Preprojective Algebras Interacting with Singularities, Cohen-Macaulay Modules and Weighted Projective Lines

ABSTRACTS for Monday/Tuesday (Oct. 3)

Chan, Daniel (University of New South Wales)
Algebraic stacks in the representation theory of finite dimensional algebras

Abstract:
When classifying modules over an algebra, one often finds infinite continuous families of modules and so geometry enters their study. The resulting geometric objects are usually not varieties but are moduli stacks. In this talk, we will give an introduction to algebraic stacks aimed at representation theorists, concentrating on the example of moduli stacks and weighted projective lines.

Their importance will be examined in the following context. Given a tilting bundle T on a projective stack X, the endomorphism algebra A of T is relatively easy to study because it is derived equivalent to X. This naturally raises the question, given an arbitrary finite dimensional algebra, how might one produce a projective stack which is derived equivalent to it (assuming of course it exists)? We propose a first approximate answer: the moduli stack of Serre stable representations and show how it re-interprets Geigle-Lenzing's derived equivalence between canonical algebras and weighted projective lines. This final part is joint work with Boris Lerner.

Ebeling, Wolfgang (Leibniz Universität Hannover)
Strange duality between hypersurface and complete intersection singularities

Abstract: This is a continuation of the talk of A.Takahashi and based on joint work with him. We shall discuss a hierarchy of singularities and its relation to chains of triangulated categories associated to weighted projective lines. We shall introduce the notion of a virtual singularity. We shall derive the strange duality between virtual singularities and complete intersection singularities observed by the speaker and Wall from the mirror symmetry introduced in the previous talk.

Faber, Eleonore (University of Michigan)
A McKay correspondence for reflection groups

Abstract:
The classical McKay correspondence relates the geometry of so-called Kleinian surface singularities with the representation theory of finite subgroups of $SL(2,C)$.
M. Auslander observed an algebraic version of this correspondence: let G be a finite subgroup of $SL(2,k)$ for a field k whose characteristic does not divide the order of G. The group acts linearly on the polynomial ring $S=k[x,y]$ and then the so-called skew group algebra $A=G*S$ can be seen as an incarnation of the correspondence. In particular, A is isomorphic to the endomorphism ring of S over the corresponding Kleinian surface singularity.
Moreover, it is known that A is Morita-equivalent to the preprojective algebra of an extended Dynkin diagram.

We want to establish an analogous result when G in $GL(n,K)$ is a finite group generated by reflections, assuming that the characteristic of k does not divide the order of the group. Therefore we consider again the skew group algebra $A=G*S$, where S is the polynomial ring in n variables, and its quotient A/AeA, where e is the idempotent in A corresponding to the trivial representation. With D the coordinate ring of the discriminant of the group action on S, we show that the ring A/AeA is the endomorphism ring of the direct image of the coordinate ring of the associated hyperplane arrangement.
In this way one obtains a noncommutative resolution of singularities of that discriminant, a hypersurface that is singular in codimension one. This is joint work with Ragnar-Olaf Buchweitz and Colin Ingalls.

Herschend, Martin (Uppsala University)
Higher preprojective algebras (provisional title)

Abstract: To any finite dimensional algebra of global dimension *d* we can associate its higher preprojective algebra. This generalizes the classical notion of preprojective algebras of hereditary algebras. In my talk I will introduce this construction and discuss some of its basic properties. I will also show how the preprojective algebra can be computed in some interesting cases. Finally the relevance of higher preprojective algebras to higher dimensional Auslander-Reiten theory will be discussed. This last part of the talk is based on joint work with Osamu Iyama and Steffen Oppermann.

Hille, Lutz (Universität Münster)
Weighted projective spaces, crepant resolutions and tilting (jt. w. R. Buchweitz)

Abstract: We consider three types of 'weighted projective spaces', the classical one, the one in the sense of Baer, Geigle and Lenzing, and the toric stacks. The classical one is defined by a quotient of a C^*-action defined by a sequence of weights $(q_0,...,q_n)$. If this sequence is reduced, then the associated toric variety isFano, precisely when each q_i devides the sum $q := \sum q_j$. Moreover, let Y be a crepant resolution of a Fano X. Associated to the weights, there is also a weighted projective space P in the sense of Baer. This is also a toric stack in the sense of Borisov, Hu and Kawamata. The latter admits a full, strongly exceptional sequence of line bundles $O(i)$ for$ i=0,...,q-1$. We construct a full, strongly exceptional sequence of line bundles on any crepant resolution Y of X, so that the endomorphism algebra coincides with the one on P. Consequently, Y and $P% have equivalent derived categories of coherent sheaves. We close with some examples and some applications.

Iyama, Osamu (Nagoya University)
Tilting theory for Geigle-Lenzing complete intersections

Abstract: We discuss Cohen-Macaulay representations of a Geigle-Lenzing complete intersection *R* in dimension *d+1*. The stable category always has a tilting object.
We discuss when there exists a tilting object whose endomorphism algebra has global dimension at most *d*. In this case *R* must be Fano, and moreover *d*-Cohen-Macaulay finite. Moreover this often gives a tilting bundle on the Geigle-Lenzing projective space whose endomorphism algebra has global dimension *d*.
This is a part of joint work with Herschend, Minamoto and Oppermann.

