Harmonic Analysis, $\overline{\partial}$, and CR Geometry
Oct 18 – Oct 23, 2015

MEALS

*Breakfast: 7:30–9:00 am, Restaurant Hotel Hacienda Los Laureles, Monday–Friday
*Lunch: 13:30–15:00 pm, Restaurant Hotel Hacienda Los Laureles, Monday–Friday
*Dinner: 19:00–21:00 pm, Restaurant Hotel Hacienda Los Laureles, Monday–Thursday
*Dinner: 19:30–22:00 pm, Restaurant Hotel Hacienda Los Laureles, Sunday only
*Continuous Coffee Breaks: Conference Room San Felipe, Hotel Hacienda Los Laureles

MEETING ROOMS

All lectures will be held in the Conference Room San Felipe at Hotel Hacienda Los Laureles. An LCD projector, laptop, document camera and blackboards are available for presentations.

SCHEDULE

Sunday

14:00	Check-in begins (front desk at your assigned hotel - open 24 hours)
19:30–22:00	Dinner, Restaurant Hotel Hacienda Los Laureles
20:30	Informal gathering Hotel Hacienda Los Laureles (if desired)
	A welcome drink will be served by the hotel.

Monday

7:30–9:00	Breakfast
9:45–10:00	Introduction and Welcome
10:00-10:50	Gerardo Mendoza
11:00–11:30	COFFEE
11:30–12:20	Yuan Yuan
12:30–13:20	Loredana Lanzani
	Group Photo
13:30–15:00	Lunch
15:00–15:50	Brian Street
16:00–16:30	COFFEE
16:30–17:20	Jingzhi Tie
19:00–21:00	Dinner

Tuesday

7:30–9:00	Breakfast
10:00-10:50	Christine Laurent
11:00–11:30	COFFEE
11:30–12:20	Ilya Kossovskiy
12:30–13:20	Nordine Mir
13:30–15:00	Lunch
15:00–19:00	Free afternoon
19:00–21:00	Dinner

Wednesday

7:30–9:00	Breakfast
9:00-10:50	Phillip Harrington
10:00-10:50	Jeffrey McNeal
11:00–11:30	COFFEE
11:30–12:20	Andrew Raich
12:30–13:20	Sonmez Sahutoglu
13:30–15:00	Lunch
15:00–15:50	Friedrich Haslinger
16:00–16:30	COFFEE
16:30–17:20	Soledad Benguria
19:00–21:00	Dinner

Thursday

7:30–9:00	Breakfast
10:00-10:50	Xianghong Gong
11:00–11:30	COFFEE
11:30–12:20	Takeo Ohsawa
12:30–13:20	Howard Jacobowitz
13:30–15:00	Lunch
15:00–15:50	Rafael Herrera
16:00–16:30	COFFEE
16:30–17:20	Po Lam Yung
19:00–21:00	Dinner

Friday

	Breakfast
7:30–9:00	
9:00–13:30	Informal discussions
	Coffee Break
11:30–13:30	Lunch
Checkout	
(by 12 noon)	

** 5-day workshop participants are welcome to use Hotel Hacienda Los Laureles facilities until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. **

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ABSTRACTS
(in alphabetic order by speaker surname)

Speaker: **Soledad Benguria** (University of Wisconsin-Madison)
Title: *An application of John ellipsoids to the Szego kernel on unbounded convex domains*
Abstract: I use classical convexity tools, in particular John ellipsoids, to obtain a size estimate for the Szego kernel on the boundary of a class of unbounded convex domains in $\mathbb{C}^n$. Given a polynomial $b : \mathbb{R}^n \to \mathbb{R}$ satisfying a certain growth condition, I consider domains of the type $\Omega_b = \{z \in \mathbb{C}^{n+1} : \text{Im}[z_{n+1}] > b(\text{Re}[z_1], \ldots, \text{Re}[z_n])\}$.

