Kinetic equations: recent developments and novel applications

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1 Overview of the Field

Kinetic theory has become a powerful mathematical tool to describe the dynamics of many interacting particle systems. Starting with Boltzmann's seminal idea of replacing the large number of particles by their statistical distribution in phase space, kinetic theory quickly spread into different fields such as plasma physics and semiconductors. In the last years, it made its way into applications in the life and social sciences, for example to model the dynamics of animal flocks or the trading behavior in large economic markets. The complex nature of all these problems results in numerous challenging questions in mathematical research - ranging from mathematical modeling to analysis and numerical simulations. The proposed workshop aimed at bringing together experts working in different fields, presenting recent developments and promoting the emergence of innovative research ideas.

2 Recent Developments and Open Problems

Kinetic equations have been used in different fields of applied mathematics to describe the dynamics of large interacting particle systems. Classical applications include the dynamics of dilute gases or the transport of charged particles in plasmas as well as semiconductor crystals. These classic kinetic models, such as the Boltzmann equation or the Vlasov-Poisson equation, have been studied for many decades now, however many analytic and numerical questions are still open. Within the last years, kinetic models become a popular tool to describe the dynamics of a large number of individuals - for example biological cells, herding animals, pedestrians and even traders in large financial markets. These models have an even more complex nature, due to their coupling to other PDEs or their formulation on discrete structures, such as graphs. While a significant progress has been made over the last years, there remain many open and challenging problems at all levels of the modeling cycle. Examples include the derivation and analysis of kinetic equations, the qualitative behavior of solutions as well as the development of efficient numerical schemes.
Research on the applied aspects of kinetic theory becomes more and more popular in recent years, topics include:

- New applications of kinetic models in the life and social sciences, with a particular focus on special challenges, such as the non-conservative nature of these problems, the formulation of models on discrete structures or the connection to available data.

- Derivation of kinetic models from many body systems and their connections to hydrodynamics and aggregation-diffusion equations by asymptotic limits.
- Numerical methods for collisional kinetic equations of the Boltzmann type, and collisionless kinetic equations of the Vlasov type.

Several questions around these topics are important for the community. In the following we specify a number of examples:

- Interactions among individuals in the life and social sciences are in general not clearly defined, and highly nonlinear. This as well as the multiscale nature of the problems often lead to the formation of complex macroscopic patterns. Kinetic theory has been used successfully to establish mathematical connections between the individual behavior and macroscopic phenomena. However many questions in mathematical and numerical analysis about the asymptotic and long time behavior of solutions, which correspond to aggregation and segregation, are still open. Hence understanding the large time behavior of these systems as well as developing numerical schemes which capture the asymptotic and multiscale behavior is of great importance.
- The development of numerical methods for collisional kinetic equations, such as the Boltzmann and Landau equations, as well as collisionless equations, such as the Vlasov-Poisson equation, is one of the main discussion points for this workshop. These equations often contain high-dimensional, nonlinear, nonlocal collision operators that are difficult to approximate. Furthermore, the multiscale nature of these equations presents essential difficulites in designing efficient time and spatial discretization methods. Moreoever, some structures of the equations, such as the asymototic limit, conservation of mass/energy, positivity, entropy-decay, etc. are also important in numerical approximations. Ideas borrowed from the understanding of the complex particle systems and the derivation of kinetic models may also be helpful.
- In socio-economic applications the individual dynamics are often determined by game theoretical and optimal control approaches. This coupling leads to new classes of problems, such as for example Boltzmann mean-field games, which have a highly nonlinear structure due to their forward-backward in time coupling. Another challenge are social networks, which strongly influence the interactions among individuals. Again the highly nonlinear coupling as well as the formulation on discrete structures pose significant challenges for the analysis and simulations.
- The rigorous derivation of kinetic models is one of the most classical areas of research in kinetic theory. There is a renovated interest in this direction in view of the new applications mentioned above. Moreover, the interplay of different scales at the level of numerical methods might lead to new approximation algorithms.
- Individual interactions in socio-economic and biological applications are often not based on fundamental principles but rather empirical forces, which involve stochastic components. Here methods developed in the field of uncertainty quantification and multiscale kinetic problems can be particularly useful to identify the main driving forces and describe the system consistently.