Leuschke, Graham (Syracuse University)
Cohen-Macaulay modules I

Abstract: This talk will give an overview of the theory of maximal Cohen-Macaulay (MCM) modules, focusing on the background and history of the subject, focusing on (but not limited to) representation-theoretic aspects.

Leuschke, Graham (Syracuse University)
Cohen-Macaulay modules II

Abstract: This talk will report on some aspects of cluster tilting theory for MCM modules, one of the most active and influential sectors of the MCM theory.

Meltzer, Hagen (University of Szczecin)
Weighted projective lines

Abstract:

Miro-Roig, Rosa-Maria (Universitat de Barcelona)
The representation type of a projective variety

Abstract:

In my talk, I will construct families of non-isomorphic Arithmetically Cohen Macaulay (ACM)
sheaves (i.e., sheaves without intermediate cohomology) on projective varieties. Since the seminal result by Horrocks characterizing ACM bundles on P^n as those that split into a sum of line bundles, an important amount of research has been devoted to the study of ACM on a given variety.

ACM sheaves also provide a criterium to determine the complexity of the underlying variety.
More concretely, this complexity can be studied in terms of the dimension and number of families of indecomposable ACM sheaves that it supports, namely, its representation type. Along this lines, a variety that admits only a finite number of indecomposable ACM sheaves (up to twist and isomorphism) are called of finite representation type. These varieties are completely classified: They are either three or less reduced points in P^2, a projective space P_k^n, a smooth quadric hypersurface $X\subset P^n$, a cubic scroll in $P_k^4 $, the Veronese surface in P_k^5 or a rational normal curve.

On the other extreme of complexity we would find the varieties of wild representation type,
namely, varieties for which there exist *r*-dimensional families of non-isomorphic indecomposable ACM sheaves for arbitrary large *r*. In the case of dimension one, it is known that curves of wild representation type are exactly those of genus larger or equal than two. In my talk, I will give a brief account of the known results in higher dimension.

Izuru Mori (Shizuoka University)
n-regular modules over n-representation infinite algebras

Abstract: Recently, Herschend, Iyama and Oppermann introduced the notion of n-representation infinite algebra, extending the notion of hereditary algebra of infinite representation type to an algebra of higher global dimension n, and the notion of n-regular module over such an algebra. A typical example of such an algebras is the Beilinson algebra R of an AS-regular algebra A. In this setting, it is known that the preprojective algebra of R is graded Morita equivalent to A. As in the hereditary case, studying n-regular modules is essential to understand such an algebra. In this talk, we will show that if A satisfies Cohen-Macaulay property, then the category of n-regular modules over the Beilinson algebra R is abelian, and isomorphism classes of simple n-regular modules over R are parameterized by closed points of the noncommutative projective scheme associated to A. We will see by examples that such application of noncommutative algebraic geometry to representation theory is very powerful.

Oppermann, Steffen (Norwegian University of Science and Technology)
Geigle-Lenzing spaces and d-canonical algebras

Abstract: This talk is based on joint work with Herschend, Iyama, and Minamoto, and in part work by Iyama and Lerner.

Weighted projective lines were introduced by Geigle and Lenzing in 1987, and have since proven to be useful in representation theory. One key property is that they have nice tilting bundles, whose endomorphism rings are precisely Ringel's canonical algebras.

The aim of this talk is to generalize the above to a *d*-dimensional setup: We introduce Geigle-Lenzing spaces as *d*-dimensional analogs of weighted projective lines, and show that there is an analogous derived equivalence to *d*-canonical algebras. We exploit this derived equivalence to study both sheaves on the Geigle-Lenzing space and the module category of the *d*-canonical algebra.

Takahashi, Atsushi (Osaka University)
Introduction to the mirror symmetry between weighted projective lines and cusp singularities

Abstract:
This is an introductory talk to the one of W. Ebeling. We introduce the notion of an invertible polynomial and its Berglund-Hübsch transpose. We define Dolgachev and Gabrielov numbers for them. We derive a mirror symmetry between weighted projective lines and cusp singularities. We relate this to homological mirror symmetry and to Orlov's semi-orthogonal decomposition of the singularity category.

Ueda, Kazushi: (Osaka University)
Moduli of relations of quivers

Abstract:
The derived category of coherent sheaves on an algebraic stack admitting a tilting object is described by a quiver with relations. We introduce moduli spaces of relations of quivers,
which describe not necessarily commutative deformations of the stack.
If the stack is a weighted projective line, the moduli of relations of the quiver
is the configuration space of points on the projective line. In the talk, we will discuss our joint work with Tarig Abdelgadir and Shinnosuke Okawa on moduli spaces of relations of quivers
associated with the projective plane, the quadric surface, and cubic surfaces.