Mathematical and Conceptual Aspects of Quantum Theory

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1 Overview of the Field

Interest in the foundations of quantum theory has considerably increased in recent years, fueled by new research directions on the one hand and an increasing cross-fertilization and relevance to other fields on the other hand. Among new research directions are the operational axiomatic approaches, categorical and diagrammatic formalizations, generalized probabilistic theories, a drive into the relativistic realm and into quantum field theory among others. At the same time the boom in quantum information theory, due in part to the prospect of practical quantum computation, has impacted the neighboring field of foundations and vice versa. While the challenge of quantum gravity remains unresolved, it appears increasingly likely that progress at the foundations of quantum theory might be key. Conversely, contexts without a determined background metric (such as indefinite causal structures) are increasingly investigated in quantum foundations.

Along with the physical concepts in quantum foundations, also the mathematics used to express them has experienced a transformation. Mathematical axiomatizations are increasingly important; category theory, in particular monoidal categories and higher categories have entered the field; topological quantum field theory appears in various guises, functional analysis gains relevance with attention on systems with infinitely many degrees of freedom; and convex analysis is crucial in models generalizing both classical and quantum theory.

These developments have the potential to yield progress on problems that have long lacked conceptual breakthroughs, such as quantum gravity or a mathematically rigorous formulation of quantum field theory. However, sociologically speaking the research communities that could and should participate in these developments are fragmented. E.g. there is a quantum foundations community traditionally restricted to finite degrees of freedom and non-relativistic settings, only gradually opening to challenges from quantum field theory and quantum gravity. There is a quantum gravity community traditionally largely ignoring or postponing vital foundational questions, while at the same time divided into areas with little communication between each other (such as loops vs. strings). There is a quantum field theory community with different camps (such as Lagrangian/path integral vs. algebraic) and with little appreciation for foundations and a limited perspective on quantum gravity. And while the work of the community of quantum optics not only allows us to understand in detail the interaction between radiation fields and matter, and to take advantage of the phase transitions in such systems to design basic elements for quantum computing and information, it usually disregards certain relativistic phenomena and covariant formalisms. On the other hand there are relevant contributions from topological quantum field theory, monoidal category theory, higher categories, information theory that are only beginning to get integrated.

Mexico suffers particularly from fragmentation, with many fields such as foundations of quantum theory or mathematical approaches to quantum field theory severely underdeveloped. In these fields and many related ones there either exist only isolated researchers scattered at different places or perhaps there is a single dedicated group nationwide.

2 Scope of the workshop

Rather than focusing on progress in any one particular community of the foundations of quantum theory, quantum information theory, quantum gravity, mathematical quantum theory, etc. this workshop brought together leading and early career researchers from all these different communities. The workshop covered the broad subject area of mathematical and conceptual aspects of quantum theory with an emphasis on foundations. In this way it was possible to foster communications and cross-fertilization between the different areas and thus overcome the mentioned traditional barriers and limitations of perspective. That is, rather than consider the specialized in-depth development of any one research direction, the participants were encouraged to focus in the workshop on the actual or potential relations between these, both in their formal presentations and in informal discussions. A dedicated effort was also made to contribute to the formation of a Mexican research community throughout the workshop and at the same time connect it with the international research community.

Due to the pandemic, the workshop followed a hybrid format. 15 participants were physically present at CMO, while 29 participants connected remotely. This presented particular challenges. A key idea of the BIRS conferences has been to foster spontaneous interaction and discussion between participants by having them share a common space and common activities (including meals) during most of the duration of the conference. This has worked quite well before the pandemic and lead to many fruitful encounters and started new collaborations, as one of the organizers of this workshop can attest from previous experiences. It has also meant that formally organized discussion sessions were not a necessity in the workshops, even though there would often be such sessions included. In contrast, with the hybrid format remote participants would be excluded from spontaneous gatherings limiting the corresponding benefits. To partially compensate for this, three formal discussion sessions were included in the program of the present workshop. In this way the remote participants were able to participate at least in these discussion sessions, allowing them to interact both with physically present participants as well as with other remote participants. A particular problem originated from the diverse global locations of the remote participants and the resulting challenges to bridge the different time zones. In particular, 11 of the 29 remote participants where located in Europe, with a typical seven-hour temporal displacement. This meant that these participants were only allocated speaking slots in the first morning sessions and were not able to attend talks and discussion sessions in the later afternoon. In total there were 23 talks, each of a duration of 40 minuted plus question time. Of these, 8 talks were given in-person, while 15 talks were given remotely. Seven of the talks were given by participants based in Mexico (six in-person, one remotely).