Speaker: **Xianghong Gong** (University of Wisconsin-Madison)
Title: *The $\bar{\partial}$-equation on variable strictly pseudoconvex domains*
Abstract: We investigate regularity properties of the $\bar{\partial}$-equation on domains in a complex euclidean space that depend on a parameter. Both the interior regularity and the regularity in the parameter are obtained for a smooth family of pseudoconvex domains. The boundary regularity and the regularity in the parameter are also obtained for smoothly bounded strongly pseudoconvex domains. This is joint work with Kangtae Kim.

Speaker: **Phillip Harrington** (University of Arkansas)
Title: *Bounded Plurisubharmonic Exhaustion Functions in $\mathbb{CP}^n$*
Abstract: It is frequently helpful in the study of pseudoconvex domains to have a defining function r with the property that $-(-r)^s$ is plurisubharmonic for some $s > 0$. In Stein manifolds, Diederich and Fornaess showed that this is always possible on bounded pseudoconvex domains with C^2 boundaries. Ohsawa and Sibony proved the corresponding result in $\mathbb{CP}^n$, using a result of Takeuchi to substitute for the lack of a global strictly plurisubharmonic function. In this talk, we will examine the possibility of extending the Ohsawa-Sibony result to domains with Lipschitz boundaries.

Speaker: **Friedrich Haslinger** (University of Vienna)
Title: *On some spectral properties of the weighted $\bar{\partial}$-Neumann problem*
Abstract: We derive a necessary condition for compactness of the weighted $\bar{\partial}$-Neumann operator on the space $L^2(\mathbb{C}^n, e^{-\varphi})$, under the assumption that the corresponding weighted Bergman space of entire functions has infinite dimension. Moreover, we compute the essential spectrum of the complex Laplacian for *decoupled* weights, $\varphi(z) = \varphi_1(z_1) + \cdots + \varphi_n(z_n)$, and investigate (non-) compactness of the $\bar{\partial}$-Neumann operator in this case. More can be said if every $\Delta\varphi_j$ defines a nontrivial doubling measure. Joint work with Franz Berger (University of Vienna).

Speaker: **Rafael Herrera** (Centro de Investigacion en Matematicas)
Title: *CR structures and twisted spin geometry*
Abstract: I will describe a geometrical setup using twisted spin geometry to characterize CR structures of arbitrary codimension, thus generalizing some of the work carried out by H. Baum for strictly pseudo-convex codimension-1 CR-manifolds..

Speaker: **Howard Jacobowitz** (Rutgers University-Camden)
Title: *Left invariant CR and pseudo-hermitian structures on S^3*
Abstract: The CR and pseudo-hermitian structures on S^3 invariant under the action of $SU(2)$ are classified using the moving frames method of Cartan and Webster, respectively. Let

$$L = \overline{w}\partial_z - \overline{z}\partial_w \text{ and } \theta = -i(\overline{z}dz + \overline{w}dw)$$

and let μ be a complex constant different from ± 1. Then $L + \mu\bar{L}$ defines all left invariant CR structures on S^3 with the CR distribution $\theta^\perp$ (except for that corresponding to $\bar{L}$).

Theorem 1 *CR structures given by μ_1 and μ_2 are equivalent if and only if either $|\mu_1| = |\mu_2|$ or $|\mu_1| = |\mu_2|^{-1}$. If $|\mu_1| = |\mu_2|$ the CR equivalence can be chosen to preserve θ and so the two structures are also equivalent in the pseudo-hermitian sense.*

If time permits, conjugate CR structures will be introduced in a more general context.

Speaker: **Ilya Kossovskiy** (University of Vienna)
Title: *Sphericity of a Real Hypersurface via Projective Geometry*
Abstract: Identifying the 3-sphere among real hypersurfaces in 2-dimensional complex space is a problem which attracted a lot of attention of experts in CR-geometry since the work of Cartan and Chern-Moser. It is well known that the sphericity of a real hypersurface amounts to vanishing of its special - CR - curvature. However, the latter is extremely hard to compute explicitly, and even more difficult to identify geometrically. In connection with that, we have recently discovered an amusing (unknown) fact on a very simple geometric identification of sphericity. It employs the Segre family of a real hypersurface, as introduced by Webster. We prove that a real-analytic hypersurface in 2-dimensional complex space is spherical if and only if its Segre family satisfies one of the configuration theorems of Projective Geometry: the Desargues theorem. Such a characterizations is important in that it does not presume any conditions on a real hypersurface (e.g. existence of additional symmetries), neither it requires any special choice of coordinates to characterize the sphericity. In this talk, we will show an elegant proof of the above described Segre-Desargues theorem.