3 Presentation Highlights

The presentations gave an overview on new developments in the classic theory of kinetic equations as well as novel applications in the life and social sciences. The workshop brought together experts from different fields related to kinetic theory including numerical methods, applied analysis, and modeling. Their broad spectrum provided a unique environment to exchange ideas and make substantial progress in developing mathematical theories for different applications.
In the following we give an overview of the workshop presentations; classifying them into numerical methods, applied analysis, and modeling. However, several talks covered all three aspects.

3.1 Numerical Methods

There were 9 talks focusing on numerical methods for kinetic models - discussing classical challenges related to physics as well as novel models which include uncertainty. Important aspects in the construction of computations schemes for kinetic equation include their potential multiscale nature, their low rank structure, hypocoercivity as well as the preservation of physical properties, such as positivity. Other very recent and promising developments are based on consensus dynamics in stochastic particle system, which have applications in high-dimensional optimisation problems.

Qi Wang: An Optimal Mass Transport Method for Random Genetic Drift. In population genetics, genetic drift describes random fluctuations in the numbers of gene variants (alleles) over time. Mathematical modeling of genetic drift such as the Wright– Fisher model employs a discrete stochastic process to model dynamics of finite populations at the individual level such that each copy of the gene of the new generation is selected independently and randomly from the whole gene pool of the previous generation. In the limit of a large population, these processes can be approximated by the diffusion that describes the probability of fixation of a mutant with frequency-independent fitness. While the continuum framework makes a systematic qualitative and quantitative analysis of the discrete model possible thanks to tools from modern analysis, it also inherits degenerate diffusion from the discrete stochastic process that conveys to the blow-up into Dirac-delta singularities hence bringing great challenges to the analytical and numerical studies. In this talk, we describe and analyze an optimal mass transport method for a random genetic drift problem driven by a Moran process formulated as a degenerate reaction- advection-diffusion equation. The proposed numerical method can quantitatively capture to the fullest possible extent the development of Dirac-delta singularities for genetic segregation on one hand, and preserves several sets of biologically relevant and computationally favored properties of the random genetic drift on the other. Moreover, the numerical scheme exponentially converges to the unique numerical stationary state in time at a rate independent of the mesh size up to a mesh error. Numerical evidence is given to illustrate and support these properties, and to demonstrate the spatiotemporal dynamics of random generic drift. This talk is based on a joint work with J. A. Carrillo and L. Chen.

Weiran Sun: Asymptotic preserving method for multiscale Levy-Fokker-Planck. In this talk, we present a recent result that shows an operator splitting scheme for the multiscale Levy-Fokker-Planck equation is asymptotic preserving (AP). The analysis is carried out by separating the parameter domain, which generalizes the traditional AP method when estimates are performed in a uniform way.

Mattia Zanella: Uncertainty quantification for kinetic equations of emergent phenomena. Kinetic equations play a leading role in the modelling of large systems of interacting particles/agents with a recognized effectiveness in describing real world phenomena ranging from plasma physics to multi-agent dynamics. The derivation of these models has often to deal with physical, or even social, forces that are deduced empirically and of which we have limited information. Hence, to produce realistic descriptions of the underlying systems it is of paramount importance to consider the effects of uncertain quantities as a structural feature in the modelling process.In this talk, we focus on a class of numerical methods that guarantee the preservation of main physical properties of kinetic models with uncertainties. In contrast to a direct application of classical uncertainty quantification methods typically leading to the loss of positivity of the numerical solution of the problem, we discuss the construction of novel particle Galerkin schemes that are capable of achieving high accuracy in the random space without losing nonnegativity of the solution . Applications of the developed methods are presented for the Boltzmann equation and for kinetic equations of plasmas with uncertainties.

Francis Filbet: On a discrete framework of hypocoercivity for kinetic equations. We study a class of spatial discretizations for the Vlasov-Fokker-Planck equation written as an hyperbolic system using Hermite polynomials.Then we propose a specific finite volume discretization for which we can prove hypocoercive estimates. This allows us to explore the long time behavior of the numerical solution and the asymptotitc limit (diffusive limit).

