locally equivalent forms which are not globally similar. The proof involves defining an invariant (equivalent to Bartle's invariant) that can detect such a counterexample.

Discussion afterwards indicated this yields a explicit computation for the Tate-Šafarevič group for the projective unitary group.

R. Parimala. The determination of conditions under which a quasi-projective variety having a zero cycle of degree one has a rational point is a problem of long standing. This is, of course, true for conics and elliptic curves. It is clear that certain restrictions should be assumed. The most reasonable varieties to consider, where a positive answer may occur, are homogeneous spaces of a connected linear algebraic group. The answer is known to be positive for torsors of some groups by work of E. Bayer and H.W. Lenstra Jr., M. Rost, V. Chernousov, and S. Garibaldi, and over number fields by the work of J. –J. Sansuc. Parimala discussed this problem and indicated a possible way to construct a counterexample for the non-projective case over a Laurent series field over a p-adic field.

This led to much discussion after the lecture as to whether this approach would produce such a counterexample. It was determined that it could not without modification.

A. Pfister. As many of the talks demonstrated, new and sophisticated methodologies are developing to attack problems in quadratic form theory. The question always arises of whether one can obtain some of the results by more elementary means. For example, Karpenko simplified some of Vishik's proofs and talked about this at the conference. One of the first results one proves in quadratic form theory is to determine when an element is a norm from a quadratic extension. Although the usual proof of this is not deep, Pfister presented a proof that is completely elementary, and independent of the Brauer group or cohomology. The motivation for doing this is to enable Merkurjev's theorem in Milnor K-theory to be presented in an introductory course in quadratic forms.

Z. Reichstein. Reichstein lectured on his joint work with N. Lemire and V. Popov on Cayley groups. The exponential map is a crucial tool in studying Lie groups, but suffers from the fact that it is not algebraic. One would like to find an algebraic analogue even in the case of characteristic zero. By analogy with the classical Cayley map for the orthogonal group, a natural candidate would be the following: Let G be an algebraic group over a field k. Does there exist a G-equivariant birational isomorphism $\mathrm{Lie}\, G \dashrightarrow G$? If such a birational map exists, call G a *Cayley* group. Suppose that k is algebraically closed of characteristic zero. Luna asked in 1980 which G are Cayley? For such a field k, Reichstein determines which simple groups are Cayley and also which are *stably Cayley*, i.e., $G \times (k^*)^n$ is Cayley for some n.

M. Rost. Morley's Theorem states that the (appropriately chosen) three points of intersection of trisectors of the angles of a triangle form an equilateral triangle. (cf. that the bisectors of angles of a triangle meet in a point.) All known proofs eventually rest on computation using the Euclidean metric, the most recent by A. Connes [6]. Rost talked on this theorem and Connes's proof. He also indicated how one could formulate it in group cohomology independent of any Euclidean structure.

D. Saltman. The notion of "trialitarian algebra" was introduced in [11]. The underlying structure is a central simple algebra with an orthogonal involution, of degree 8 over a cubic étale algebra. The trialitarian condition relates this algebra to its Clifford algebra. M.-A. Knus, R. Parimala and R. Sridharan constructed a generic trialitarian algebra, defined using the invariants of the group $T = PG0_8^+ \rtimes S_3$ where $PG0_8^+$ is a group of projective proper similitudes [12]. This theory is parallel to the theory of central simple algebras but instead of PGL_n, T is used. In his lecture, Saltman described the center of the generic trialitarian algebra as the field of multiplicative invariants of the Weyl group $((S_2)^3 \rtimes S_4) \rtimes S_3$. He also constructed analogues of Azumaya algebras and Brauer factor sets and described trialitarian algebras in terms of Brauer factor sets, in a way similar to the classical Brauer construction of central simple algebras. In so doing, he defined a new type of cocycle attached to a pair of groups $H \subset G$, which he named $G-H$ cocycles, and which are used to describe their associated Azumaya crossed products.

A. Schultz, J. Swallow. Let $K^\times$ be the multiplicative group of the field K. Already in 1947, I.R. Šafarevič in his influential paper on p-extensions [23] realized that the growth of $\dim_{\mathbb{F}_p} K^\times/K^{\times p}$ where K runs over the finite Galois p-extensions of F can yield important information about the Galois group $G_F(p)$ of the maximal p-extension of a field F. In fact Šafarevič was able to show that if F is a local field not containing a primitive p^{th} root of unity then $G_F(p)$ is a free pro-p-group using information about $\dim_{\mathbb{F}_p} K^\times/K^{\times p}$ as above.

In the 1960s D.K. Fadeev and Z.I. Borevič [5] succeeded in classifying possible $\mathrm{Gal}(K/F)$-modules $K^\times/K^{\times p}$ for cyclic extensions of local fields of degree p^n.

In recent work, Mináč, Schultz and Swallow classified all Galois $G(K/F)$-modules $K^\times/K^{\times p}$ for cyclic extensions K/F of degree p^n where char $F \neq p$. Their description relies upon arithmetical invariants associated with K/F.

This work and other related work have already been used by Mináč and Swallow for finding conditions for the solution of specific Galois embedding problems and providing explicit solutions when they exist. They also determined which arithmetic invariants attached to cyclic extensions K/F of degree p, which are used for the classification of a Galois module $K^\times/K^{\times p}$, are actually realizable for a suitable extension K/F.

These investigations are closely related to previous investigations of Galois modules attached to fields with Galois groups of exponent 2 by A. Adem, W. Gao, D. Karagueuzian and J. Mináč [1]. These results can also possibly be useful in determining the Galois $G(K/F)$-module $\mathrm{K}^*(K)/p\mathrm{K}^*(K)$ (where $\mathrm{K}^*(K)$ is the Milnor ring of the field K).

Conclusion

During the last decade the revolutionary methods of motivic homotopy theory have intervened in the algebraic theory of quadratic forms. Many long-standing conjectures have been solved, as evidenced by this conference. These new methods affirm that even in a subject as well worked-over as the algebraic theory of quadratic forms, significant progress on interesting problems, often in unexpected directions, is still possible, and provide convincing evidence of continuing progress in the future. Of course these methods are producing striking results in many other fields as well, and it may be that those in quadratic forms will, as they have often done in the past, foreshadow similar and analogous progress in fields such as algebraic groups and Galois cohomology.

Nevertheless many important open problems remain in the algebraic theory of quadratic forms—for example, description of the generic discrete invariant of a quadratic form—which will be attacked by means of motivic methods as well as by more traditional techniques.

The conference provided a marvellous venue for exchange of ideas and the establishment of collaboration. One of the participants told us that he had never before come away from a conference with so many new ideas for his research. The BIRS facilities were greatly appreciated by all of us, and we extend our sincere gratitude to the management and staff of the institute for enabling us to have an extraordinarily successful conference.

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