3 Presentations and topics

The broadness of the scope of workshop topics and participating research communities means that a thematic presentation with in-depth treatment of each particular research direction in this report is not practical. Instead, The following exposition is structured by presentation. That is, for each talk, a small section providing a short summary of the main contribution is included. Where convenient, this summary is expanded with additional comments on the particular research topic and/or its relation to others presented.

3.1 Day 1

3.1.1 G. Mena Marugan: "Quantum unitary dynamics in nonstationary spacetimes"

Quantum field theory in curved spacetime is relevant to understanding the physics of quantum fields in astronomical and cosmological contexts. Moreover, it is an important stepping stone on the way from ordinary quantum field theory in Minkowski space towards a theory of quantum gravity. While the subject started in the 1960s and has considerable maturity, there are still many unresolved questions. In particular, on the subject of quantization, i.e., obtaining from a classical field theory a corresponding quantum field theory, it is still only partially understood under what circumstances unitarity of the quantum theory can be guaranteed. The speaker addresses the question of unitary implementation of the dynamics for scalar fields in nonstationary spacetimes describing cosmological scenarios [1, 2]. Together with invariance under spatial isometries, the requirement of a unitary evolution singles out a rescaling of the scalar field and a unitary equivalence class of Fock representations for the associated canonical commutation relations. Moreover, this criterion also provides a privileged quantization for the unscaled field, even though the associated dynamics is not unitarily implementable in that case. The relation between the initial data that determine the Fock representations in the rescaled and unscaled descriptions and the relation with adiabatic states is also addressed.

3.1.2 K. Życzkowski: "Thirty six entangled officers of Euler: quantum solution of a classically impossible problem"

In this refreshing talk, the speaker revisited Euler's impossible puzzle, asking how to arrange thirty-six officers, of six different ranks and from six different regiments, in a six by six square, so that no two ranks or regiments appear in the same row or column. Already since the early 1900s it is known, thanks to the work of Tarry, that this particular "magic square" is impossible, although it has since been shown that a solution exists for any other dimension (*i.e.*, changing the number six above to any other number) greater than two. The renewed efforts allowed for quantum states for the officers, including superpositions of the above definite-rank and definite-regiment classical ones, replacing the distinctness of the thirty-six uniforms with orthogonality of the states in the Hilbert space, and the "no repeat" requirement with suitable partial trace conditions. In a recent paper [3, 4], coauthored, apart from the speaker, by a collaboration of Polish and Indian physicists, a solution to the quantum version of the puzzle was found explicitly, with the help of computers, involving entangled states. Apart from the intrinsic beauty of the result, there are also possible applications to quantum error correction.

3.1.3 C. Anastopoulos: "Information in quantum field theory: the challenge of measurements"

Quantum information theory (QIT) has made its appearance in several QFT contexts, ranging from the black hole information paradox to holography. But, traditionally, QIT has been developed in a non-relativistic setting, *e.g.*, focusing exclusively on spacelike correlations, while a fully relativistic theory ought to treat timelike correlations on the same footing. The speaker argued for the need to develop a consistent operational theory of QFT measurements, that would permit extraction of quantum information in a relativistic setting. A summary of challenges associated to quantum measurements in QFT was presented, followed by the introduction of the *quantum temporal probabilities* method, based on QFT correlation functions [5, 6]. A number of applications, to photo-detection, black hole information, and non-equilibrium QFTs were also developed.

3.1.4 A. Klimov: "Global view on quantum properties of many-body systems"

Visualizing quantum states is a powerful way to get intuition about their properties. The speaker started his talk by pinpointing several problems standard visualizations have in the case of large-N multipartite systems. He then proceeded to propose a new visualization that only reflects information about collective observables, the latter defined as invariant under arbitrary particle permutations, and which is therefore useful in the analysis of macroscopic features of the state. Underlying the new visualization is a discrete distribution in a 3D space of symmetric measurements, the analytical properties of which permit a characterization of quantum correlations in the large N limit [7].