Weizhu Bao: Uniform error bounds on numerical methods for long-time dynamics of dispersive PDEs. In this talk, I report our recent work of error estimates on different numerical methods for the long- time dynamics of dispersive PDEs with small potential or weak nonlinearity, such as the Schroedinger equation with small potential, the nonlinear Schroedinger equation with weak nonlinearity, the nonlinear Klein-Gordon equation with weak nonlinearity, the Dirac equation with small electromagnetic potential, and the nonlinear

Dirac equation with weak nonlinearity, etc. By introducing a new technique of regularity compensation oscillatory (RCO), we can establish improved uniform error bounds on time-splitting methods for dispersive PDEs with small potentials and/or weak nonlinearity. This talk is based on joint works with Yongyong Cai and Yue Feng.

Mária Lukácová: Hybrid multiscale methods for polymeric fluids. I present our recent results on hybrid multiscale methods for polymeric fluids. I will concentrate on a class of kinetic models for polymeric fluids motivated by the Peterlin dumbbell theories for dilute polymer solutions with a nonlinear spring law. The polymer molecules are suspended in an incompressible viscous Newtonian fluid confined to a bounded domain in two or three space dimensions. The unsteady motion of the solvent is described by the incompressible Navier– Stokes equations with the elastic extra stress tensor appearing as a forcing term in the momentum equation. The elastic stress tensor is defined by Kramer's expression through the probability density function that satisfies the corresponding Fokker–Planck equation. In this case a coefficient depending on the average length of polymer molecules appears in the latter equation. I present the main steps of the proof of the existence of global-in-time weak solutions to the kinetic Peterlin model. Numerical simulations are obtained by the conservative scheme for a high-dimensional Fokker–Planck equation that models polymer molecules. The hybrid kinetic- continuum scheme combines the Lagrange–Galerkin method for the solvent and the Hermite spectral method for the Fokker-Planck equation together with a space splitting approach. Several numerical experiments will be presented to illustrate the performance of the scheme, and to confirm the conservation of mass at the discrete level. If time permits I can mention recent results of the hybrid molecular dynamics-continuum methods.

Lorenzo Pareschi: On consensus-driven stochastic particle systems for global optimization. In this talk we will review some recent results on the global minimization of a non-convex high- dimensional objective function by gradient-free stochastic particle methods. In the first part we will focus on the relations between the well-known particle swarm optimization (PSO) method and the recently introduced consensus-based optimization (CBO). In the second part we consider some recent extensions to multi-objective problems with uniform approximation of the Pareto front. Rigorous results regarding the mean-field limit and the convergence of the methods to the global minimum are presented together with some applications to machine learning problems.

Thomas Rey: Projective integration for kinetic equations. Projective integration has been recently proposed as a valuable alternative to fully implicit and micro- macro methods for providing light, nonintrusive and almost AP integrators for collisional kinetic equations. I shall present in this talk fully explicit projective integration and telescopic projective integration schemes for a broad class of kinetic equation, including (but not only) the Boltzmann equation. The methods employ a sequence of small forward-Euler steps, interspersed with large extrapolation steps. The telescopic approach repeats said extrapolations as the basis for an even larger step. This hierarchy renders the computational complexity of the method essentially independent of the stiffness of the problem, which permits the efficient solution of equations in the hyperbolic scaling with very small Knudsen numbers.

Jingmei Qiu: A locally macroscopically conservative (LoMaC) low rank high order tensor approach for nonlinear Vlasov equations. We propose a conservative adaptive low-rank tensor approach to approximate nonlinear Vlasov solutions. The approach takes advantage of the fact that the differential operators in the Vlasov equation is tensor friendly, based on which we propose to dynamically and adaptively build up low-rank solution basis by adding new basis functions from discretization of the PDE, and removing basis from an SVD-type truncation procedure. For the discretization, we adopt a high order finite difference spatial discretization and a second order strong stability preserving multi- step time discretization. While the SVD truncation will destroy the conservation properties of the full rank conservative scheme, we further develop low rank schemes with local mass, momentum and energy conservation for the corresponding macroscopic equations. The mass and momentum conservation are achieved by a conservative SVD truncation, while the energy conservation is achieved by replacing the energy component of the kinetic solution by the ones obtained from conservative schemes for macroscopic energy equation. Hierarchical Tucker decomposition is adopted for high dimensional problems, overcoming the curse of dimensionality. An extensive set of linear and nonlinear Vlasov examples are performed to show the high order spatial and temporal convergence of the algorithm, the significant CPU and storage savings of the proposed low-rank approach especially for high dimensional problems, as the local conservation of macroscopic mass, momentum and energy.

