36QP Conference and CMO–BIRS Workshop (15w5086) Quantum Markov Semigroups in Analysis, Physics and Probability
Abstracts

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Local equilibrium states, Dynamical detailed balance and Markov semi-groups of stochastic limit type

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My talk reports the results of a joint research with Franco Fagnola and Roberto Quezada, begun several years ago with the goal of extending, to some important class of nonequilibrium steady states (NESS), the results obtained in the attempts to connect equilibrium states with special classes of Markov generators.
These investigations have a long history, both in physics and mathematics, starting with a famous paper of Glauber in 1963 who constructed a phenomenological Markov generator leaving invariant the Gibbs measure.
In the theory of stochastic limit one deduces, from realistic Hamiltonian interactions, a natural class of nonequilibrium stationary states and the associated generators (hence the name *stochastic limit type generators*). The relations between the two are more subtle than in the equilibrium case since they involve *microcurrents*, that are identically zero in the equilibrium case. The deductive basis of the connections, between states and generators, is the *principle of similarity*: one of the basic principles of the stochastic limit theory.
The Markov generators emerging from this theory have a very special structure which gives, contrarily to the more general but abstract GKSL form, an immediately intuitive picture of the microinteractions that build up the whole dynamics.
In particular we associate to each such generator, its *interaction graph* and show that the structure of the invariant states strongly depends on the topology of this graph.
After the publication of the paper:
L. Accardi, F. Fagnola, R. Quezada:
"Weighted Detailed Balance and Local KMS Condition for NonEquilibrium Stationary States"
Busseikenkyu, Perspectives of Nonequilibrium Statistical Physics, Dedicated to the memory of Shuichi Tasaki 97 (3) (2011/2012) 318356
Kyoto University Research Information Repository
several groups in the world (in particular in Poland and in France) have begun to study the class of *stochastic limit type generators*.

Decoherence free subspaces of a Quantum Markov Semigroup

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We give a full characterisation of decoherence free subspaces of a given quantum Markov semigroup with generator in a generalised Lindbald form which is valid also for infinite dimensional systems. Our results, extending those available in the literature concerning finite dimensional systems, are illustrated by some examples.

The form of the generator of a quantum Markov Semigroup

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Quantum Markov semigroups arise in the study of irreversible open quantum systems. The form of the generator of a quantum Markov semigroup has been of interest since Lindblad gave a useful form in the case when the quantum Markov semigroup is uniformly continuous, a condition which is not true in many important examples. In a recent joint work with Matthew Ziemke, it is shown that without assuming uniform continuity, the generator of a quantum Markov semigroup can be written in a form similar to Lindblad's form. While results exist in this direction, our result has different and minimal assumptions.

Free Subordination and Belinschi-Nica Semigroup

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We consider the Belinschi-Nica semigroup of homomorphisms and realize it as a free multiplicative subordination. This realization allows to define more general semigroups which are also homomorphism with respect to multiplicative subordination. For this semigroups we show that a differential equation holds generalizing the complex Burgers equation.

A generalization of the q-deformed Bargmann representation

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A Bargmann representation of a probability measure on $\mathbb{R}$ will be discussed. Especially, we shall focus on the representation associated with the (α, q)-Fock space introduced recently by Bozejko-Ejsmont-Hasebe, which is a generalization of the q-Fock space by Bożejko-Speicher. As a result, our result generalizes that of van Leeuwen-Maassen. This talk is based on the joint work with T.Hasebe (Hokkaido Univ.).