3.1.5 J. Orendain: "Equivariant functorial quantum field theory"

Topological Quantum Field Theory (TQFT) emerged at the end of the 1980s as a categorical and axiomatic framework inspired by the structural properties of quantum field theory [8]. It played a big part in the revolution happening at that time in algebraic topology, knot theory and low dimensional topology. Unfortunately, initial hopes that it might also serve as a rigorous and axiomatic foundation for quantum field theory have not been borne out. Learning from the failure with a modified set of axioms a new attempt was initiated by R. Oeckl with General Boundary Quantum Field Theory (GBQFT) [9]. While GBQFT has seen considerable development [10, 11], it has lacked the elegant and powerful categorical formulation of TQFT. The present talk was concerned with a presentation of GBQFT with a view towards recent work of the speaker with R. Oeckl on Equivariant Functorial Quantum Field Theory (EFQFT) as a categorification of GBQFT.

3.2 Day 2

3.2.1 F. Finster: "An introduction to causal fermion systems and the causal action principle"

The theory of causal fermion systems is an alternative approach to describe fundamental physics [12, 13]. It gives quantum mechanics, general relativity and quantum field theory as limiting cases and is therefore a candidate for a unified physical theory. Moreover, causal fermion systems provide a general framework for modelling and analyzing non-smooth spacetime structures. The dynamics of a causal fermion system is described by a nonlinear variational principle, the causal action principle. This talk provided a simple introduction, with an emphasis on the underlying concepts. The connection to quantum field theory was also outlined.

3.2.2 A. Ibort: "Schwinger picture of quantum mechanics: groupoids"

The speaker presented a new picture of Quantum Mechanics, based on the theory of groupoids, which provides the mathematical background for Schwinger's algebra of selective measurements and helps to understand its scope and eventual applications. The kinematical background was described using elementary notions from category theory, in particular the notion of 2-groupoids as well as their representations. Some basic results were presented, and the relation with the standard Dirac-Schrödinger and Born-Jordan-Heisenberg pictures was succinctly discussed [14].

3.2.3 O. Oreshkov: "Quantum processes on time-delocalized systems"

It has been shown that it is theoretically possible for there to exist quantum and classical processes in which the operations performed by separate parties do not occur in a well-defined causal order [15]. A central question is whether and how such processes can be realized in practice. In order to provide a rigorous argument for the notion that certain such processes have a realization in standard quantum theory, the concept of time-delocalized quantum subsystem has been introduced. In this talk, the speaker discussed the concept of time-delocalized subsystem and its relevance to the question of realizability of processes with indefinite causal order in standard quantum theory and quantum gravity [16]. He explained how, given a description of an experiment in the form of a (generally cyclic) circuit, the experiment can be described with respect to an arbitrary alternative choice of (sub)systems, which is obtained by a transformation akin to a spatio-temporal change of basis. This provides a simple and very general notion of transformation between different equivalent descriptions of an experiment. He showed how the quantum SWITCH [17] can be seen as realizable on time-delocalized systems in standard quantum mechanics and that all unitarily extendible tripartite processes with indefinite causal order admit such realizations [18]. Remarkably, this includes processes violating causal inequalities, whose physical realizability has been a central open problem. The speaker discussed the meaning of causal inequality violation in this setting and argued that it is a meaningful concept to show the absence of a definite causal order between the variables of interest. He closed with some speculation on the link between time-delocalized systems and quantum reference frames.

3.2.4 R. Sorkin: Spacelike correlations do not imply superluminal causation (so what are they telling us?)"

In the first part of the talk, titled "Nonlocality rescues causality", the speaker argued that even if superluminal causation is excluded, arbitrary spacelike correlations are still thinkable, including maximal violations of the CHSH inequality (to put the audience at ease, the speaker quoted his collaborator as agreeing that the claim "clearly is not obviously false"). The speaker further claimed that such correlations say nothing about faster-than-light influences. Certain overused terms where clearly defined: *causality*, to be identified with relativistic causality, means that no cause can act outside its future lightcone, while *locality* dictates that no cause can act outside its immediate spacetime neighborhood, the two concepts being logically distinct so that, *e.g.*, tachyons violate the former but not the latter. In the second part of the talk, titled "Does some analogue of "factorization" hold in nature?", the speaker touched upon the concepts of *factorization* (a statement of stochastic independence that makes sense classically) and *persistence of zero* (roughly speaking, the requirement that if an event A has zero probability of occurring, every other "composite" event A *and* B, with B

not in A's past, also has zero probability). A final conclusion stated in the abstract but for which time was insufficient during the talk was that quantum theories are *causal* but *non-local*.