3.2 Applied Analysis

The presentations included novel analytical results for classic kinetic models, such as the Vlasov-Riesz system or the Boltzmann equation as well as new models arising in the life and social sciences. Examples of the latter include integrate and fire neuronal model in neuron science, alignment model for collective behaviors, and models for opinion formation. The presented results focused on various aspects, such as the asymptotic limits, singularity, long time convergence, log Sobolev inequalities, etc.

Young-Pil Choi: The Vlasov-Riesz system: existence and singularity formation. In this talk, we discuss the Cauchy problem for the Vlasov-Riesz system, which is a Vlasov equation featuring interaction potentials generalizing various previously studied cases, including the Coulomb and Manev potentials. For the first time, we extend the local theory of classical solutions to interaction potentials which are more singular than that for the Manev. Then, we obtain finite-time singularity formation for solutions with various attractive interaction potentials, extending the well-known singularity formation result for attractive Vlasov-Poisson. Our local well- posedness and singularity formation results extend to cases with linear diffusion and damping in velocity.

Maria José Caceres: Nonlinear Noisy Leaky Integrate and Fire neuronal models. In recent decades, kinetic theory has been used to model the collective behavior of large ensembles of neurons in a neural network. A macroscopic representation of the dynamics of the network was derived directly from the microscopic dynamics of individual neurons. Therefore, different approaches led to different PDE models for the probability density function of neuronal membrane potentials and synaptic conductances. In this talk we analyze some of the simplest of that family: Nonlinear Noisy Leaky Integrate and Fire neuronal (NNLIF) models. NNLIF models have been studied from a mathematical point of view; at the microscopic level, using Stochastic Differential Equations and at mesoscopic/macroscopic level, through the mean field limits using nonlinear Fokker-Planck type equations. The considerable amount of publications and unanswered questions on these models reveal their high mathematical complexity, despite their simplicity.

Changhui Tan: Sticky-particle Cucker-Smale dynamics and the entropic selection principle for the Euler-alignment system. In this talk, I will discuss weak solutions to the Euler-alignment system for collective behaviors. An entropic selection principle will be introduced to isolate a unique weak solution. Interestingly, the solution can be constructed and approximated by the Cucker-Smale dynamics with sticky particle collision rules. I will show an analytical convergence result, as well as the asymptotic flocking behavior to the entropy solution. This is joint work with Trevor Leslie.

Matias Delgadino: Phase transitions and log Sobolev inequalities. In this talk, we will study the mean field limit of weakly interacting diffusions for confining and interaction potentials that are not necessarily convex. We explore the relationship between the large N limit of the constant in the logarithmic Sobolev inequality (LSI) for the N-particle system and the presence or absence of phase transitions for the mean field limit. The non-degeneracy of the LSI constant will be shown to have far reaching consequences, especially in the context of uniform-in-time propagation of chaos and the behaviour of equilibrium fluctuations. This will be done by employing techniques from the theory of gradient flows in the 2-Wasserstein distance, specifically the Riemannian calculus on the space of probability measures.

Eitan Tadmor: Hydrodynamic alignment with pressure. Alignment reflects steering towards a weighted average heading. We study the swarming behavior of hydrodynamic -alignment, based on -graph Laplacians and weighted by a general family of symmetric communication kernels. This extends the classical alignment model corresponding to . The main new aspect here is the long time emergence behavior for a general class of pressure tensors without a closure assumption, beyond the mere requirement that they form an energy dissipative process. We refer to such pressure laws as 'entropic', and prove the flocking of -alignment hydrodynamics, driven by singular kernels with general class of entropic pressure tensors. We extend these findings to systems of multi-species, proving their long-time flocking behavior for connected arrays of multi-species, with self-interactions governed by entropic pressure laws and driven by fractional -alignment.