Speaker: **Loredana Lanzani** (Syracuse University)
Title: *Hardy Spaces of holomorphic functions for domains in $\mathbb{C}^n$ with minimal smoothness*
Abstract: I will present recent joint work [LS-1] with E. M. Stein concerning representations and density results for the classical Hardy spaces of boundary values of holomorphic functions, in the non-classical context of domains with non-smooth boundary.

References

[LS-1] Lanzani L. and Stein E. M., *Hardy Spaces of Holomorphic functions for domains in $\mathbb{C}^n$ with minimal smoothness*, ArXiv: 1506.03748.

[LS-2] Lanzani L. and Stein E. M., *The Cauchy-Szegő projection for domains with minimal smoothness*, ArXiv: 1506.03965.

Speaker: **Christine Laurent** (Institut Fourier)
Title: *Dolbeault cohomology of the Chinese coin*
Abstract: In $\mathbb{C}^2$, we study the Dolbeault cohomology of the annulus between the bidisc and a large ball. As an application, we get W^1 estimates for the Cauchy-Riemann equation in the bidisc.

Speaker: **Jeffrey McNeal** (The Ohio State University)
Title: *The Bergman projection on generalized Hartogs triangles*
Abstract: We introduce a class of domains in $\mathbb{C}^2$, that interpolate between the classical Hartogs triangle and the product domain $D \times D^*$, and discuss their Bergman theory. The main result is that the Bergman projection, B, of these domains is only bounded on L^p for a restricted range of p that shrinks to 2 as the domains fill out $D \times D^*$. This is somewhat surprising as B on $D \times D^*$ is known to map L^p to itself boundedly for all $1 < p < \infty$. The results are joint work with Luke Edholm.

Speaker: **Gerardo Mendoza** (Temple University)
Title: *Microlocal infinitesimal deformations of CR structures*
The equation expressing infinitesimal deformations of a strictly pseudoconvex almost CR structure of hypersurface type in dimension 5 consists locally of 4 first order equations in 6 unknowns. The study of this underdetermined system leads to a second order selfadjoint system whose characteristic variety is the span of the contact structure of the almost CR manifold and is hypoelliptic on one side of the characteristic variety. The talk will focus on the microlocal structure of this system, but will include all arguments leading to the expression of the system.

Speaker: **Nordine Mir** (Texas A&M University at Qatar)
Title: *Artin approximation and CR geometry*
Abstract: In 1968, Artin proved his famous approximation theorem : given any system of real-analytic equations, if there exists a formal solution to such a system at a given point, then there exists a real-analytic solution that is as close as we want in the Krull topology to the formal solution. One question that naturally thereafter arises is whether the conclusion of Artin's approximation theorem is still preserved if the system of equations is coupled with a specific PDE. In 1978, Milman investigated such a question when the PDE consists of the standard Cauchy-Riemann operator in $\mathbb{R}^{2n} \simeq \mathbb{C}^n$: he showed that any formal solution of a system of real-analytic equations and of the standard CR equations in $\mathbb{C}^n$ can be approximated (in the Krull topology) by a sequence of convergent solutions of the system of analytic and CR equations. In this talk, we will discuss recent results generalizing Milman's theorem when the standard Cauchy-Riemann operator in $\mathbb{C}^n$ is replaced by the tangential Cauchy-Riemann operator associated to a real-analytic CR manifold.

Speaker: **Takeo Ohsawa** (Nagoya University)
Title: *Background and open questions on L^2 extension theorems*
Abstract: In these decades, there appeared many L^2 extension theorems and their proofs. One of them is closely related to the study of the asymptotic behavior of the Bergman kernel. Old and new results will be reviewed related to the Bergman kernel, harmonic analysis and potential theory.