3.2.5 J. A. Zapata: "Parametrized field theory and gluing"

The speaker presented recent work where he pairs the definition of parametrized field theory with the concept of a cubical omega groupoid [19]. He started with a brief introduction to cubical omega groupoids and their role in homotopy. This new language allows to state well known features of field theory simply, gives a slightly different interpretation to boundary conditions and gluing, suggests extensions of field theory, and brings a homotopical point of view to discretization and coarse graining. The speaker was also proposing to use this as a new ingredient in GBQFT and its categorification, see the talk 3.1.5 by J. Orendain.

3.2.6 E. Martin-Martinez: "The geometry of spacetime from quantum measurements"

Citing earlier work by A. Kempf [20], the speaker showed how the vacuum fluctuations of a quantum field have information about the background spacetime geometry, permitting recovery of the latter from the Feynman propagator. Consequently, he argued, the geometry of spacetime can be recovered from local measurements of quantum particle detectors coupled to the field. First the field's correlation function is recovered from detector measurements, and then the invariant spacetime interval (and, hence, the metric) is deduced from the correlation function. The suggestion that emerges is that particle detectors are the appropriate operational substitutes for the classical rulers and clocks of general relativity [21].

3.3 Day 3

3.3.1 J. Martin: "Anticoherent spin states: from their properties to their preparation"

The last twenty years or so have witnessed a surge in the interest in the space of quantum states, its geometrical structures, and various distinguished elements, usually characterized as extremal under suitably defined criteria. Thus, in the space of spin-s states, one may look for *coherent states*, defined as those that maximize the modulus of the spin expectation value, and generally considered as the most "classical" spin states. In 2006, Zimba proposed to look in the opposite direction, searching for states with vanishing spin expectation value, which he termed *anticoherent* [22] — these, in many ways, are the most "quantum" spin states available, and have been shown to provide the solution to a host of extremal problems. Their study becomes particularly intriguing when Majorana's stellar representation of spin states is employed, in which a "constellation" of $2s$ points on the unit sphere is uniquely associated to each spin-s state. Coherent states correspond to completely degenerate constellations, where all $2s$ points coincide, while anticoherent states tend to have their "stars" uniformly distributed over the sphere.

The speaker presented a review of several years worth of exploration of anticoherent spin states. He first summarized some of their properties, notably that, when considered as symmetrized states of $2s$ spin-1/2 subsystems, they are maximally entangled with respect to appropriate bipartitions, and also that they lack low-lying components in their multipolar expansion. A significant contribution to their study has been the introduction of *anticoherence measures*, which are postulated to satisfy a series of properties, and which permit the quantification of anticoherence in a continuous manner. An important development has been the realization that, in almost every SLOCC class, there is a unique (up to local unitaries) anticoherent state, so the latter can serve as representative of the class [23]. Apart from their relevance in theoretical considerations, anticoherent states have been shown to be optimal in rotation detection [24, 25, 26] and also exhibit superdecoherence [27]. The final part of the talk dealt with a method for producing anticoherent states with a control hamiltonian, involving rotations and squeezing in a fixed direction, with a couple of fascinating animations showing how the procedure gives rise to platonic anticoherent states.

3.3.2 D. Braun: "Stochastic emulation of quantum algorithms"

Searching for a substitute for the state vector of a quantum system that would facilitate simulations, the speaker introduced higher-order partial derivatives of a probability distribution of particle positions, arguing that they share basic properties with the state vector so that, discretizing the position space of n qubits allows the description of their quantum state by $2(n+1)$ classical stochastic bits. A universal gate set was subsequently mapped to a set of stochastic maps, allowing the translation of quantum algorithms to classical stochastic ones. Several examples of well-known quantum algorithms were presented, along with an analysis of the way the number of realizations scales with the number of qubits.

3.3.3 E. Nahmad-Achar: "Finite matter-radiation systems"

The singular role of radiation-matter systems in present day application-oriented physics was the starting point of the speaker, who went on to introduce phase diagrams for n-level atoms interacting dipolarly with a multimode radiation field in a cavity. He then showed that the superradiant region of the parameter space divides itself into monochromatic regions, where only one mode of the electromagnetic field dominates. Recursive application of a reduction scheme maps the n-level problem to a collection of 2-level Dicke atoms. An additional simplification occurs by truncating the infinite-dimensional Hilbert space of the problem, while fidelity-based criteria, put in place to monitor the accuracy of the approximations, show an excellent agreement with numerically obtained solutions.