Renjun Duan: Uniform shear flow governed by the Boltzmann equation. For a rarefied gas, the uniform shear flow is characterized at a macroscopic level as a state where the horizontal velocity is linear along its normal direction while the density and temperature remain spatially uniform. Due to the shearing motion that induces the viscous heat, the total energy and hence temperature monotonically increase in time. It is more fundamental to understand the change of energy under the effect of shear forces at the kinetic level where the

gas motion is governed by the nonlinear Boltzmann equation. In this context, the USF state is defined as the one that is spatially homogeneous when the velocities of particles are refereed to a Lagrangian frame moving with the given macro shearing velocity. In the talk I will present recent results on existence and regularity of solutions to the spatially homogeneous Boltzmann equation with a shear force for the uniform shear flow.
Pedro Aceves Sanchez: Fractional diffusion limit of a linear kinetic transport equation in a bounded and unbounded domain. In recent years, the study of evolution equations featuring a fractional Laplacian has received much attention due to the fact that they have been successfully applied to the modeling of a wide variety of phenomena, ranging from biology, and physics to finance. The stochastic process behind fractional operators is linked, in the whole space, to an
$alpha$-stable process as opposed to the Laplacian one which is connected to a Brownian stochastic process. Evolution equations involving fractional Laplacians offer new exciting and very challenging mathematical problems. There are several equivalent definitions of the fractional Laplacian in the whole domain, however, in a bounded domain, there are several options depending on the stochastic process considered. In this talk, we shall present results on the rigorous passage from a velocity jumping stochastic process in a bounded domain to a macroscopic evolution equation featuring a fractional Laplace operator. More precisely, we shall consider the long-time/small mean-free path asymptotic behavior of the solutions of a re-scaled linear kinetic transport equation in a smooth bounded domain. In addition, we will also present some results regarding the entire domain.
Jian-Guo Liu: A selection principle for weak KAM solutions via Freidlin-Wentzell large deviation principle of invariant measures We will give a gentle introduction of weak KAM theory and then reinterpret Freidlin-Wentzell's variational construction of the rate function in the large deviation principle for invariant measures from the weak KAM perspective. We will use one-dimensional irreversible diffusion process on torus to illustrate some essential concepts in the weak KAM theory such as the Peierls barrier, the projected Mather/Aubry/Mane sets. Freidlin-Wentzell's variational formulas for both a self- consistent boundary data at each local attractors and for the rate function are formulated as the global adjustment for the boundary data and the local trimming from the lifted Peierls barriers. Based on this, we proved the Freidlin-Wentzell's rate function is a weak KAM solution to the corresponding stationary Hamilton-Jacobi equation satisfying the selected boundary data on projected Aubry set, which is also the maximal Lipschitz continuous viscosity solution. The rate function is the selected unique weak KAM solution and also serves as the global energy landscape of the original stochastic process. A probability interpretation of the global energy landscape from the weak KAM perspective will also be discussed. This is a joint work with Yuan Gao from Purdue University.

3.3 Modelling

Many physical or social system can be modelled using large interacting particle systems. Eleven workshop talks focused on modeling aspect of interacting particle systems, with applications ranging from economics to game theory and control theory. Very recent developments combined interacting particle systems with control, graph neural networks as well as other techniques in data science. This connection also highlights the timeliness and rapid developments in the field.

Sebastien Motsch: Using kinetic theory to study econophysics. In this talk, we discuss money exchange models where individuals randomly exchange dollars with each others according to (simple) rules. The goal is to link the individual behavior and the wealth distribution of an entire population. Mathematically, starting from a simple money exchange model, we study the limit of the dynamics in two regimes: i) letting the number of players goes to infinity, ii) studying the long time behavior. Our strategy is to first show that the dynamics becomes deterministic as the number of players goes to infinity proving the so-called propagation of chaos (i.e. a refined version of the Ïaw of large number ̈). Then, we use energy method to study the long time behavior of the derived deterministic dynamics. In many cases, we are able to show that the dynamics converge to an equilibrium distribution. But there is also an interesting scenario where the dynamics only converge weakly since all the money is Ïostäs time goes to infinity. This is a joint work with Fei Cao.
Shi Jin: Allen-Cahn Message Passing with Attractive and Repulsive Forces for Graph Neural Networks. Neural message passing is a basic feature extraction unit for graph-structured data considering neighboring node features in network propagation from one layer to the next. We model such process by an inter-