An Information Complexity index for Probability Measures on $\mathbb{R}$ with all moments

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We prove that, each probability meassure on $\mathbb{R}$, with all moments, is canonically associated with (i) a $*$–Lie algebra; (ii) a complexity index labeled by pairs of natural integers. The measures with complexity index $(0, K)$ consist of two disjoint classes: that of all measures with finite support and the semi–circle–arcsine class. The class $(1, 0)$ includes the Gaussian and Poisson measures and the associated $*$–Lie algebra is the Heisenberg algebra. The class $(2, 0)$ includes the non standard (i.e. neither Gaussian nor Poisson) Meixner distributions and the associated $*$–Lie algebra is a central extension of $sl(2, \mathbb{R})$. Starting from $n = 3$, the $*$–Lie algebra associated to the class $(n, 0)$ is infinite dimensional. This produces a new class of infinite dimensional $*$–Lie algebras, canonically associated to probability measures, which does not seem to be present in the literature.
(**Joint work with Luigi Accardi and Mohamed Rhaima**)

Units of quantum dynamical semigroups

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The concept of *unit* plays an important role in classifying product systems and semigroups of endomorphisms (E_0-semigroups). A one parameter semigroup of the form $X \mapsto C_t X C_t^*$ is said to be a unit of a given quantum dynamical semigroup τ, if it is dominated by τ up to scaling, that is, there exists a positive scalar q such that

$$X \mapsto \tau_t(X) - e^{-qt} C_t X C_t^*$$

is completely positive for all t. We explore various implications of existence of a unit for a quantum dynamical semigroup.

A Quantum Kac Walk and its Kinetic Limit (Part II)

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We present recent results on models for quantum systems of N particles undergoing random binary collisions, focusing on propagation of chaos and the rate of convergence to equilibrium. These questions arise from the work of Mark Kac and his investigation into the probabilistic structure underlying the Boltzmann equation. Recently, the quantum mechanical variation on Kac's question has begun to be investigated. In this case, the Kac Master equation becomes an evolution equation of Lindblad type, while the corresponding Boltzmann equation is a novel sort of non-linear evolution equation for a density matrix.There are novel difficulties due to the fact that in quantum mechanics, conditional probability is not always well-defined. Nonetheless, a substantial quantum analog of of the Kac program can be carried out, and this leads to an interesting and novel class of quantum kinetic equations. This is joint work with Michael Loss and Maria Carvalho.

A Quantum Kac Walk and its Kinetic Limit (Part I)

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We present recent results on models for quantum systems of N particles undergoing random binary collisions, focusing on propagation of chaos and the rate of convergence to equilibrium. These questions arise from the work of Mark Kac and his investigation into the probabilistic structure underlying the Boltzmann equation. Recently, the quantum mechanical variation on Kac's question has begun to be investigated. In this case, the Kac Master

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Structure of norm-continuous quantum Markov semigroups and their invariant states

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In this talk, we present recent results on the structure of norm-continuous quantum Markov semigroups with atomic fixed point algebra or atomic decoherence-free subalgebra providing a natural decomposition of a Markovian quantum open system into its irreducible components and noiseless components. We also discuss new characterisations of the structure of invariant states and decoherence-free subsystems with applications to quantum Markov semigroups arising from the stochastic limit. This talk is based on joint with J. Bolaños, J. Deschamps, E. Sasso and V. Umanità.

References

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Symmetries and ergodic properties in Quantum Probability

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By using the standard techniques of Operator Algebras, we study the general structure of the stochastic processes on the two-sides chain (whose parameter is then $\mathbb{Z}$) in the quantum setting, together with their natural symmetries such as the invariance under the shift and the permutations. The ergodic properties of stationary and exchangeable processes are investigated for many interesting cases arising from physics, and including those arising from free probability. Among that, we treat in detail the Bose/Fermi alternative (by describing the quantum generalisation of the De Finetti and Hewitt-Savage Theorems), and more generally the q-canonical commutation relations $-1 < q < 1$, where $q = 0$ corresponds to the free (or Boltzmann) case. In the last situation, we show that the set of shift-invariant states (corresponding to the stationary processes) is made of a singleton.