3.3.4 A. Kempf: "Information theory *vs.* quantum gravity"

Information theory is agnostic about the subject matter of the information that it studies, and it is, therefore, by its nature very versatile. It is thus unsurprising that information theory provides useful tools throughout engineering and physics including even quantum gravity. But the fact that information theory applies equally to all subject matter may also indicate that information theory may be more than versatile, namely universal. The speaker suggests, that information theory could be universal, with quantum gravity emerging from it [28]. The talk consisted first in a brief discussion of tools that information theory can provide to quantum gravity. Secondly, the speaker addressed the question of how spacetime, matter and their dynamics could be emergent from information theory. An important role in this plays the relation between quantum field theoretic correlators and spacetime geometry, a topic that was treated in depth by Eduardo Martin-Martinez in his talk 3.2.6.

3.4 Day 4

3.4.1 D. Oriti: "The universe as a quantum many-body system, cosmology as its hydrodynamics"

In the Group Field Theory (GFT) formulation of quantum gravity [29], which is based on Loop Quantum Gravity, the universe is described as a quantum many-body system with basic entities being quantum simplices, glued to form extended structures by entanglement. Quantum gravity states are then generalized tensor networks, and exhibit a discrete entanglement/geometry correspondence. The emergent cosmological dynamics for the same system takes the form of condensate hydrodynamic equations on superspace, thus a non-linear extension of quantum cosmology [30]. This prompts the exploration of general maps between the hydrodynamics of quantum fluids and cosmology, which had in fact appeared independently in the mathematical physics literature, further corroborated by the discovery of hidden symmetries in cosmological dynamics, which match those of condensate hydrodynamics. A key ingredient is the relational understanding of space and time, which makes superspace the natural arena for gravitational dynamics, as opposed to the "spacetime" manifold. These results, and the perspective they suggest, have also potential implications for analogue gravity systems in the lab.

3.4.2 M. Reisenberger: "Insights from trying to teach an honest quantum mechanics course: problems with the textbook postulates and the solutions to almost all of them within standard QM"

The speaker started with the claim that the standard textbook postulates of quantum mechanics (based mostly on von Neumann's original treatment [31]) are baffling, ambiguous, and in some ways plain wrong. But solutions to these problems are known and for the most part are incorporated in standard quantum mechanics as it is practiced. The talk provided a review of the most glaring defects of the textbook postulates, and the solutions to almost all of them, leaving as open only a small number of genuinely unresolved and/or controversial issues.

3.4.3 L. Hardy: "Causality constraints in quantum field theory from an operational perspective"

Consider an operational formulation of Quantum Field Theory in which operators are associated with arbitrary regions of spacetime (such as the Positive Formalism [32], see the following Talk 3.4.4 by R. Oeckl). If we jiggle some input to one part of the boundary of this region, we want to be sure no information is transmitted to a second part of the boundary if this second part lies outside of the forward light cone of the first part. This leads to a constraint on the allowed operators that can be associated with any given arbitrary region of spacetime. What are these constraints? The speaker summarized an approach to this building on the work discussed in [33] (see Part III and the appendix). This approach takes the iterative conditions of Chribella, D'Ariano, and Perinotti [34] imposed on the operators associated with quantum combs as the basis for causality conditions associated with operators associated with arbitrary regions of spacetime.

3.4.4 R. Oeckl: "The positive formalism"

The Positive formalism (PF) is an operational framework for physical theories that subsumes a range of frameworks for classical and quantum theories, including the standard formulation of quantum theory [32]. It grew out of the General Boundary Formulation (GBF) and the axiomatic system of General Boundary Quantum Field Theory (GBQFT) [9], see the talk 3.1.5 by J. Orendain. This talk provided an overview of the PF, emphasizing that it contains in particular a timeless formulation of quantum theory, such as required for constructing a quantum theory of gravity. An important focus was the causality condition of the standard formulation of quantum theory and how it must be relaxed when no fixed time background is available. This was illustrated with the example of the black hole bounce time [35]. The speaker also pointed out, that a different solution to the problem addressed by L. Hardy in the previous talk 3.4.3 was proposed in a talk at a conference at the Perimeter Institute in 2018 [36].