acting particle system with attractive and repulsive forces and the Allen-Cahn force arising in the modeling of phase transition. The dynamics of the system is a reaction- diffusion process which can separate particles without blowing up. This induces an Allen-Cahn message passing (ACMP) for graph neural networks where the numerical iteration for the particle system solution constitutes the message passing propagation. ACMP which has a simple implementation with a neural ODE solver can propel the network depth up to one hundred of layers with theoretically proven strictly positive lower bound of the Dirichlet energy. It thus provides a deep model of GNNs circumventing the common GNN problem of oversmoothing. GNNs with ACMP achieve state of the art performance for real-world node classification tasks on both homophilic and heterophilic datasets.

Domenec Ruiz I Balet: The interplay between control and deep learning. This talk will be about Neural Ordinary Differential Equations from a control theoretical perspective. We will see how they have strong simultaneous control properties that allow interpolation and approximation results. The simultaneous control problem of the differential equations can be interpreted as a bilinear-type control of the associated continuity equation, from which controllability results will be presented. Finally, we will also introduce the so-called Momentum ResNet, a variation that leads to a kinetic equation.

Sui Tang: Bridging the interacting particle models and data science via Gaussian process. System of interacting particles that display a wide variety of collective behaviors are ubiquitous in science and engineering, such as self-propelled particles, flocking of birds, milling of fish. Modeling interacting particle systems by a system of differential equations plays an essential role in exploring how individual behavior engenders collective behaviors, which is one of the most fundamental and important problems in various disciplines. Although the recent theoretical and numerical study bring a flood of models that can reproduce many macroscopical qualitative collective patterns of the observed dynamics, the quantitative study towards matching the well- developed models to observational data is scarce. We consider the data-driven discovery of macroscopic particle models with latent interactions. We propose a scalable learning approach that models the latent interactions as Gaussian processes, which provides an uncertainty-aware modeling of interacting particle systems. We introduce an operator-theoretic framework to provide a detailed analysis of recoverability conditions, and establish statistical optimality of the proposed approach. Numerical results on prototype systems and real data demonstrate the effectiveness of the proposed approach.

Bertram Düring: On a kinetic Elo rating model for players with dynamical strength. We propose and study a new kinetic rating model for a large number of players, which is motivated by the well-known Elo rating system. Each player is characterised by an intrinsic strength and a rating, which are both updated after each game. We state and analyse the respective Boltzmann- type equation and derive the corresponding nonlinear, nonlocal Fokker Planck equation. We investigate the existence of solutions to the Fokker-Planck equation and discuss their behaviour in the long time limit. Furthermore, we illustrate the dynamics of the Boltzmann and Fokker-Planck equation with various numerical experiments.

Dante Kalise: Learning feedback laws for collective dynamics. In this talk we will discuss the construction of feedback strategies for controlling a large population of agents towards consensus. To overcome the curse of dimensionality associated to controlling the agent ensemble, we consider a mean field formulation of the consensus control problem. Although such a formulation is designed to be independent of the number of agents, it is computationally feasible only for moderate intrinsic dimensions of the agents' space. For this reason, the consensus problem is approached at the kinetic level by means of a quasi-invariant limit of controlled binary interactions as approximation of the mean field PDE. The need for an efficient solver for the binary interaction control problem motivates the use of a deep learning approaches to encode a high-dimensional binary feedback map.

Hangjie Ji: Dynamics of thin liquid films on vertical cylindrical fibres. Thin liquid films flowing down vertical fibres spontaneously exhibit complex interfacial dynamics, creating irregular wavy patterns and traveling liquid droplets. Such dynamics is a fundamental component in many engineering applications, including mass and heat exchangers for thermal desalination and water vapor and particle capture. Recent experiments present a wealth of new dynamics that illustrate the need for more advanced theory. In this talk, I will first present a study of a full lubrication model that includes slip boundary conditions, nonlinear curvature terms, and a film stabilization term. This model better explains the observed velocity and stability of traveling droplets in experiments and their transition to isolated droplets. Next, I will discuss thermally driven droplet coalescence induced by a temperature field along the fibre. To characterize the flow regime transition induced by varying nozzle geometries, I will also present a study of a weighted residual integral boundary-layer model that incorporates moderate inertia. I will conclude by discussing opportunities for developing kinetic equation-based models for vapor and particle capture in fibre coating dynamics.