Speaker: **Andrew Raich** (University of Arkansas)
Title: *The $\bar{\partial}$-problem on unbounded domains*
Abstract: In this talk, I will discuss recent progress with Phil Harrington on understanding the $\bar{\partial}$-problem in L^2 and L^2-Sobolev spaces on not necessarily pseudoconvex unbounded domains. I will discuss the challenges in moving from bounded to unbounded domains, and in particular, finding good defining functions, derivatives, and Sobolev spaces in which to work. Time permitting, I will conclude with a short explanation of how infinity can be thought of as a part of the boundary of the domain at which the metric and domain are poorly behaved.

Speaker: **Sonmez Sahutoglu** (University of Toledo)
Title: *Estimating the failure of compactness of the $\overline{\partial}$-Neumann operator*
Abstract: The essential norm of an operator is its distance to compact operators. In this talk we give estimates for the essential norm of the $\overline{\partial}$-Neumann operator on convex domains and on worm domains. This is joint work with Zeljko Cuckovic.

Speaker: **Brian Street** (University of Wisconsin-Madison)
Title: *Mutliparameter singular integrals*
Abstract: This talk concerns a theory of "multiparameter singular integrals." The Calderon-Zygmund theory of singular integrals is a well developed and general theory of singular integrals–in it, singular integrals are associated to an underlying family of "balls" B(x,r) on the ambient space. We talk about generalizations where these balls depend on more than one "radius" parameter $B(x, r_1, r_2, \ldots, r_k)$. These generalizations contain the classical "product theory" of singular integrals as well as the well-studied "flag

kernels," but also include more general examples. Depending on the assumptions one places on the balls, different aspects of the Calderon-Zygmund theory generalize. These operators were partially motivated by questions relating to the Kohn Laplacian on weakly pseudoconvex domain of finite type.

Speaker: **Jingzhi Tie** (University of Georgia)
Title: *Yau's Gradient Estimate and Liouville Theorem for Positive Pseudoharmonic Functions in a Complete Pseudohermitian manifold*
Abstract: I will introduce the basic notion of pseudhermitian manifold first and derive the sub-gradient estimate for positive pseudoharmonic functions in a complete pseudohermitian $(2n+1)$-manifold (M, J, θ) which satisfies the CR sub-Laplacian comparison property. It is served as the CR analogue of Yau's gradient estimate. Secondly, we obtain the Bishop-type sub-Laplacian comparison theorem in a class of complete noncompact pseudohermitian manifolds. Finally we will show the natural analogue of Liouville-type theorems for the sub-Laplacian in a complete pseudohermitian manifold of vanishing pseudohermitian torsion tensors and nonnegative pseudohermitian Ricci curvature tensors. This a joint project with Shu-Cheng Chang and Ting-Jung Kuo of National Taiwan University.

Speaker: **Yuan Yuan** (Syracuse University)
Title: *On local holomorphic maps between bounded symmetric domains*
Abstract: The study of local holomorphic maps preserving invariant forms between bounded symmetric domains has two motivations. One is from number theory, and the other one is rigidity/classification in the differential geometric point of view. I will describe the developments on the extension and the rigidity for such maps and explain the connection to algebraic geometry, CR geometry and etc. The talk is based on several works, including joint works with Huang, Xiao and Zhang.

Speaker: **Po Lam Yung** (The Chinese University of Hong Kong)
Title: *Some borderline Sobolev inequalities for the complex hyperbolic space*
Abstract: It is well known that the Sobolev space $W^{1,n}$ on $\mathbb{R}^n$ does not embed into L^∞ when n is at least 2. About 10 years ago, Bourgain, Brezis, van Schaftingen, Lanzani and Stein have found some remedies for the failure of this Sobolev embedding on $\mathbb{R}^n$. Since then these remedies have been studied in a number of different settings. In this talk, we will present a version of these remedies in the context of some symmetric spaces. Among them we have a result on the complex hyperbolic space, and the key is some analysis on the Heisenberg group. This is joint work with Sagun Chanillo and Jean van Schaftingen.