3.4.5 A. Corichi: "Geometry of quantum theory and squeezed states: an application to QFT"

The talk started with a brief review of the relevance of geometric considerations in the formulation of quantum mechanics, in particular the natural metric, symplectic and derived complex structure of the quantum state space. It was then pointed out that the starting point for the construction could be taken to be the classical phase space, where one may freely choose a metric, giving rise to a vacuum state. In the example of the simple harmonic oscillator, choosing a simple euclidean metric with respect to rescaled axes gives rise to squeezed states. This point of view is adopted while dealing with inflationary cosmology, with the field and the dual momentum modes replacing position and momentum. As a result of squeezing, the field freezes out, while the fluctuations of the dual momentum grow unboundedly [37]. The speaker further pointed out that the standard observable quantities in cosmology only involve the quantum field itself, not its dual momentum, so the extreme reduction in its uncertainty furnished by the squeezing essentially render it classical.

3.5 Day 5

3.5.1 C. Brukner: "Falling through masses in superposition: quantum reference frames for indefinite metrics"

A major weakness of the current physical theories available for dealing with gravitational phenomena in a quantum context, *i.e.*, quantum mechanics and general relativity, is that they fail to provide a convincing description of the gravitational field of a mass in a quantum superposition of position eigenstates. To deal with this intriguing problem, the speaker proposed the idea of a quantum reference frame transformation, which, intuitively, extends the familiar Lie group element concept to the case where the parameter of the corresponding transformation is an operator, rather than a simple number. Thus, applying a (quantum) translation to a localized state, one may end up with a bilocalized one — importantly, the inverse transformation is also possible. The speaker argued that one can then transform the original problem mentioned above, in which it seems unavoidable that the resulting metric will be in a superposition of states (*indefinite*, in the speaker's words), to one where the metric is localized in (Wheeler's) superspace [38] (termed *definite* by the speaker), in which case standard techniques provide a satisfactory description.

3.5.2 E. Serrano-Ensástiga: "Quantum rotosensors of multiqudit systems"

The talk explored the metrological problem of measuring rotations of multipartite quantum states. The case of symmetric spin-1/2 multipartite states has been previously analyzed, relying heavily on their stellar representation (see talk 3.3.1). But when the spin-1/2 factor states are replaced by general qudits, Majorana's recipe is no longer applicable, and an appropriate generalization is called for. The speaker presented such a generalization for symmetric multiqudit states [39], and then used it to identify optimal rotosensors for a variety of optimization criteria, involving the quantum Fischer information, and the quantum Cramer-Rao bound. In the case of rotations about a given axis, generalizations of the GHZ states are shown to be optimal rotosensors. The case of averaging over all possible rotation axes was also considered, and the solution was found to involve what are known as 2-anticoherent states [22]. Similar techniques were employed to the case of antisymmetric multipartite states (*e.g.* Slater determinants) [40], which are ubiquitous in molecular quantum mechanics, the study of Grassmanians, *etc*.

3.5.3 O. Müller: "No-go theorems and loopholes for functors between physically relevant categories"

In this talk, three no-go theorems were explored for the existence of functors between categories that are important for fundamental physics. The first concerns spinors: It is shown how spinors cannot and how they can be defined on metric-independent bundles [41]. The second concerns natural metrics on categories of Lorentzian metrics [42]. The third and most central one concerns quantization relations (relations between classical and quantum observables) [43]. It was shown, in a vast generalization of the Groenewold-van Hove theorem, how minimal assumptions (quantum objects as operators on a Hilbert space and the von Neumann rule) already exclude linearity of any quantization relation. Also, an experiment was suggested to test the degree of linearity of the quantization relation.

4 Scientific Progress Made and Outcome

What we identify as the main scientific goal achieved in this workshop is the amalgamation of a plethora of approaches to aspects of quantum theory, in search of breakthroughs in long-standing roadblocks, most notably quantum gravity and foundational questions in quantum mechanics. Experts from the communities of quantum gravity, quantum information theory, mathematical quantum theory, *etc.* where brought together under ideal conditions to interact and see the problems they pursue through a different set of eyes. In broad geographical terms, the workshop bridged North America with Europe. On a finer, local scale, it brought together groups from Mexico City, Morelia and Guadalajara. Other types of bridges were also extended: senior and mid-career scientists where joined by postdocs and even doctoral students — for the latter, due to the pandemic, this was the first international event they ever attended. The pressing need to maintain such interactions suggests the adoption of a regular series of similar encounters, a prospect we are currently exploring.