Shigeru Takata: A kinetic model for the phase transition and its numerical simulation. In this talk, we will first present a simplified kinetic model for the phase transition and then discuss some properties related to thermodynamics. Some results of numerical simulations will be presented as well. Thermodynamical and mechanical surface tensions are evaluated numerically and are found to agree well with each other.

Jonathan Franceschi: Kinetic Models in Epidemiology and Social Sciences. Recent years have shown us that both viral diseases and misinformation can spread massively within a population and have pernicious consequences. It is therefore crucial to study these phenomena and develop strategies to mitigate their damages. Thus, in this talk, we will see some recently introduced kinetic models, which borrow from classical epidemiological theories to study the dissemination of fake news and contagious illnesses. Our focus is to start from microscopic interactions to devise later a model of the evolution of observable (i.e., macroscopic) quantities. We thus show how to construct a closed hierarchy of macroscopic equations through a limit procedure in the spirit of classical kinetic theory. Our overview will be complemented by an interface between the obtained models and available data as well as optimal control strategies and uncertainty quantification techniques on them. Simona Mancini: Modeling decision making for neuronal interactions We study a non gradient Fokker-Planck equation arising in the modeling of visual cortex neuronal activity. A slow-fast reduction permits to simplify the equation and to reduce computational costs. The model is validate comparing numerical results and experimental ones. Other applications and recent developments will be also addressed.

Alethea Barbaro: A novel model for phase separation In this talk, I will discuss an interacting particle model for different groups of agents moving on a lattice. Each agent leaves its own group's marking on the lattice as it moves; the agents preferentially avoid other groups' graffiti. This model exhibits phase separation and coarsening over time for certain parameter values. We formally derive a system of convection-diffusion equations with cross-diffusion. We use this system to identify the critical parameter for the model. Using two energy functionals for a simplified version of the system, we also have proven a weak- stability result and shown using this result that the system does not allow segregated solutions. We will discuss possible reasons for this potentially contradictory result as well as giving an overview of the state of the art of the current anlytical results

Elisa Calzola: A data-driven kinetic model for opinion dynamics and contacts. We present a kinetic model for opinion dynamics depending on the presence of social media contacts. We first model the contact dynamics, starting from the single agent's evolution of social contacts, then we restore to kinetic collision-like models in order to derive the Boltzmann equation satisfied by the density of the contacts. We then study the evolution of the distribution of opinions, that obeys a bilinear Boltzmann-like equation. We were able to collect data from Twitter in order to estimate the parameters appearing in our model and we conclude our work with some numerical simulations.

4 Scientific Progress Made

The workshop provided an excellent chance for researchers to present and discuss their research in an informal setting. There were lively discussions with the online participants as well as the speakers at the CMO. The schedule provided enough opportunities for discussions and working.

We believe that the workshop was the starting point for several projects and collaborations and that the cross fertilization between communities as well as the mix of junior and more senior participants lead to a promising outcome.

Discussions on parameter estimation for models in kinetic theory, application of kinetic theory tools in neural networks and machine learning, and uncertainty quantification approaches to mathematical biology are just examples of the new discussions and directions in which several groups got together during the days at Oaxaca.

5 Outcome of the Meeting

The workshop was a great continuation of recent activities in the very dynamic kinetic theory community. It connected researchers from the United States with colleagues in Europe, Asia and South America. It featured researchers at different seniority levels, ranging from PhD students to well-established experts in the field. The workshop aimed at providing a platform to discuss ideas and work on various aspects of kinetic equations in an informal setting. We believe that this setting and exchange will lead to new research directions and collaborations and the wider dissemination of kinetic theory on the international level.

The workshop also strengthened the kinetic research community and we believe that it will also result in future activities in the United States and Canada as well as on the international level. Current examples include the recently approved Newton Research Network KineticNet in the UK (in which several of the workshop organisers as well as participants are involved) as well as the planned submission of a special proposal semester on kinetic theory at IPAM.

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