The universal situation arising from free probability and corresponding to free product C^*-algebra (with or without an identity) is also treated in detail. More precisely, we connect some natural algebraic properties of the stochastic process, like that to being a *product state* or a *block-singleton* one, to natural ergodic properties of the state (on the free product C^*-algebra) naturally associated to the process under consideration, like the weak clustering, and the property of *convergence to the equilibrium* respectively. The ergodic properties of the class of the so-called Haagerup states on the group C^*-algebra of the free group on infinitely many generators are investigated in full generality.

Finally, we also specialise our investigation to the study of ergodic properties of stationary processes arising from the monotone (firstly defined investigated by Lu and Muraki), and Boolean processes. We apply the results for achieving the structure of stationary states, and in the Boolean case, we present a De Finetti theorem too.

On conditionally positive functions and functionals

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Let A be a unital $*$algebra and $\varepsilon : A \to \mathbb{C}$ a unital $*$homorphism. A functional $\psi : A \to \mathbb{C}$ is called *conditionally positive* (or a *generating functional*) on (A, ε) if (a) $\psi(1) = 0$, (b) ψ is hermition, (c) ψ is positive on the kernel of ε. Examples are the linear extensions to the group algebra of negative type functions on groups. Conditionally positive functions classify Lévy processes on groups or involutive bialgebras. I will present new results about the relation between conditionally positive functions and 1cocycles, and about the existence of a decomposition of such functions into a maximal Gaussian part and a Gaussfree remainder. Based on joint work with Biswarup Das, Malte Gerhold, Anna Kula, Adam Skalski, Andreas Thom.

The Laguerre Distribution and some of its moments

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The Laguerre Distribution appears in the context of the chaotic scattering (see for instance, [1, 2]) as the joint distribution of the reciprocals of the eigenvalues of the so called Wigner-Smith time delay times matrix. Such eigenvalues are known as proper delay times. Their join moments are needed to determine the statistical fluctuations of different transport properties through chaotic cavities like the quantum dots. In this talk, we compute some of these joint moments in a more a or less direct way, as it is presented in [4]; on the other hand, we present advances of a new work where we compute some moments with the aid of a technique based on multiple integration of Vandermonde type functions, as exposed in [3].

References

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Operator amenability vs symmetric operator amenability

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In this talk we discuss the symmetric operator amenability of operator algebras. For completely contractive Banach algebras, we give equivalent conditions to symmetric operator amenability for operator algebras and prove some structural properties of symmetric operator amenability. We also discuss about the existence of complete bounded Jordan derivation from symmetrically operator amenable Banach algebra into operator bimodule

Wick Algebras of Admissible White Noise Operators

and Applications

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A quantum extension of the Girsanov theorem is formulated as an implementation problem for admissible white noise operators. For a systematic study of implementation problems, we first start with a detailed study of admissible white noise operators with constructions of Wick algebras. Secondly, we develop the concept of quantum white noise derivatives of admissible operators and as applications, we study general forms of implementation problem for admissible white noise operators. Finally, an implementation problem with solution as a quantum martingale is applied to quantum Girsanov theorem. This talk is based on a series of joint works with Nobuaki Obata.

Review of mixing time tools in quantum information and many body theory

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I will provide an overview of the recent results involving the mixing time analysis of quantum dynamical semigroups, and their application to problems in quantum information theory and manybody physics. These results include quantum functional inequalities, such as Poincare Nash and Log-Sobolev inequalities. The applications include: analysis of quantum memories, quantum algorithms for state preparation and sampling, and quantum shannon theory.

Convex structures arising from quantum information theory

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In order to distinguish entanglement from separability, we may have to understand the boundary of the convex set of all separable states, which consists of faces. As the first step, we look for faces which are affinely isomorphic to simplices, in various levels. In the course of discussion, we need some techniques from various fields of mathematics; combinatorics, algebraic geometry as well as functional analysis. We also explore simplex-like convex sets arising in this way.