The hybrid format imposed by the pandemic was seen to present both advantages and limitations: on the one hand, invitations could be blissfully extended to all latitudes and longitudes, without any regard to travel logistics, and people who for practical reasons (*e.g.*, teaching obligations) would not be able to attend, managed to do so, enriching the experience for all attendees. On the other hand, much of the interaction among in-person attendees simply cannot be extended via zoom, even the finer dynamics of a live discussion are heavily distorted when experienced through cameras and microphones. In that regard, the technical support at all times from dedicated personnel, as well as the quality of the audiovisual equipment used were crucial to eliminating distractions and allowed focusing on the content of the talks. An obvious improvement would be the acquisition of at least a second (and, ideally, a third) wireless microphone, to be used during discussions. Also, the number of the speakers in the lecture hall could be doubled, and their placement improved. We consider it quite probable, and certainly desirable, that some degree of hybridism remains after the end of the pandemic, although the ideal proportions of in-person *vs* remote participants would be different from the ones experienced during the workshop.

COVID protocols were followed strictly, luckily without any particular effort on the part of the organizers — even so, a number of infections, which, fortunately, all proved mild, were reported the week following the event. The organizers kept all attendees, as well as CMO, informed of these cases via email. It would probably be a good idea to include an explicit set of instructions on how to deal with such cases in the booklet supplied to the organizers, in particular, clarify who informs whom, how, and when, including details such as whether the names of the infected persons should be communicated.

In summary, despite organizational difficulties related to the pandemic, the workshop succeeded in fostering collaborations within a hitherto fragmented community, and hopefully will mark the beginning of a more integrated approach to the major conceptual conundrums mentioned above. Like all other workshops held in these singular times, the event will also contribute in shaping the format of future events, long after the pandemic is over, which will probably bear the marks of this particular period, in the form of a yet-to-be-determined degree of hybridism.

References

[1] J. Cortez, G. A. Mena Marugán, J. M. Velhinho, *Quantum unitary dynamics in cosmological spacetimes*, Annals of Physics **363** (2015) 36–47.

[2] B. E. Navascués, G. A. M. Marugán, T. Thiemann, *Hamiltonian diagonalization in hybrid quantum cosmology*, Class. Quantum Grav. **36** (2019) 185010.

[3] D. Garisto, *Euler's 243-year-old impossible puzzle gets a quantum solution*, Quanta Magazine (2022).

[4] S. A. Rather, et al., *Thirty-six entangled officers of Euler*, Phys. Rev. Lett. **128** (2022) 080507.

[5] C. Anastopoulos, N. Savvidou, *Coherences of accelerated detectors and the local character of the Unruh effect*, J. Math. Phys. **53** (2012) 012107.

[6] C. Anastopoulos, N. Savvidou, *Time of arrival probabilities for general particle detectors*, Phys. Rev. A **86** (2012) 012111.

[7] A. Klimov, C. Muñoz, *Macroscopic features of quantum fluctuations in large N qubit system*, Phys. Rev. A **89** (2014) 052130.

[8] M. Atiyah, *Topological quantum field theories*, Inst. Hautes Études Sci. Publ. Math. **68** (1988) 175–186.

[9] R. Oeckl, *General boundary quantum field theory: Foundations and probability interpretation*, Adv. Theor. Math. Phys. **12** (2008) 319–352, `hep-th/0509122`.

[10] R. Oeckl, *Holomorphic Quantization of Linear Field Theory in the General Boundary Formulation*, SIGMA **8** (2012) 050, 31 pages, `1009.5615`.

[11] R. Oeckl, *Free Fermi and Bose Fields in TQFT and GBF*, SIGMA **9** (2013) 028, 46 pages, `1208.5038v2`.

[12] F. Finster, *The Continuum Limit of Causal Fermion Systems*, *Fundamental Theories of Physics*, vol. 186, Springer, Cham, 2016.

[13] F. Finster, N. Kamran, *Spinors on Singular Spaces and the Topology of Causal Fermion Systems*, *Memoirs of the American Mathematical Society*, vol. 259, American Mathematical Society, 2019.

[14] F. M. Ciaglia, A. Ibort, G. Marmo, *Schwinger's picture of quantum mechanics I: groupoids*, Int. J. Geom. Meth. Mod. Phys. **16** (2019), no. 08.

[15] L. Hardy, *Probability Theories with Dynamic Causal Structure: A New Framework for Quantum Gravity*, 2005, `gr-qc/0509120`.

[16] O. Oreshkov, F. Costa, Č. Brukner, *Quantum correlations with no causal order*, Nature Communications **3** (2012) 1092, `1105.4464`.

[17] G. Chiribella, G. M. D'Ariano, P. Perinotti, B. Valiron, *Quantum computations without definite causal structure*, Phys. Rev. A **88** (2013) 022318.