A kinetic reaction model: decay to equilibrium and macroscopic limit

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We propose a kinetic relaxation-model to describe a generation-recombination reaction of two species. Decay to the stationary solution is studied by using hypocoercivity of the linearized equations. Exponential decay of small perturbations can be shown for the full nonlinear problem. We will give a short comparison of recent methods for tackling such problems [1, 2, 3]. If time allows we will also discuss the macroscopic/fast-reaction limit and sketch a rigorous derivation using Entropy decay estimates. The macroscopic limit, in our case, is a nonlinear diffusion equation for the difference of the position densities.

This talk is based on joint work with C. Schmeiser (University of Vienna).

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Dynamical properties of a mean field laser equation

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The talk is based on a joint work with Franco Fagnola, Politecnico di Milano. We examine the dynamics of the solution of a non-linear quantum master equation describing a quantum system formed by two level atoms and a single mode of an optical cavity that are coupled to two reservoirs. First, we establish the well-posedness of the non-linear quantum master equation under study. Then, we present a bifurcation analysis of the solution structure.

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Quantum White Noise Derivatives and Characterization of White Noise Operators

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Quantum white noise calculus provides a framework of operator theory on (Boson) Fock space by means of infinite dimensional distribution theory based on a Gelfand triple $\mathcal{W} \subset \Gamma(H) \subset \mathcal{W}^*$. It is natural to think of the derivatives of a white noise operator $\Xi = \Xi(a_s, a_t^*; s, t \in T)$ with respect to the pointwise annihilation and creation operators a_s, a_t^*, formally denoted by

$$D_t^+ = \frac{\delta \Xi}{\delta a_t^*}, \qquad D_t^- = \frac{\delta \Xi}{\delta a_t}.$$

In fact, the quantum white noise derivatives are defined by

$$D_\zeta^+ \Xi = [a(\zeta), \Xi] = a(\zeta)\Xi - \Xi a(\zeta), \qquad D_\zeta^- \Xi = -[a^*(\zeta), \Xi] = \Xi a^*(\zeta) - a^*(\zeta)\Xi,$$

for any white noise operator $\Xi \in \mathcal{L}(\mathcal{W}, \mathcal{W}^*)$ and $\zeta \in E$. Then $D_\zeta^\pm$ are Wick derivations on $\mathcal{L}(\mathcal{W}, \mathcal{W}^*)$ and white noise operators are characterized by differential equations of Wick type.

In this talk we will review the fundamentals on quantum white noise derivatives and show their applications: (i) the implementation problem for the canonical commutation relation and derivation of Bogoliubov transform; (ii) calculating the normal-ordered form of the composition of the exponential of a quadratic annihilation operator and its adjoint.

Our work is based on the long-term collaboration with Un Cig Ji. Further recent achievements e.g., quantun Girsanov transform will be discussed by his talk.

Quantum Dynamical Semigroups – their perturbations

AND THE STABILITY THE MARKOV/CONSERVATIVITY PROPERTY

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A theory of perturbation of a quantum dynamical semigroup by another is developed ; the total semigroup is constructed . Some results on the conservativity of the total semigroup are obtained

THE COMMUTING FAMILY OF DUNKL OPERATORS VIEWED FROM A NONCOMMUTATIVE PERSPECTIVE

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In a joint paper with M. Durdevich the commutative family of operators introduced by Charles Dunkl in harmonic analysis is viewed in terms of a covariant derivative in a quantum (i.e., noncommutative) principal bundle equipped with a quantum connection. This relates two previously independent theories in an unexpected way. In particular, the well known commutativity of the Dunkl operators is seen for the first time to be a consequence of a quantum connection having (quantum!) curvature zero.