[18] J. Wechs, C. Branciard, O. Oreshkov, *Existence of processes violating causal inequalities on time-delocalised subsystems*, 2022, 2201.11832.

[19] R. Brown, P. J. Higgins, *On the algebra of cubes*, Journal of Pure and Applied Algebra **21** (1981) 233–260.

[20] A. Kempf, *Replacing the notion of spacetime distance by the notion of correlation*, Front. Phys. **9** (2021) 655857.

[21] T. Rick Perche, E. Martín-Martinez, *The geometry of spacetime from quantum measurements*, Phys. Rev. D **105** (2022) 066011.

[22] J. Zimba, *Anticoherent Spin States via the Majorana Representation*, Electronic Journal of Theoretical Physics **3** (2006), no. 10 143–156.

[23] F. Verstraete, J. Dehaene, B. De Moor, *Normal forms and entanglement measures for multipartite quantum states*, Phys. Rev. A **68** (2003) 012103.

[24] C. Chryssomalakos, H. Hernández-Coronado, *Optimal quantum Rotosensors*, Phys. Rev. A **95** (2017) 052125.

[25] J. Martin, S. Weigert, O. Giraud, *Optimal Detection of Rotations about Unknown Axes by Coherent and Anticoherent States*, Quantum **4** (2020) 285.

[26] A. Z. Goldberg, A. B. Klimov, G. Leuchs, L. L. Sánchez-Soto, *Rotation sensing at the ultimate limit*, J. Phys. Photonics **3** (2021) 022008.

[27] J. Denis, J. Martin, *Extreme depolarization for any spin*, Phys. Rev. Res. **4** (2022) 013178.

[28] A. Kempf, *Replacing the Notion of Spacetime Distance by the Notion of Correlation*, Frontiers in Physics **9** (2021).

[29] D. Oriti, *The group field theory approach to Quantum Gravity*, Approaches to Quantum Gravity: Toward a New Understanding of Space, Time and Matter, (ed. D. Oriti), Cambridge University Press, 2009, pp. 310–331.

[30] L. Marchetti, D. Oriti, *Effective dynamics of scalar cosmological perturbations from quantum gravity*, J. Cosmol. Astropart. Phys. **2022** (2022) 004.

[31] J. von Neumann, *Mathematische Grundlagen der Quantenmechanik*, Springer, Berlin, 1932.

[32] R. Oeckl, *A local and operational framework for the foundations of physics*, Adv. Theor. Math. Phys. **23** (2019) 437–592, 1610.09052v3.

[33] L. Hardy, *The Construction Interpretation: Conceptual Roads to Quantum Gravity*, 2018, 1807.10980.

[34] G. Chiribella, G. M. D'Ariano, P. Perinotti, *Theoretical framework for quantum networks*, Phys. Rev. A **80** (2009) 022339.

[35] R. Oeckl, *A predictive framework for quantum gravity and black hole to white hole transition*, Phys. Lett. A **382** (2018) 2622–2625, 1804.02428.

[36] R. Oeckl, *Local quantum operations and causality*, Talk at Perimeter Institute, Waterloo, Canada, 30 July 2018, URL https://pirsa.org/18070050.

[37] A. Ashtekar, A. Corichi, A. Kesavan, *Emergence of classical behavior in the early universe*, Phys. Rev. D **102** (2020) 023512.

[38] A.-C. de la Hamette, V. Kabel, E. Castro-Ruiz, u. Brukner, *Falling through masses in superposition: quantum reference frames for indefinite metrics*, 2021, 2112.11473v1.

[39] C. Chryssomalakos, et al., *Symmetric multiqudit states: Stars, entanglement, and rotosensors*, Phys. Rev. A **104** (2021) 012407, `arXiv:2103.02786`.

[40] C. Chryssomalakos, E. Guzmán-González, L. Hanotel, E. Serrano-Ensástiga, *Stellar representation of multipartite antisymmetric states*, Commun. Math. Phys. **381** (2021) 735–764, `arXiv:1909.02592`.

[41] O. Müller, N. Nowaczyk, *A universal spinor bundle and the Einstein–Dirac–Maxwell equation as a variational theory*, Lett. Math. Phys **107** (2017) 933–961.

[42] O. Müller, *Lorentzian Gromov-Hausdorff theory and finiteness results*, 2019, `1912.00988`.

[43] O. Müller, *On the von Neumann rule in quantization*, J. Math. Phys. **60** (2019) 082104.