A HÖLDER-YOUNG INEQUALITY FOR NORMS OF GENERALIZED WICK PRODUCTS

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Joint work with: Paolo Da Pelo and Alberto Lanconelli, from the University of Bari, Italy

Unlike Riemann integral, the stochastic integral depends on the way the sample points are chosen in the subintervals of a partition. Thus for example, in Itô integral the sample points are the leftend points of the subintervals of a partition, while in Stratonovich integral they are the midpoints. One can imagine a stochastic integral in which the sample points are always chosen one quarter away from the leftend points and three quarters from the right endpoints of the subintervals of a partition. To each of these stochastic integrals it corresponds a particular generalized Wick product. This generalized Wick product links the integrand to the stochastic infinitesimal dBt. We present a Hölder inequality for the norms of these generalized Wick products. In order to bound the bilinear generalized Wick product, we need to use also the second quantization operators.

Thermalization time bounds for Pauli stabilizer Hamiltonians

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We prove a lower bound to the spectral gap of the Davies generator for general N qubit commuting Pauli Hamiltonians. We expect this bound to provide the correct asymptotic scaling of the gap with the systems size up to a factor of 1/N in the low temperature regime. We derive rigorous thermalization time bounds, also called mixing time bounds, for the Davies generators of these Hamiltonians. Davies generators are given in the form of a Lindblad equation and are known to converge to the Gibbs distribution of the particular Hamiltonian for which they are derived. The bound on the spectral gap essentially depends on a single number μ referred to as the generalized energy barrier. When any local defect can be grown into a logical operator of a stabilizer code S by applying single qubit Pauli operators and in turn any Pauli operator can be decomposed into a product of the clusters of such excitations, μ corresponds to the largest energy barrier of the canonical logical operators. The main conclusion that can be drawn from the result is, that the presence of an energy barrier for the logical operators is in fact, although not sufficient, a necessary condition for a thermally stable quantum memory when we assume the full Davies dynamics as noise model.

Linear growth for supports of multiplicative c.f. independent random variables.

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In conditionally free independence (c.f. independence) one uses two states to measure the independence between families of algebras. In this talk we present some similarities and differences between the concepts of free independence and conditionally free independence, taking a look at the support of conditionally free multiplicative convolution of non-commutative random variables.

A transference result of L_p continuity from the Jacobi setting

to the Hermite and Laguerre setting

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In this talk we develop a transference method to obtain the L_p-continuity of the Gaussian-Riesz transform and the L_p-continuity of the Laguerre-Riesz transform, in dimension one, from the L_p-continuity of the Jacobi-Riesz transform using the well known asymptotic relations between Jacobi polynomials and Hermite and Laguerre polynomials. Also, time permitting we will also discuss the case of the Littlewood-Paley g functions i.e. The L_p-continuity the Gaussian-Littlewood-Paley g function and the L_p-continuity of the Laguerre-Littlewood-Paley g function from the L_p-continuity of the Jacobi-Littlewood-Paley g function.

Multi-dimensional orthogonal polynomials

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We investigate multi-dimensional orthogonal polynomials. Based on the previous work by Accardi-Barhoumi-Dhahri we construct creation, annihilation , and preservation operators (CAP operators) on an interacting Fock space for a given orthogonal system. We also consider the reconstruction theorem. Given CAP operators on an interacting Fock space, we give sufficient conditions for the operators so that they define a probability measure and moreover they are reconstructed from the measure. We discuss also the deficiency rank of the Jacobi operators and its relation to the support of the measure. We will consider some examples. This is a joint work with Ameur Dhahri and Nobuaki Obata

The closedness of the generator of a semigroup

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In a recent work with George Androulakis, we studied semigroups of bounded operators on a Banach space whose members are continuous with respect to various weak topologies and we give sufficient conditions for the generator of the semigroup to be closed with respect to the topologies involved. Since the proofs of these results involve the Laplace transforms of the semigroup, we first give sufficient conditions for Pettis integrability of vector-valued functions with respect to scalar measures. In this talk, we will discuss sufficient conditions for Pettis integrability of vector-valued functions then use these results to give sufficient conditions for the generator of a semigroup to be closed with respect to various topologies, given certain continuity assumptions for the semigroup.